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REPORT OF COMMITTEE ON COLORIMETRY FOR 1920–21

BY
L. T. TROLAND, CHAIRMAN[1]

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[1] The Chairman of the Committee wishes to acknowledge his indebtedness to various members of his committee as follows: to Mr. Priest for a great mass of data, helpful criticisms and the suggestions drawn from his preliminary committee report of 1919 (not published, but copy may be borrowed from Bureau of Standards Library, Cf. J. OP. SOC., 4, p. 186; 1920); to Mr. Weaver for his elaborate computations of chromatic excitation values for many different classes of stimuli as well as for the development of the data upon which these computations were based; to Mr. Jones and to Dr. Ives for data and valuable criticisms.

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I. INTRODUCTION

That the nomenclature and standards of color science are in an extremely unsatisfactory condition is manifest to practically all workers in this field. It is the purpose of the present report to take an initial step towards remedying this state of affairs. That the result cannot be final as regards either nomenclature or standards is a natural consequence of the pioneer character of the effort.

The terminology which is proposed in the following pages represents an endeavor to crystallize the consensus of usage among experts, but where experts disagree and extant terms are vague, it has been deemed wise to introduce certain innovations. While the recommendations of this report are tentative, it is hoped that their careful consideration will assist in the clarification of ideas and the eventual unification of nomenclature. It is desired that all interested persons present their objections directly to the Committee, with a view to the resolution of possible disagreements. Every relevant idea will thus be thrown into the "melting pot" and the final product should be maximally satisfactory to all concerned.

In addition to its attempt (1) to outline a clear terminology, the following report endeavors (2) to summarize in usable form the best available psychophysical data relating color to its stimulus conditions, (3) to formulate or to define certain standard color stimuli—or intensity distributions of radiant energy (or allied quantities), (4) to outline briefly the principal methods of color measurement and (5) to establish fundamentally the relations between their respective scales. A detailed analysis of the techniques and terminologies of the special methods together with a discussion of the best instruments available, or proposed, for applying them, are reserved for a later report.

The incompleteness and imperfection of the data and methods of color science are only too apparent to the Committee, which

NOTE: The numbers within brackets in the text and footnotes refer to books and papers listed in the appended bibliography. The full face numbers are the serial bibliography numbers while succeeding numbers in ordinary type represent the pages in the given book or article to which reference is specifically made.

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recognizes clearly that much must be added and many changes be made before the work can attain its final goal: the production of an authoritative and satisfactory text on colorimetrics. The present draft may serve not only as a presentation of the possessions, but also of the *needs* of the science, and become a stimulus—as well as an aid—to new contributions.

II. NOMENCLATURE

The discussion of the general terminology of colorimetrics may be divided into three sections dealing respectively with (1) psychological terms, (2) stimulus terms, and (3) psychophysical terms. In the present part of the report we shall consider only general conceptions, the detailed terminology and symbolism involved in special methods of color designation being presented—if at all—in connection with the discussion of the several methods.

1. PSYCHOLOGICAL TERMS

A. COLOR.—[2]**Color is the general name for all sensations[3] arising from the activity of the retina of the eye and its attached**

[2] The definition of the term *color* which is advocated in the present report is the result of very careful consideration and protracted debate between various members of the Committee. It is unfortunate that in common speech the word color is employed, in different contexts, with at least two different meanings which are mutually inconsistent. The most common usage of the word makes it denote visual qualities which possess hue or have a finite degree of saturation, thus excluding all members of the gray series, including black and white. The second common usage of the word color is in harmony with the one recommended in the present report and causes it to embrace all visual qualities within its meaning. This second usage is most frequently found in the interrogative mood. For example, if we ask, "What is the color of a house?" it is as legitimate an answer to say "white" or "gray" as to say "red" or "green." On the other hand, the statement "the woman wore a colored dress" evidently excludes grays from the intended meaning. Such terms as color-photography, color-blindness, etc., have a similarly restricted meaning.

It is scarcely admissible in a scientific terminology to employ one term in two distinct and closely allied senses, since this will inevitably lead to confusion. Consequently, it is necessary to reject one of the common-speech meanings of the word color. A careful study of the situation shows, however, that the rejection of either meaning must result, in the beginning, in certain perplexities. If we employ color in the broader sense we not only sacrifice a well recognized distinctive term for the hue-saturation aspects of visual experience but we also seem to discard a large number of terms derived from the Greek root *chroma* which have been used in the same sense. On the other hand, if we define color in the restricted sense to exclude the gray series we find it necessary to exclude all considerations of brilliance from the field of colorimetry.

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nervous mechanisms, this activity being, in nearly every case in the normal individual, a specific response to radiant energy of certain wave-lengths and intensities. It may be exemplified by an enumeration of characteristic instances, such as red, yellow, blue, black, white, gray, pink, etc.

This means that if we are asked to specify the color of a gray object we must state that it has no color, and hence lies outside of our province. Similarly, we should be compelled to affirm that certain browns are identical in color with certain yellows, oranges, and reds because they possess the same hue and saturation, although their brilliances are quite different. The necessity of reactions of this sort on the part of the scientific colorimetrician would cause serious embarrassment in practice. It seems necessary to permit a certain degree of overlapping of the provinces of colorimetry and photometry, and possibly it would be desirable to include the latter under the former as a special branch.

A way out of this dilemma appears possible to the chairman if we can decide to employ the Greek root, *chroma*, in a different sense from the Latin root, *color*. There seems to be no particular etymological reason for regarding these roots as exact equivalents, and it is in line with economy of terminology to differentiate between their technical meanings. We therefore propose that the root, *color*, and its derivatives be employed hereafter to designate all visual qualities, including those of the gray series as well as those possessing hue and saturation. (The German equivalent, *Farbe*, is already used in this sense.) The root, *chroma*, and its derivatives, on the other hand, will be used to designate visual qualities possessing hue and saturation and excluding the gray series, with its terminal members, black and white. Such a separation of meanings is far more defensible etymologically than many distinctions which have been formally established in scientific nomenclature; for example, the distinction between *physics*, the general science of material properties, and *physiology*, the special science of vital processes, both of which terms must be considered to have the same etymological significance because of the common Greek root which they contain.

In harmony with the above general recommendation the following subsidiary developments may be indicated. The word, *chroma*, may be substituted bodily for the word, color, when the latter is intended in the restricted sense, thus red, green, pink, lavender, etc., are chromas, while black, any gray, and white are not chromas, although all of these qualities are correctly designated as colors. This usage of the term chroma may involve a slight confusion with its use by some authorities as a synonym for saturation but it will be noted that the change involved is only a small one, being simply the substitution of a qualitative for a quantitative meaning in practically the same context. The present report recommends that chroma be not used as an equivalent of saturation. If we recognize the suggested distinction between the Greek and Latin roots it is not a contradiction in terms to speak of an "achromatic color" nor is it a tautology to refer to a "chromatic color." (cf. German: *bunten Farben*.)

The distinction in question has the advantage of preserving all of the derivatives of the root, chroma, in their accepted meanings, and there are so many of these derivatives so firmly fixed in scientific discussion as to make it practically impossible to eliminate or to modify them. All such terms as chromatic, achromatic, chromaphore,

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It is imposible to identify color with radiant energy, or with wave-lengths of radiant energy, although radiant energy is the adequate stimulus for color. This is because color is known to depend upon the presence and character of the perceiving individual and because it is directly recognized to be something radically different in kind from its stimuli. Consequently, nothing but confusion can result from the use of the word "color" as a synonym

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monochromatic, dichromatic, trichromatic, photochromatic, etc., will be taken to refer to colors possessing hue and saturation or to the stimulus or organic conditions underlying the production of such colors. The root, chroma, and its derivatives provide us with a well established and hence constantly available means for differentiating between color in a restricted sense and members of the gray series, while "color" and its derivatives provide us with a means for designating both of these meanings together.

Some difficulties of course arise and must be met courageously by bold changes in usage. Fortunately the cases in question are not very important. For example, "chromatics" can no longer be regarded as synonymous with "colorimetry," chromatics being strictly the science of hue and saturation coördinate with photometry, if the latter is also regarded as a sub-division of colorimetry. The term "colorless" cannot be regarded as the equivalent of "achromatic" and must be taken to indicate complete transparency as well as achromaticity in an object. This is probably already the most common meaning of the term. The equivalent of the phrase "a colored object" in the common restricted usage of the term color would be "a chromed object." The phrase "color vision" becomes redundant and must be replaced by "chromatic vision." The terms relating to "color-blindness" may need some revision but the most common forms of this disorder are already designated as *partial* color-blindness, a designation quite in harmony with our usage of the term color. However, "total color-blindness" would be the equivalent of "complete blindness" on this basis and hence the word *achromatopia*, already in use, will be necessary in this instance.

It is the opinion of the Committee that the above suggestions, although necessitating a number of radical changes, involve a minimum of such changes among the possibilities which are open to us in improving the nomenclature of color science. However, the recommendations of the present report are intended to be tentative and the Committee will be glad to listen to alternative proposals and to objections to the particular form taken by the present suggestions, which represent a compromise between strongly opposed factions, all well represented in the Committee.

[3] The word sensation is used here to stand for an elementary form of experience or consciousness normally depending upon the operation of a sense organ. Although the existence of any sensation rests upon the operation of the nervous system, this should not lead us to *localize* it in that system. Although color is not a physical entity, it obviously exists outside of us on the surfaces of objects as we see them, such visual objects or perceptions being themselves nothing but arrangements of color areas in space. This statement, however, should not be misinterpreted to mean that the colors are physical or are located on physical objects. There is no reason for supposing that visual objects are identical or coincident with the objects of physical science.

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of "wave-length" or "wave-length constitution."[4] Color cannot be identified with or reduced to terms of any purely physical conception; it is fundamentally a psychological category.[5]

B. The Three Attributes of Color.—The nature of any color can be completely specified psychologically in terms of three fundamental attributes, this specification taking the form of an immediate description of the color, as such, *without any reference whatsoever to the stimulus.* The names employed for these three attributes by different authorities vary widely and frequently are such as to refer not only to properties of the color but also to related properties of the stimulus. Hence it seems necessary, in the interests of unambiguous thinking, to introduce certain refinements and possibly some innovations in terminology at this point. The Committee suggests the following nomenclature.

(*a*) **Brilliance[6] is that attribute of any color in respect of which it may be classed as equivalent to some member of a series of grays ranging between black and white.** Synonymous terms, as used by various writers, are "luminosity" (Abney, **4**, 4, 86) (Rood, **89**, 33) (Troland, **93**, 948), "brightness" (Luckiesh, **55**, 1) (Helmholtz, **21**, 243–245), "tint" (Titchener, **92**, 61–64), "value" (Munsell, **61**, 12–13), and "visual brightness" (Nutting, **63**, 300).

(*b*) **Hue is that attribute of certain colors in respect of which they differ characteristically from the gray of the same brilliance and which permits them to be classed as reddish, yellowish, greenish, or bluish.** There is a very satisfactory agreement among authorities regarding the usage of this term, which seems not to have been corrupted by any definite physical application.

(*c*) **Saturation is that attribute of all colors possessing a hue,**

[4] As, e.g. in the English Translation of Planck's "Theory of Heat Radiation," (74).

[5] On the definition of color as a psychological entity see: (69, 1), (73, 21–23), (94).

[6] The substitution of the word "brilliance" for the commonly used "brightness" and "luminosity" is necessitated by the fact that both of the latter terms have received technical definitions in connection with photometric measurements. It is impossible either to discard these technical definitions or to identify them with the definition here offered for the term "brilliance."

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which determines their degree of difference from a gray of the same brilliance. Synonymous terms, as used by various writers, are "purity"[7] (Rood, **89,** 32; Nutting, **67,** 139; Abney, **4,** 4) and "chroma" (Munsell, **61,** 12–14; Titchener, **92,** 62–63).

(*d*) Auxiliary Terms: The term *chromaticity* may be used to characterize a color qualitatively without reference to its brilliance. Chromaticity is determined by hue and saturation together, a gray being specified by the statement that it has *no* chromaticity.[8]

(*e*) Interdependence of the Attributes: All colors except absolute black exhibit brilliance, but grays have zero saturation, and hence no hue. All colors which exhibit a hue must also exhibit saturation, and *vice versa*.

(*f*) *Species of Colors*. Colors can be classified into *chromatic* and *achromatic* species, according as they do or do not exhibit hue, respectively. The former may be designated briefly as *chromas* (including colors of all finite degrees of saturation) and the latter as *grays* (including black and white).

Median gray (="mid-gray") is the middle member of a series of grays in which each member differs from its immediate neighbors by the least perceptible difference, and of which black and white are the terminal members. This gray furnishes the most practicable reference point for the achromatic as well as for the chromatic series of colors.

Median colors are all colors equivalent in brilliance to median gray, including the latter.

Tints and shades are colors, including grays and chromas, which are respectively lighter or darker than median gray.

D. PSYCHOLOGICAL PRIMARIES.—The psychologically primary colors are those which are necessary and sufficient, in minimum number, for the description of all colors by introspective analysis.

[7] Many of the authorities mentioned fail to distinguish between the subjective attribute of color, which is designated in the present report by the term saturation, and the ratio of homogeneous to total radiation in the stimulus, or the *purity*.

[8] The term *chromaticity* as applied to a color is a natural substitute for the term *quality* which is sometimes employed to distinguish that aspect of a color which excludes its "intensity." The use of *quality* in this context is undesirable on account of the more general meaning which it possesses in psychology.

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For normal vision these primaries are: black, white, red, yellow, green, and blue. (**73**, 251–252; **94**, 21). Red and yellow may be grouped together under the designation of "warm," while green and blue may be classified under the designation of "cold" primary chromas.

E. THE MEASUREMENT OF COLOR.–The three attributes of color can be treated as quantities and specified numerically, if all discriminable colors are conceived to be arranged into a system such that neighboring members differ from one another in each of the three attributes by just noticeable degrees (or threshold steps). (**92**, 207–215; **13**, 1–10). Such a system (*vide infra*) is necessarily three-dimensional (**61**, 18–31), and three ordinal values, representing the positions of a given color in the several dimensions are needed to define the color. The spectral chroma scale, considered more in detail below, is an application of this principle of color measurement to the study of the dependency of chromaticity upon wave-length.

2. STIMULUS TERMS

A. RADIANT ENERGY.–The adequate stimulus of color consists of radiant energy of certain frequencies or wave-lengths which have various stimulus values depending on the type of visual response system under consideration. The term "radiation" is often employed as a brief equivalent of "radiant energy," although this usage tends to confuse the *process* of radiation with the outcome of the process.[9]

B. THE PHYSICAL SPECTRUM is an arrangement of radiant energies in order of their respective frequencies or wave-lengths. It should not be confused with the *color spectrum* which is a series of colors aroused by part of the physical spectrum.

C. SPECTRAL DISTRIBUTIONS. The properties inherent in any sample of radiant energy which determine its capacity as a color stimulus are completely specified by its *spectral distribution*, which expresses the "intensity" for any frequency (or wave-length) as a function of the frequency (or wave-length) in question.

[9] Cf. (**37**).

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(*a*) When plotted in the form of a curve, the *ordinates* of a spectral distribution represent "intensity" per abscissa unit (frequency or wave-length, as the case may be); and the intensity concept, for the essential case of the incidence of the radiant energy upon the retina, will be: energy per second per unit area. To be completely specific, the function must express absolute values, but this is often difficult in practice.

(*b*) The *wave-length unit* which is ordinarily employed in colorimetrics is the millimicron which is correctly symbolized by $m\mu$ (not $\mu\mu$).[10]

(*c*) It is to be noted that wave-length, strictly interpreted, does not furnish a reliable specification of the color-stimulating capacities of radiant energy, as the response of the visual system depends upon *frequency*, while wave-length may vary independently of frequency. Since wave-lengths can only be interpreted in colorimetrics as reciprocal representations of frequency, it would be desirable theoretically to employ frequency directly in formulating spectral distributions. A suggested unit of frequency is the *fresnel*, defined as one vibration per trillionth (10^{-12}) of a second. Table 1 provides means for interconverting between millimicrons and fresnels.

(*d*) Spectral distributions of *transmission*, *reflection*, *luminosity*, etc., which are often employed to specify "color," may be regarded as constituents or as developments of the essential distribution function (*vide infra*).

D. HOMOGENEOUS RADIANT ENERGY—for the purposes of colorimetrics—is radiant energy, sensibly all of the intensity of which lies within a single spectral region so small as to exhibit—under the conditions most favorable for discrimination—no perceptible hue difference within the region.

E. PURITY—The purity of any sample of radiant energy, with respect to any one of its constituents, may be defined in general as the ratio of the intensity of this constituent to the total intensity of the sample. By *physical purity* we may mean such a

[10] On the use of the symbolism $m\mu$ instead of $\mu\mu$ see C. E. Guillame, Unités et Étalons, p. 7, Paris, 1893; also Soc. Fran. de Phys., Recueil de constants physiques, p. 1; B. S. Tech. Pap. **119**, p. 7.

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ratio in which the intensity is measured in energy terms, while *photometric purity* may be defined as a similar quantity based upon evaluations in terms of light units. Although the choice of the particular constituent with respect to which the purity is to be estimated is necessarily more or less arbitrary, we may define —as a special case of considerable importance—the *colorimetric purity*, which is the ratio in luminosity terms, between the dominant homogeneous constituent and the total sample, where the "dominant homogeneous constituent" comprises a range of wavelengths not greater than that corresponding to a single chromaticity threshold in the given spectral region, and has a dominant hue identical with that of the total sample, the intensity of the homogeneous constituent being arbitrarily so adjusted with respect to the total intensity that it can be mixed with "gray light" in such proportions as to yield a color-match with the total sample. This last definition corresponds with that of "per cent. white" in the method of colorimetry by "monochromatic analysis," but evidently involves psychophysical considerations in so essential a manner as to have very little physical significance.

F. MODE OF INCIDENCE—The color which is evoked by any adequate stimulus depends not only upon the spectral distribution of the latter, but also upon certain further conditions which may be called those of its *mode of incidence*. These conditions include: (1) the type of color system possessed by the observer, (2) the portion of the retinal field stimulated, (3) the size of the field, (4) the momentary state of adaptation of the optic nervous mechanism, and (5) the excitation processes in adjacent visual areas. In accurate work these factors require specification. In general, we assume pure cone vision of the normal trichromatic system, central fixation, a size not exceeding three degrees, and a gray contrast field of the same apparent brightness as the given stimulus light. (*vide infra*, for conditions of pure cone vision.)

3. PSYCHOPHYSICAL TERMS

This section deals with the terminology of the relation between color and its stimulus. The study of this relation constitutes the science of *color sensation*.

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TABLE 1

Equivalents in Terms of Fresnels of Wave-lengths in Millimicrons

Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}$ 10^{12}	Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}$ 10^{12}	Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}$ 10^{12}
400	750.0	He 492.2	609.5	600	500.0
Hg 404.7	741.3	500	600.0	610	491.8
Hg 407.8	735.7	He 501.6	598.1	Hg 615.2	487.7
410	731.7	510	588.3	620	483.9
420	714.3	520	576.9	630	476.2
430	697.8	530	566.1	640	468.8
H 434.1	691.1	540	555.6	650	461.6
Hg 435.8	688.4	Hg 546.1	549.4	H 656.3	457.1
440	681.9	550	545.4	660	454.6
He 447.2	670.8	560	535.8	He 667.8	449.3
450	666.6	570	526.2	670	447.8
460	652.2	Hg 576.9	520.0	680	441.2
470	638.4	Hg 579.1	518.0	690	434.8
He 471.3	636.5	580	517.2	700	428.6
480	625.0	He 587.6	510.5	He 706.5	424.6
H 486.2	617.1	590	508.5	710	422.6
490	612.3				
Hg 491.6	610.2				

A. PSYCHOPHYSICAL FUNCTIONS.–There are two important, general types of psychophysical functions which occur in colorimetrics, (1) a type which expresses a direct relation of dependency between a psychological color attribute (*vide supra*)—measured in threshold steps—and a stimulus variable, and (2) a type which formulates relations between two or more stimulus variables, such relations depending upon and expressing the conditions for the *equation* in one or more psychological dimensions, of the colors due to different stimuli.[11]

B. COLOR EXCITATIONS AND PHYSIOLOGICAL PRIMARIES.–An example of the second type of function appears in the color excitation curves for various visual color systems, which curves show in what proportions of intensity a number of selected color

[11] A further type specifies the stimulus conditions for just-noticeable (or otherwise standardized) differences between the colors evoked by compared stimuli. On certain assumptions, functions of this type can be integrated to yield those of type (1).

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stimuli must be mixed to match the homogeneous stimuli of the spectrum. (48; 4, 15, 223–247).

(*a*) Any set of component stimuli, thus applied, may be regarded as physiological primaries, but when they are so chosen for any or all visual color systems as to account for the maximum number of facts they may be called the *fundamental physiological primaries*. Such primaries are stimuli, not colors; although they may properly be said to "have" a color. They are of the *additive* type because the stimuli are *added* to produce the required effects.

(*b*) The corresponding *subtractive*, or "pigment," primaries are determined roughly by the spectrophotometric complementaries of the additive primaries. However, they do not consist of radiant energy but of absorbing mechanisms of one sort or another. In general they absorb from a "gray light" spectral distribution—of a certain intensity—(*vide infra*) portions which are as nearly as possible equivalent in color mixture value to the respective, additive primaries.

C. Visibility, Light, and Luminosity.[12] Another psychophysical function of the second type is the so-called *visibility curve*, which expresses reciprocally the intensities of radiant energy of different frequencies (or wave-lengths) which are required to match a standard in brilliance alone. The most recent average visibility values are given in Table 2.

(*a*) The conception of *light*, which is fundamental to *photometry*, is a psychophysical quantity defined as the product of the *absolute* power and visibility measures for any given sample of radiant energy.

(*b*) *Relative* light quantities are called *luminosities*.

D. Complementary Stimuli are stimuli which when mixed additively in certain required proportions evoke a gray.

(*a*) *Complementary colors* are the colors evoked by these stimuli, in the given proportions, separately.

E. The definition of color as a strictly psychological entity does not preclude the legitimate use of such convenient expressions

[12] For photometric concepts, see the report of the Committee on Nomenclature and Standards of the I. E. S. published each year in the Transactions of the Illuminating Engineering Society.

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TABLE 2
Average Normal Visibility Values
Adopted as Standard by the Illuminating Engineering Society and the Optical Society of America 1919–1920.[13]

Wave-Lengths	Adopted Mean I. E. S.	Absolute Visibility
400	0.0004	0.27
10	.0012	0.80
20	.0040	2.7
30	.0116	7.72
40	.023	15.3
450	0.038	25.3
60	.060	40.0
70	.091	60.7
80	.139	92.6
90	.208	139
500	0.323	216
10	.484	322
20	.670	446
30	.836	557
40	.942	629
550	0.993	662
60	.996	663
70	.952	641
80	.870	580
90	.757	502
600	0.631	421
10	.503	336
20	.380	253
30	.262	175
40	.170	113
650	0.103	68.8
60	.059	39.3
70	.030	20.0
80	.016	10.7
90	.0081	5.4
700	0.0041	2.7
10	.0021	1.4
20	.0010	0.67
30	.00052	0.35
40	.00025	0.17
750	0.00012	0.08
60	.00006	0.04
Wave-Length of maximum	556	

[13] Cf. JOUR. OPT. Soc. of America, 4, p. 58; 1920.

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as "*the color of light*," "*the color of a material*," "*spectral colors*," etc., since such expressions may be taken to imply a psychophysical linkage between stimulus and color which is reliable under normal conditions.

III. STANDARD PSYCHOPHYSICAL DATA

The purpose of the ensuing Part of this Report is to present data on certain laws or conditions which are fundamental to visual response and, in particular, to the science of colorimetrics. These data refer mainly to psychophysical relations of the two types defined in the preceding Part, but also to certain purely psychological and purely physiological laws. A great deal remains to be discovered and made definite in this field, and the following statements merely represent the best determinations available when the best are often far from good. Reference should be had to the Report of the Committee on Visual Sensitometry for a more detailed treatment of visual laws which are of interest mainly to the photometrician or the illuminating engineer.

1. THE PSYCHOLOGICAL COLOR SOLID

The three-fold attributive nature of color permits the symbolic arrangement of all possible colors in the form of a geometrical, or quasi-geometrical solid, neighboring members being separated by just noticeable differences. At present there are not sufficient data to permit of an accurate construction, but the following approximations would appear to be determined.[14]

A. General Coördinates.—Since the hues are cyclical in their resemblances, they will be represented most appropriately by angular measures with reference to a fixed point and line in any plane. The saturation must then be determined—on the general principle of polar coördinates—by a distance in the plane from the fixed point, or pole. The third dimension, of brilliance, consequently becomes a distance perpendicular to the selected plane.

B. Reference Axes.—The most natural origin of coördinates is the point on the axis of the figure which represents the median gray (*vide supra*), the axis itself (standing for all of the achromatic

[14] For discussions of the Psychological Color Solid see (92, 60–64; 61, 22–25).

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colors) serving as the reference line for saturation, while a plane passing through the axis and the extreme spectral red forms the reference plane for hue. (See Fig. 1.)

C. BOUNDARIES.—It is certain that the bounding surfaces of such a color solid will have an irregular contour. The axial dimen-

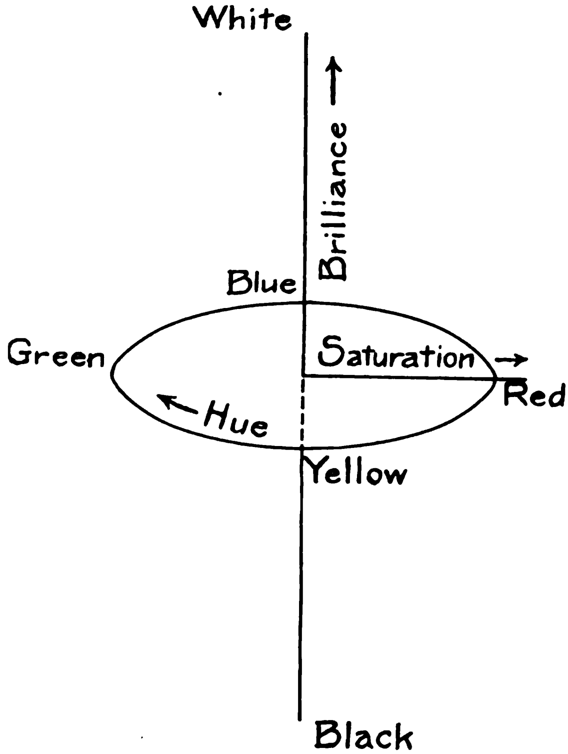

FIG. 1. The Dimensions of Psychological Color Solid

sion will be about thirty times the radial one at the greatest value of the latter. Both in the directions of high and low brilliance, there will be convergence towards a vertex. If the edges of planes perpendicular to the axis are regarded as determined by the saturations obtainable by *spectral stimuli*, these edges will be maximally near the axis at the yellow, receding on either side to find a constant value for the intermediaries of red and blue.

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D. PSYCHOLOGICAL PRIMARIES.—It is not legitimate to represent the psychological primaries, as Titchener does, as corners in a quadrilateral construction, since there is no correlation between psychological primacy and saturation, which such a construction implies.

2. THE SPECTRAL CHROMA SCALE

The functions which link the attributes of color, expressed in threshold units (*vide supra*) with the characteristics of the stimulus have thus far been determined only imperfectly. Reference should be had to the Report of the Committee on Visual Sensitometry (**68**) for data on the relation between brilliance and stimulus intensity. Jones (**42**), utilizing measurements made by Steindler, Nutting and himself, has determined the function connecting the chromaticity of spectral colors with wave-length. Since the spectral colors, even for conditions of pure cone vision, are by no means of equal saturation (*vide infra*) Jones' so-called "hue scale" is in reality a resultant hue-saturation scale of a very special kind. However, as an index of the law of change of chromaticity with respect to wave-lengths of homogeneous radiation, his results are of fundamental importance. They are reproduced in Table 3. Each chromaticity unit corresponds to one just noticeable difference (both in chromaticity and in wave-length), and the reciprocal of the wave-length difference is the sensibility to change in wave-length. It will be noted that this sensibility has four distinct maxima in the spectrum, the two most important ones lying at 494 and 588 $m\mu$, respectively, where the wave-length threshold is approximately 1.0 $m\mu$. Jones finds 128 just noticeable chromaticity steps in the spectrum, and about 20 additional steps in the non-spectral purples and magentas, as determined by their complementaries.

Since the hues form a cyclic series, it would seem more appropriate to express the hue scale in angular than in linear notation. If there are H hue steps in the complete cycle, the angular unit will evidently be $2\pi/H$ radians. The magnitude of this unit, however—if it is to correspond always to an integral step—must vary with the saturation, so that the linear unit is probably the more convenient. No determinations have yet been made of the

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TABLE 3
Spectral Chroma Scale

No.	λ(mμ)	dλ	No.	λ	dλ	No.	λ	dλ
1	700.0	....	44	576.5	1.4	87	490.4	1.1
2	678.0	22.0	45	75.2	1.3	88	89.4	1.0
3	65.0	13.0	46	73.7	1.5	89	88.2	1.2
4	59.0	6.0	47	71.7	2.0	90	87.0	1.2
5	54.0	5.0	48	70.1	1.8	91	85.8	1.2
6	49.5	4.5	49	68.4	1.7	92	84.5	1.3
7	46.0	3.5	50	66.6	1.8	93	83.2	1.3
8	42.8	3.2	51	64.8	1.8	94	81.7	1.5
9	40.2	2.6	52	63.0	1.8	95	80.0	1.7
10	37.8	2.4	53	61.1	1.9	96	78.2	1.8
11	35.5	2.3	54	58.6	2.5	97	76.5	1.7
12	33.1	2.4	55	57.0	2.6	98	75.0	1.5
13	30.0	3.1	56	54.4	2.6	99	72.9	2.1
14	26.5	3.5	57	51.8	1.6	100	70.5	2.4
15	23.0	3.5	58	49.1	2.7	101	68.2	2.3
16	20.0	3.0	59	46.1	3.0	102	65.8	2.4
17	17.3	2.7	60	43.0	3.1	103	63.6	2.2
18	14.9	2.4	61	39.8	3.2	104	61.2	2.4
19	12.5	2.4	62	36.5	3.4	105	58.7	2.5
20	10.2	2.3	63	33.2	3.3	106	56.5	2.2
21	08.0	2.2	64	30.1	3.1	107	54.4	2.3
22	06.0	2.0	65	27.1	3.0	108	52.1	2.3
23	04.1	1.9	66	24.2	2.9	109	50.0	2.1
24	02.3	1.8	67	21.4	2.8	110	48.0	2.0
25	600.6	1.6	68	19.1	2.3	111	46.0	2.0
26	599.0	1.6	69	16.8	2.3	112	44.2	1.8
27	97.4	1.6	70	14.6	2.2	113	42.5	1.7
28	95.9	1.5	71	12.6	2.0	114	40.8	1.7
29	94.5	1.4	72	10.6	2.0	115	39.0	1.8
30	93.1	1.4	73	08.0	1.6	116	37.2	1.8
31	91.8	1.3	74	07.0	1.0	117	35.3	1.9
32	90.5	1.3	75	05.4	1.6	118	33.3	2.0
33	89.5	1.0	76	04.0	1.4	119	31.3	2.0
34	88.5	1.0	77	02.6	1.4	120	29.3	2.1
35	87.5	1.0	78	01.3	1.3	121	27.0	2.2
36	86.4	1.1	79	500.0	1.3	122	24.8	3.2
37	85.3	1.1	80	498.7	1.3	123	22.3	2.3
38	84.0	1.3	81	97.4	1.3	124	19.7	2.8
39	82.7	1.3	82	96.1	1.3	125	16.7	3.0
40	81.5	1.2	83	94.8	1.3	126	13.8	2.9
41	80.3	1.2	84	93.7	1.1	127	10.4	3.4
42	79.1	1.2	85	92.6	1.1	128	405.8	4.6
43	77.9	1.2	86	91.5	1.1	......	....	....

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number of hue steps in cycles of color with uniform saturation, and it is quite uncertain whether the change in number of steps follows the simple geometrical analogy: $H=2\pi s$, where s is the saturation measure in threshold steps from the equivalent gray. (This clearly raises the question as to whether the space of the psychological color solid is Euclidean.) The angular magnitudes in this system would preferably be measured from an axis through the normal hue of extreme spectral red, on account of the stability of this color in relation to its stimuli, and all functions of these magnitudes will be periodic.

The positions of the psychologically primary hues in the "hue scale" are matters of considerable interest. On Jones' spectral chroma scale, taking the zero in the violet and unity at extreme spectral red, the primary blue, green and yellow lie at .24, .41, and .68, respectively, the red having a value slightly in excess of 1.00, on account of the necessity of including a slight amount of blue in the stimulus for red. (**95**). It will be seen that the separations of the primaries on this scale are by no means equal.

3. THE SATURATION SCALE

Careful determinations of the number of just noticeable saturation steps between each maximally saturated color and white have not yet been made. Nutting and Jones (**67**; **68**) find about 20 such steps for red, green and blue, the thresholds for change in the per cent. white varying with the given per cent. white as shown in Table 4.

TABLE 4

Saturation Scale Data

Per cent. white	0	10	20	30	40	50	60	70	80	90	100
Threshold white	4.7	4.6	4.5	4.4	4.2	4.0	3.7	3.4	3.0	2.5	2.1

These values, like those for wave-length sensibility, are independent of brilliance over a wide range of intensities.

The Chairman has attempted to determine the relative saturations of the spectral colors by an application of the flicker photometer. It is assumed that when any color is alternated with

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a white of the same brilliance the critical flicker frequency is a function of saturation alone, so that colors possessing the same critical frequency will be of equal saturations. Since the spectral colors have radically different frequencies, with a minimum in the yellow at 575 $m\mu$, it is necessary to add white to all except the latter to attain this equality. The per cents. of white (in terms of the total mixture) which were found necessary for the writer's right eye are shown in Table 5.

TABLE 5

Comparative Saturations of Spectral Colors

Wave-length, $m\mu$	419	438	458	479	497	517	537	556
Per cent. white	53.0	75.1	63.5	51.2	49.0	30.0	28.9	19.0
Wave-length, $m\mu$	575	595	614	634	653	673	692	......
Per cent. white	0.0	19.3	38.8	31.5	46.7	50.7	49.4	......

These figures are only tentative, as the sources of error in the experiment seem quite numerous. However, they indicate quite clearly that the differences in saturation of the spectral colors are of first order importance, so that it is highly improbable that the number of saturation steps from white is the same for them all. These conclusions appear to be corroborated by the relations of the spectral colors to white as represented on the color-mixture triangle (*vide infra*).

4. STIMULI FOR THE PSYCHOLOGICAL PRIMARIES

Westphal (98) has determined stimuli, as nearly as possible homogeneous, for arousing the psychological primaries in the average normal observer. They are: for red, extreme visible long-wave end of the spectrum plus a small amount of blue or violet; for yellow, 574.5 $m\mu$; for green, 505.5 $m\mu$; and for blue, 478.5 $m\mu$. It is interesting to note that three of these stimuli (for red, green and blue) correspond quite closely with the three fundamental physiological primaries determined by König and Dieterici.

5. THE COLOR EXCITATION FUNCTIONS

Probably the most fundamental of all the psychophysical data relating to color are the *three-color excitation curves*, which represent

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the laws of three-color mixture. Extant data on these relationships are due to Maxwell (57), Abney (4), and König and Dieterici (48), the results obtained by the first investigator, however, being at present of relatively little value. The remaining two investigations were made with quite different light sources and choices of reference points, but are capable of being reduced to a common denominator by appropriate calculations. This work has been carried out very painstakingly by Mr. E. A. Weaver of the present Committee, who finds that the two sets of data actually agree surprisingly closely, so that they may legitimately be averaged. The results, reduced to an equal energy spectrum and referred to average noon sunlight as an origin (in trilinear coördinates) are given in Table 6 and Fig. 2.

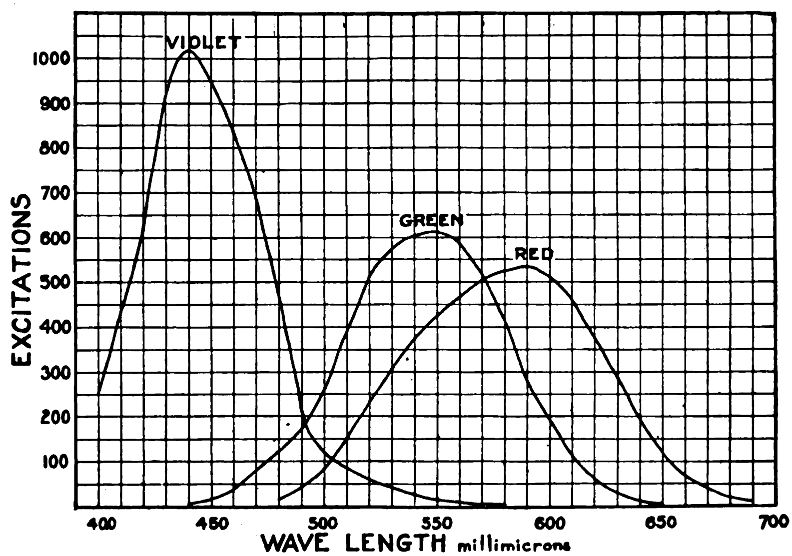

FIG. 2. Elementary Color Excitations for Different Wave Lengths
(The ordinates correspond with the values listed under "Excitations" in Table 6.)

A detailed account of the method employed in the reduction of the two sets of data will be given in a separate publication, but may be outlined briefly here as follows. Abney's luminosity curve values were first converted so as to refer to average noon sunlight

instead of to the carbon arc (as given), using Watson's data[15] for the energy distribution of the latter and Abbot's (1) data on that of the former (*vide infra*). The corresponding luminosity and percentage color excitation values were then combined as products to yield excitation values referred to sunlight, rendering them com-

TABLE 6

Spectral Colors in Terms of Elementary Excitations

Wave-Length mμ	Excitations			Percentages		Wave-Length	Excitations			Percentages	
	Red	Green	Violet	Red	Violet		Red	Green	Violet	Red	Violet
400	......	......	253	......	100	550	424	612	18	40.2	1.7
410	......	......	433	......	100	560	466	578	11	44.2	1.0
420	......	......	614	......	100	570	505	517	7	49.5	.7
430	......	......	915	......	100	580	520	415	4	55.4	.4
440	......	7	1019	......	99.3	590	535	296	...	64.3	......
450	......	16	950	......	98.3	600	510	196	...	72.2	......
460	......	38	842	......	95.7	610	462	113	...	80.4	......
470	......	81	697	......	89.6	620	375	59	...	86.3	......
480	14	122	473	2.3	77.6	630	285	29	...	90.8	......
490	41	169	220	9.5	51.1	640	195	10	...	95.1	......
500	83	260	123	17.8	26.4	650	118	3	...	97.5	......
510	151	391	87	24.0	13.8	660	68	......	...	100	......
520	233	510	61	29.0	7.6	670	40	......	...	100	......
530	307	572	43	33.3	4.7	680	22	......	...	100	......
540	373	603	29	37.1	2.9	690–750	27	......	...	100	......

These values are for an equal energy spectrum. The relative magnitudes of the three elementary excitations have been chosen so that the curves for average noon sunlight have equal areas; that is, if the percentage values are plotted on a trilinear diagram, sunlight falls in the center. The absolute excitation values are based upon a convenient arbitrary unit. The percentage values are given for the red and violet only since those for the green can be found by subtracting the sum of the other two values from 100 in each case.

parable in this respect with the values given by König and Dieterici. The respective trilinear representations of both sets of values then coincided, except for the positions of the elementary "green" excitations and the sides of the two triangles joining the green and the "violet" elementaries. Although the relation between the two green elementaries could not be determined directly, it

[15] See (5, p. 96, Table 1 and p. 97, Fig. 3).

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was established indirectly by use of the known positions of the solar white in each of the triangles, this correlation permitting the reduction of the two sets of values to three common elementaries. The red and green values of each set were now tested separately against the data of independent color matches published by Priest (79; 81) in connection with his investigations of the leucoscope and camouflage paints, which accidentally provided materials for checking the results. Both sets checked equally well, so that they were given equal weight in the computation of average values. The violets were tested by data on complementaries and that of Abney was rejected. These best values were then retransformed to terms of elementaries determined by a triangle based on extreme spectral red and violet, with its sides as closely tangent to the locus of the spectral colors as possible. Finally the excitation values were reduced to terms of an equal energy spectrum. Thus expressed, the areas under the three curves are equal for the energy distribution of average noon sunlight, the magnitudes or "weights" of the three elementaries having been so chosen as to yield the solar white with equal excitations.

When quite differently weighted, in terms of the relative powers of the three elementary processes to generate brilliance, the three chromatic curves should summate to yield the visibility curve. It is a well-known fact that in this summation the value of the violet or blue excitation is extremely small compared with that of the red and green. König and Dieterici give no data from which these specific visibility coefficients of the chromatic processes can be deduced. Abney, however, provides data[16] of this sort leading to coefficients by which the ordinates of the excitation curves must be multiplied in order that all three curves should summate to yield his own visibility curve. This latter curve, however, as derived from Abney's luminosity curve and carbon arc data, departs so widely from the average visibility function, as specified by the Standards Committee of the Illuminating Engineering Society, as to throw doubt upon the general validity of these values. Mr. Weaver and the writer have made

[16] See (4, Table 38, p. 239, and Table 34, p. 17, Columns 7 to 9). Also (17).

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new experimental determinations of the chromatic visibility coefficients for their own eyes and those of one other subject. The results not only differ from Abney's, but show such large variations among themselves as to indicate that these coefficients are subject to marked fluctuation among individuals. All three of our subjects agree perfectly on the proportions of the mixed stimuli required for the color matches (measured in energy units), but disagree on the photometry of the components. In other words, their visibility curves vary widely while their color curves remain constant. This may indicate either that the brilliance process is independent of the chromatic processes or that the latter, as constituents of the former, vary in weight without alteration in the mathematical form of their response functions. The average values for the factors obtained from our three subjects were: red, 0.370; green, 0.617; and blue, 0.012.

The stimulation values of all forms of radiant energy, simple or complex, can be completely specified in terms of the ratios of excitation of the three chromatic elementaries,—combined with one absolute measure of intensity, if the brilliance as well as the chromaticity of the color is to be taken into consideration. To arrive at the expression of the chromatic stimulation value of any given distribution of radiant energy in terms of the three elementaries, it is only necessary to multiply each of the values for the elementaries given in Table 6 by the corresponding ordinates of the distribution function throughout the spectrum and then to find the areas under the resulting three curves. The ratio of these areas determines the chromaticity of the color. The reduction of data in one system of color specification to data of another system (*vide infra*) can be accomplished with the greatest sureness of principle through the medium of a common expression in terms of the three elementaries. For the science of color the three elementaries are far more fundamental even than the spectrum, and the expression of the spectral colors in terms of these components (as in Table 6) is as natural as (say) the similar expression of various Planckian distributions (see Table 17).

It is, of course, improbable that the curves in Fig. 2 faithfully represent the actual resonance functions of the elementary chro-

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matic mechanisms in the retina. There is also little doubt, however, that the curves in question and all specifications based upon them, are potentially convertible into terms of such actual response functions. The characteristic constants of these latter functions must be determined from data auxiliary to the facts

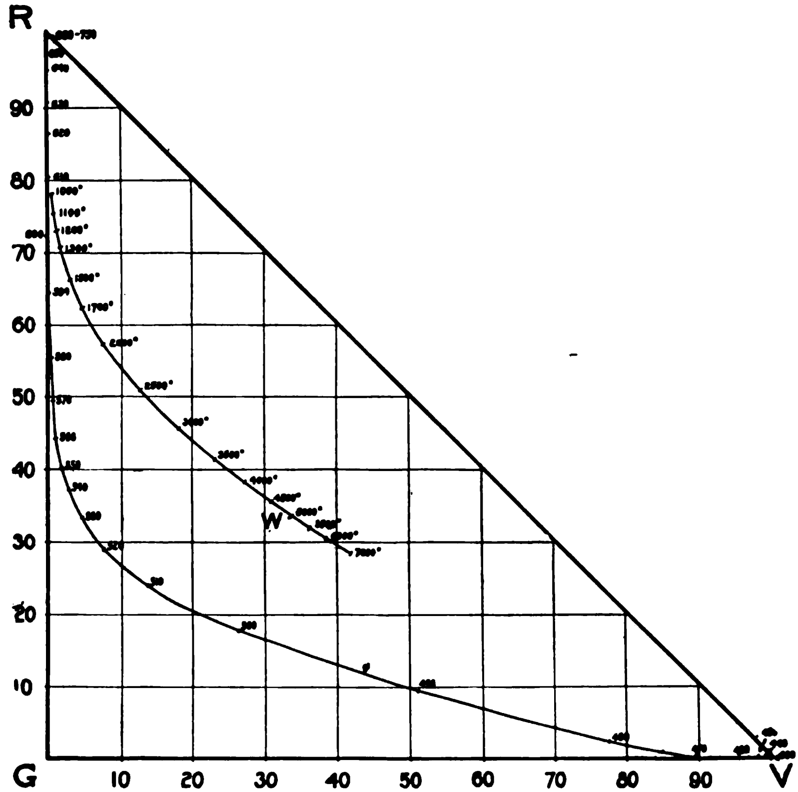

FIG. 3. Color Triangle, Showing Loci of Spectral and Black Body Excitations

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

of color-mixture for the normal eye, such as those of color-blindness and chromatic minuthesis. Attempts to arrive at so-called *fundamental excitations*, based upon these more comprehensive considerations, have been made by König, Abney, Exner (17) and others,

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but the results cannot be regarded as sufficiently final to justify their adoption in place of a maximally straightforward representation of the facts of color-mixture, such as is given in Table 6.

Fig. 3 provides a graphical representation of some of the relations based upon Mr. Weaver's analysis, in terms of trilinear coördinates or a so-called color-mixture triangle. It will be noted that the triangle here given is rectangular rather than equilateral, as is ordinarily the case. The latter form bears the simplest relation to the representation of the three color excitation values in the familiar three-dimensional Cartesian coördinates, being a

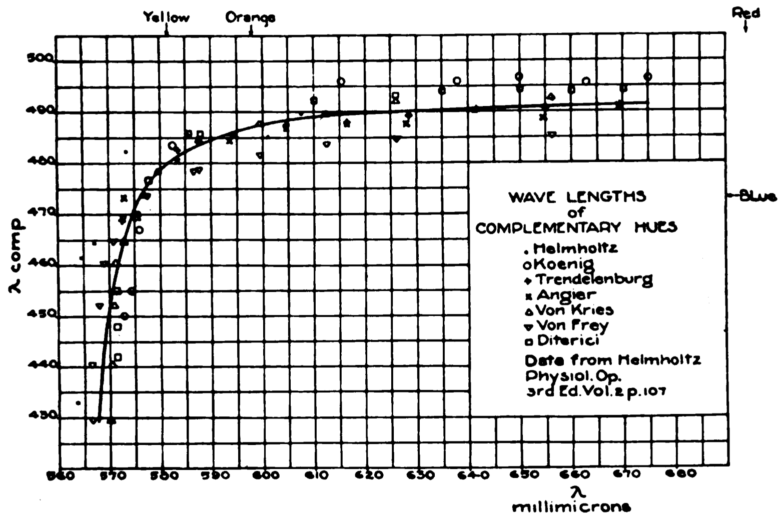

FIG. 4a. Wave-Lengths of Complimentary Hues

section of this system making equal angles with all of the reference planes. The rectangular form, however, is much easier to use either in plotting the loci of colors or in determining the excitation values of the colors lying on curves already plotted. The result of the mixture of stimuli represented by two or more points in the triangle is found by computing the position of the "center of gravity" or centroid, of the multiple-point system, in which the intensities of the components—expressed in relative units defined by the triangle—function as analogues of masses.

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6. COMPLEMENTARY COLORS AND WAVE-LENGTHS

Among the most interesting results of the mixture of different stimuli are the identification of pairs of homogeneous radiations which, with the proper ratio of intensities, yield white. Such *complementary wave-lengths* can be found in the color-mixture triangle at the two intersections of straight lines, drawn through the central white, with the locus of the spectral colors. It is

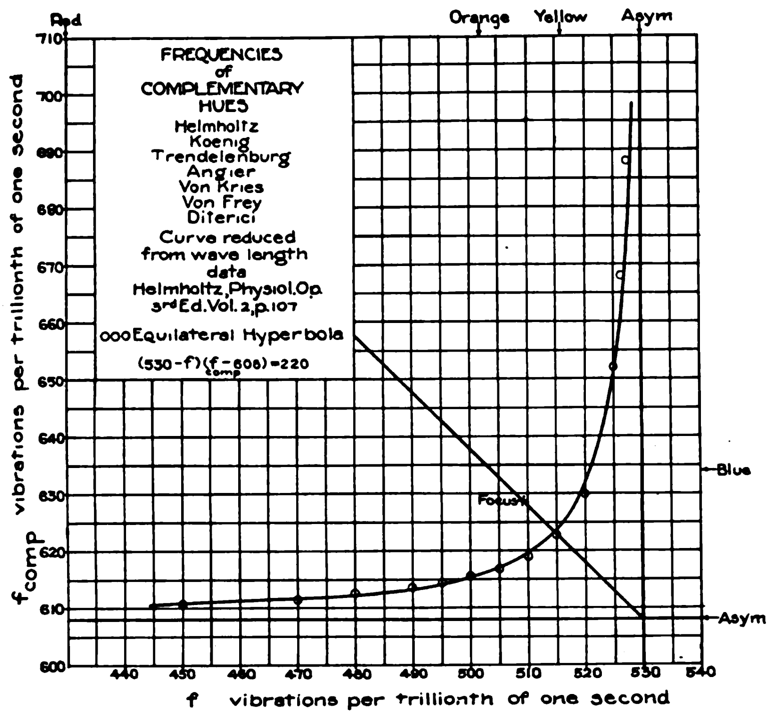

FIG. 4b. Frequencies of Complementary Hues

clear from an examination of the triangle that there is a range of spectral colors—in the green—which have no single spectral complementary, the complementaries of either end region of the spectrum being found in the other. Extant data on complementary wave-lengths are unsatisfactory because of indefiniteness in the specification of the white stimulus which was employed. Fig. 4a gives a plot of the best values which are available. In

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Fig. 4b the plot of these values in terms of frequency shows how closely their relation fits the function expressing a rectangular hyperbola, having an equation:

$$(530-f)\ (f_c-608)=220,$$

f being the given frequency and f_c its complementary. (**80**)

7. STANDARD CONDITIONS FOR PURE CONE VISION

In order to secure reliable conditions for complete color vision even in the normal observer, it is necessary to restrict the stimulus to the retinal cones, excluding the rods, which yield only achromatic colors. Pure cone vision can be secured by satisfying the following requirements.

A. CHOICE OF OBSERVERS.—Recent investigations by Abney indicate that a considerable number of individuals possess rods in the center of the retina as well as in the periphery, so that before relying upon the restriction of the stimulus to a central field, the observer should be tested for the Purkinje phenomenon in central vision.

B. SIZE OF FIELD.—The normal retina possesses no rods in an area slightly greater than three degrees in diameter, surrounding the intersection of the line of sight with the retina (**73**, **10**). Consequently, in the case of an observer known to be normal in this respect, a field of three degrees, with fixation on the center of the field, insures pure cone vision at all intensities.

C. INTENSITY.—With all observers and all field sizes, pure cone vision is obtainable at intensities above approximately one hundred photons, provided the eye has not previously been exposed for a considerable time to a much lower intensity or assuming a condition of equilibrium adaptation to the given intensity level, which should be reached within ten minutes (**75**; **34**). One hundred photons represents an external stimulus surface brightness of one hundred candles per square meter, used with a pupillary opening of one square millimeter, or equivalent conditions as regards retinal illumination.

IV. PHYSICAL STANDARDS

It is the function of the present Part of this Report to consider some physical standards which are of importance in colorimetrics.

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These standards consist in certain typical forms of stimuli to color, or in factors or functions contributory to such stimuli. Some of the standards considered below are primarily of theoretical or research interest, while others are essentially of technical significance only.

1. THE CRITERION OF HOMOGENEOUS RADIANT ENERGY

As defined in a preceding Section, the criterion of homogeneity in a stimulus, for the purposes of colorimetrics, must rest upon wave-length sensibility, and hence upon the facts which are summarized in Table 3. In general, in order to be considered homogeneous, a given sample of radiation must have a range of wave-lengths not greater than the threshold for wave-length in the given region (defined by its mid-wave-length). As seen from the Table, this varies widely for different parts of the spectrum, e.g., being 22 mμ in the extreme red (680 mμ), and 1.0 mμ for 588 mμ.

2. STANDARDS OF SPECTRAL ENERGY DISTRIBUTION

Under this caption are included curves and constants indicating the distributions of "intensity" (*vide supra*) in the physical spectrum of certain frequently encountered or critically important forms of *heterogeneous* radiant energy. These distributions are all at least approximately of the so-called "black body," or Planckian type, i.e., they are determined by a general equation of the form,

$$E = \frac{c_1}{\lambda^5\left(e^{\frac{c_2}{\lambda T}} - 1\right)}$$

where E is the energy per unit wave-length, T the absolute temperature of the source, e the base of the natural system of logarithms, λ the wave-length, and c_1 and c_2 are constants. When our concern is only with chromaticity, we need consider merely *relative* energies, and any convenient value may be adopted for c_1, such as a value which makes the maximum of the function equal to unity. The value of c_2 at present recommended is 14350 micron-degrees (19). This equation has been found to express very closely the energy distributions for the radiation

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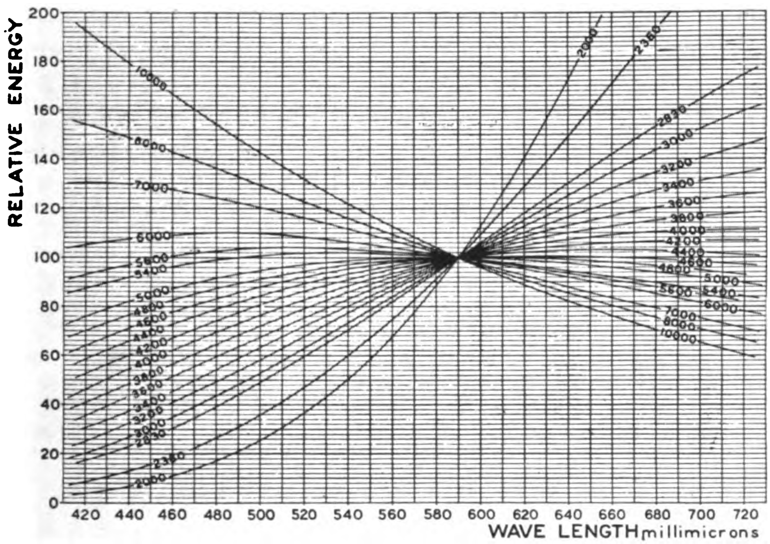
emitted by incandescent solids, such as those contained in natural and artificial illuminants, although in the majority of cases T is not the actual temperature of the material, but is a temperature determined by the distribution itself and known as the *color temperature*, this being the actual temperature of a theoretical black body which would yield that same relative distribution in the visible spectrum. Table 7 and Fig. 5 give the relative intensities for various representative wave-lengths for Planckian distributions at a considerable variety of temperatures.



Planckian Energy Distributions at Various Temperatures
Planck's constant c_2 = 14350 micron degrees.
Numbers attached to curves indicate temperatures in degrees K
Nat Bur Stands Div IV-3, March, 1921.

FIG. 5

A. AVERAGE NOON SUNLIGHT.—The most important standard of energy distribution, from the point of view of colorimetrics, is that which characterizes "daylight," since it is with respect to deviations from this distribution that the chromatic processes of vision have been adjusted by nature. Unfortunately, however, the form of this distribution is highly variable. There is, in the first place, the radical difference between sky-light and direct sunlight, the former exhibiting that marked deficiency in long-

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TABLE 7

Relative Energy of a Black Body at Various Temperatures and Wave-Lengths

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length $m\mu$	$\times 10^{-15}$	$\times 10^{-13}$	$\times 10^{-12}$	$\times 10^{-11}$	$\times 10^{-10}$	$\times 10^{-9}$	$\times 10^{-8}$	$\times 10^{-8}$	$\times 10^{-7}$	$\times 10^{-7}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$
400	26	101	727	1787	2158	1584	809	3148	994	6256	4612	7465	7482
410	54	186	1200	2728	3101	2168	1064	4005	1230	7403	5056	7872	7878
420	111	330	1926	4037	4368	2915	1378	5026	1502	8663	5782	8244	8252
430	214	568	3024	5949	6037	3856	1757	6220	1813	10038	6407	8591	8601
440	416	954	4642	8519	8204	5022	2212	7608	2163	11520	7051	8911	8923
450	766	1559	6946	11976	10965	6449	2747	9194	2555	13111	7724	9208	9222
460	1375	2490	10210	16550	14440	8171	3373	10995	2989	14798	8373	9475	9492
470	2396	3388	14740	22510	18750	10226	4096	13020	3464	16580	9041	9716	9737
480	4091	5947	20890	30140	24030	12650	4923	15280	3982	18450	9712	9933	9957
490	6776	8933	29130	39810	30420	15480	5860	17770	4544	19975	10378	10123	10151
500	10990	13169	40030	51890	38080	18760	6914	20500	5143	24410	11039	10286	10319
510	17460	19020	54170	66820	47140	22510	8087	23480	5784	24480	11688	10429	10466
520	27230	27090	72340	85020	57780	26770	9382	26690	6464	26614	12329	10544	10586
530	41740	38000	95450	107060	70160	31580	10810	30150	7180	28780	12954	10639	10686
540	62670	52550	124300	133400	84410	36950	12370	33830	7930	30980	13563	10712	10764
550	92730	71710	160200	164600	100720	42920	14050	37750	8712	33205	14148	10762	10820
560	134900	96660	203300	201200	119250	49520	15870	41900	9524	35444	14715	10797	10862
570	193500	128700	257700	243900	140060	56740	17820	46240	10364	37675	15260	10813	10882
580	274100	169400	322140	293400	163470	64640	19900	50790	11234	39920	15782	10811	10887
590	382600	220500	399000	349900	189430	73180	22110	55530	12112	42058	16280	10793	10877

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TABLE 7—*Continued*

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length $m\mu$	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-5}	x 10^{-5}
600	527500	284200	490100	414600	218200	82400	24430	60460	13012	44356	16752	10763	10853
610	718400	362700	596700	487600	249800	92300	26880	65530	13936	46542	17197	10716	10815
620	968400	458800	721500	569900	284300	102850	29450	70770	14868	48702	17618	10660	10767
630	1291500	575300	866000	661800	321800	114100	32120	76120	15808	50804	18010	10590	10702
640	1704100	715300	1032100	764000	362500	126000	34910	81620	16742	52880	18332	10510	10632
650	2226100	882400	1221300	877200	406400	138500	37790	87200	17691	54900	18713	10420	10547
660	2882000	1080200	1437500	1001600	453300	151700	40760	92850	18642	56990	19024	10323	10457
670	3697900	1312900	1681400	1138200	503600	165500	43820	98610	19590	58770	19313	10218	10362
680	4706000	1585900	1955500	1287000	557100	179900	46960	104410	20540	60640	19573	10101	10252
690	5939800	1900800	2261700	1448000	613800	194900	50160	110290	21480	62400	19809	9985	10141
700	7440500	2266700	2601900	1623000	673600	210400	53420	116140	22400	64090	20022	9861	10024
710	9251200	2686000	2979200	1810000	736700	226500	56740	121980	23210	65740	20207	9732	9903
720	11419000	3165000	3395100	2012000	803000	243000	60120	127860	24220	67320	20368	9600	9776
730	14005000	3706800	3851300	2227000	872000	260000	63520	133720	25120	68820	20511	9462	9652
740	17063000	4321700	4348600	2456000	944300	277300	66960	139570	25980	70230	20627	9323	9514
750	20663000	5013000	4891000	2700000	1019500	295100	70420	145350	26840	71600	20725	9179	9377
760	24874000	5785600	5480900	2957000	1097100	313300	73910	151100	27670	72870	20828	9035	9239

All of the values in this table were calculated by means of Wien's equation with the exception of the last column, headed "5000x10-5" and that was calculated by means of Planck's equation. The values show the correct relationships for different wave-lengths at a single temperature, and between different temperatures if multiplied by the factors which are placed at the heads of the several columns. See Forsythe, Ref. 19, pp. 330–331.

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TABLE 7

Relative Energy of a Black Body at Various Temperatures and Wave-Lengths

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length $m\mu$	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-6}	x 10^{-6}
400	26	101	727	1787	2158	1584	809	3148	994	6256	4612	7465	7482
410	54	186	1200	2728	3101	2168	1064	4005	1230	7403	5056	7872	7878
420	111	330	1926	4037	4368	2915	1378	5026	1502	8663	5782	8244	8252
430	214	568	3024	5949	6037	3856	1757	6220	1813	10038	6407	8591	8601
440	416	954	4642	8519	8204	5022	2212	7608	2163	11520	7051	8911	8923
450	766	1559	6946	11976	10965	6449	2747	9194	2555	13111	7724	9208	9222
460	1375	2490	10210	16550	14440	8171	3373	10995	2989	14798	8373	9475	9492
470	2396	3388	14740	22510	18750	10226	4096	13020	3464	16580	9041	9716	9737
480	4091	5947	20890	30140	24030	12650	4923	15280	3982	18450	9712	9933	9957
490	6776	8933	29130	39810	30420	15480	5860	17770	4544	19975	10378	10123	10151
500	10990	13169	40030	51890	38080	18760	6914	20500	5143	24410	11039	10286	10319
510	17460	19020	54170	66820	47140	22510	8087	23480	5784	24480	11688	10429	10466
520	27230	27090	72340	85020	57780	26770	9382	26690	6464	26614	12329	10544	10586
530	41740	38000	95450	107060	70160	31580	10810	30150	7180	28780	12954	10639	10686
540	62670	52550	124300	133400	84410	36950	12370	33830	7930	30980	13563	10712	10764
550	92730	71710	160200	164600	100720	42920	14050	37750	8712	33205	14148	10762	10820
560	134900	96660	203300	201200	119250	49520	15870	41900	9524	35444	14715	10797	10862
570	193500	128700	257700	243900	140060	56740	17820	46240	10364	37675	15260	10813	10882
580	274100	169400	322140	293400	163470	64640	19900	50790	11234	39920	15782	10811	10887
590	382600	220500	399000	349900	189430	73180	22110	55530	12112	42058	16280	10793	10877

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TABLE 7—*Continued*

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length mμ	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-5}	x 10^{-5}
600	527500	284200	490100	414600	218200	82400	24430	60460	13012	44356	16752	10763	10853
610	718400	362700	596700	487600	249800	92300	26880	65530	13936	46542	17197	10716	10815
620	968400	458800	721500	569900	284300	102850	29450	70770	14868	48702	17618	10660	10767
630	1291500	575300	866000	661800	321800	114100	32120	76120	15808	50804	18010	10590	10702
640	1704100	715300	1032100	764000	362500	126000	34910	81620	16742	52880	18332	10510	10632
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660	2882000	1080200	1437500	1001600	453300	151700	40760	92850	18642	56990	19024	10323	10457
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680	4706000	1585900	1955500	1287000	557100	179900	46960	104410	20540	60640	19573	10101	10252
690	5939800	1900800	2261700	1448000	613800	194900	50160	110290	21480	62400	19809	9985	10141
700	7440500	2266700	2601900	1623000	673600	210400	53420	116140	22400	64090	20022	9861	10024
710	9251200	2686000	2979200	1810000	736700	226500	56740	121980	23210	65740	20207	9732	9903
720	11419000	3165000	3395100	2012000	803000	243000	60120	127860	24220	67320	20368	9600	9776
730	14005000	3706800	3851300	2227000	872000	260000	63520	133720	25120	68820	20511	9462	9652
740	17063000	4321700	4348600	2456000	944300	277300	66960	139570	25980	70230	20627	9323	9514
750	20663000	5013000	4891000	2700000	1019500	295100	70420	145350	26840	71600	20725	9179	9377
760	24874000	5785600	5480900	2957000	1097100	313300	73910	151100	27670	72870	20828	9035	9239

All of the values in this table were calculated by means of Wien's equation with the exception of the last column, headed "5000x10^{-5}" and that was calculated by means of Planck's equation. The values show the correct relationships for different wave-lengths at a single temperature, and between different temperatures if multiplied by the factors which are placed at the heads of the several columns. See Forsythe, Ref. 19, pp. 330–331.

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wave radiations compared with the latter, which is responsible for the blue color of the sky. Direct sunlight, moreover, varies in the form of its distribution curve with the time of the day and year, and with latitude on the earth's surface. Different again, is the distribution which accompanies an overcast sky. (54, 37–39; 81).

It is necessary therefore to adopt as a standard an average curve, representing the conditions most frequently encountered.

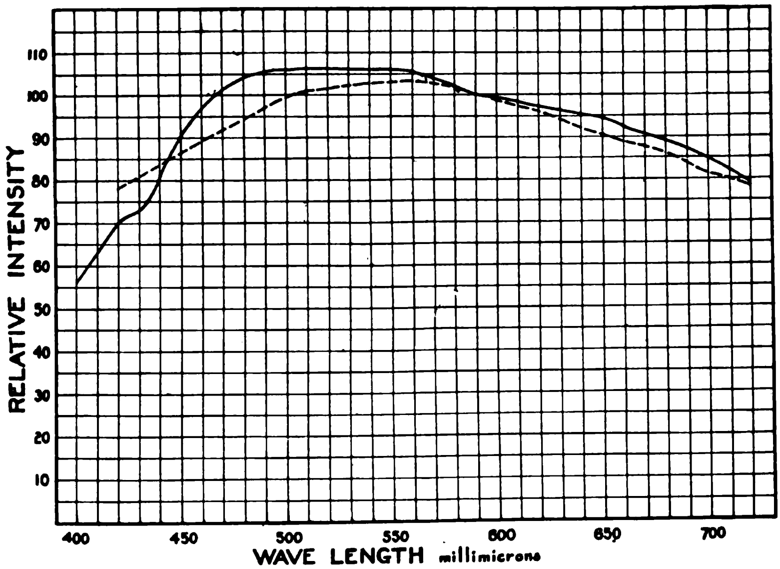

FIG. 6. Spectral Energy Distributions of Natural and Artificial Sunlight

(The solid line represents the distribution of average noon sunlight, while the broken line is that of Priest's precision artificial sunlight.)

Such an average, for noon sunlight at Washington, D. C., is given in Table 8 and the solid line in Fig. 6. It is the mean of forty determinations, half of which were made at the summer solstice (June 21) and the other half at the winter solstice (December 21), both high and low atmospheric transmissions being included. The authority is Abbot of the Smithsonian Astrophysical Observatory (1). Average noon sunlight, thus defined, corresponds roughly to a black body temperature of 5000°K. the distribution not being strictly Planckian.

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B. STANDARD ARTIFICIAL SUNLIGHT.—There are various methods for producing artificially an approximate reproduction of average noon sunlight, as above specified. The most accurate is that of Priest which consists in passing the radiation from a gas-filled tungsten lamp, operated at a color temperature of 2848°K, (about 15.6 l. p. w., for concentrated filament lamp), through a pair of crossed nicol prisms between which is inserted

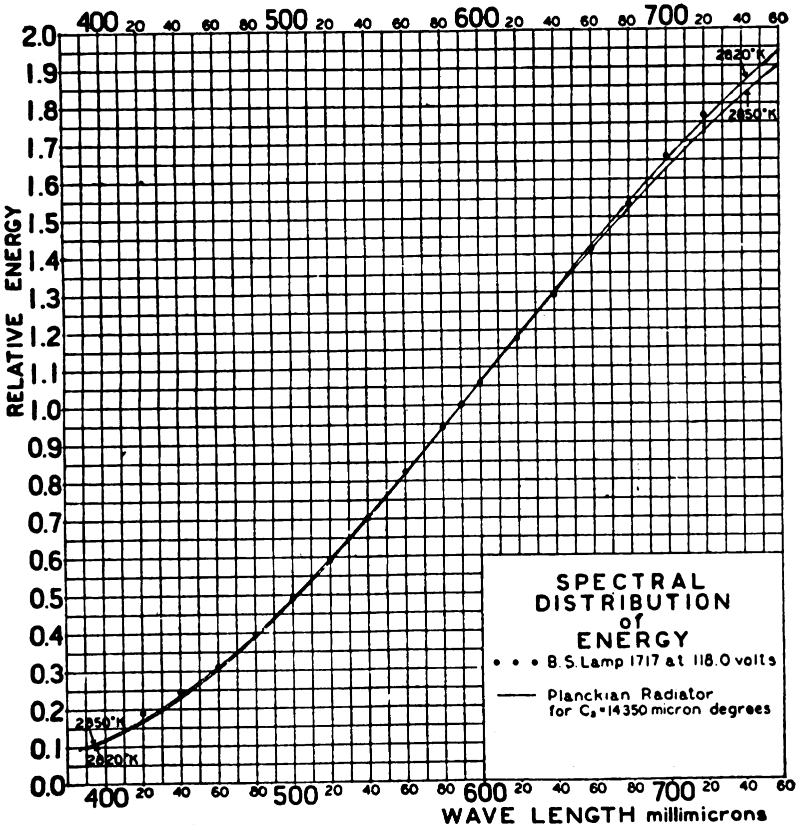

FIG. 7

a crystalline quartz plate 0.500 mm. thick with surfaces perpendicular to the optic axis of the crystal (78). The resulting energy distribution is shown, in comparison with that of actual sunlight, by the broken line in Fig. 6. The proper energy distribution for the radiation from the lamp, before passing through the nicol prisms and the quartz plate, is given in Fig. 7. Lamps yielding

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this distribution at a specified voltage can be obtained from the Bureau of Standards, Washington.

Other methods of producing artificial daylight involve the use of blue glasses or gelatine filters before standard illuminants. The most available system of this sort at the present time consists of a No. 78 Wratten photometric filter (97) manufactured by the Eastman Kodak Company and a cylindrical acetylene flame (41) produced by a standard burner (also obtainable from the same

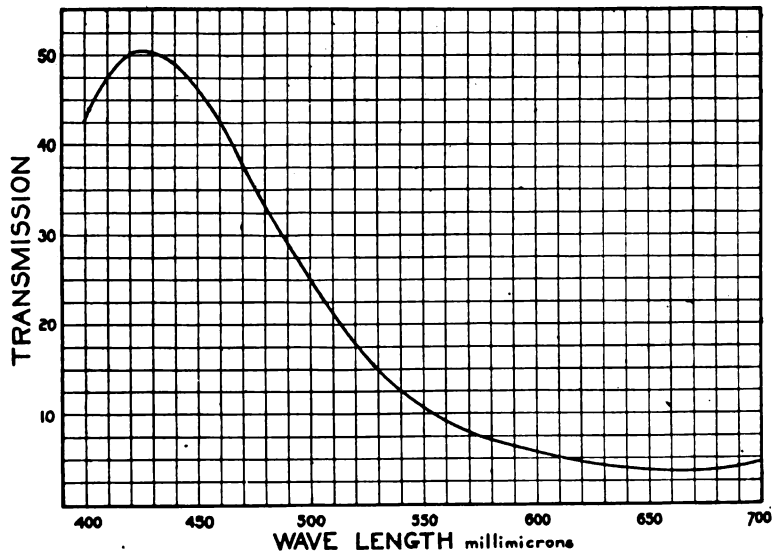

FIG. 8. Spectral Transmission of Tungsten-to-Daylight Filter (Wratten No. 78) (According to the Wratten Light Filter Booklet.)

company). This combination yields a white closely approximating average noon sunlight, and gives a very satisfactory standard white for practical purposes. Figure 8 shows the spectral transmission of the original No. 78 Wratten filter. It is planned to make the "78" and "86" series of filters in the future a quite accurate means of converting one color temperature to others, the original filters having been only approximate means to this end. The color temperature of the standard acetylene flame is 2360 degrees K., which corresponds with that of a vacuum tungsten filament burned

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TABLE 8

Relative Intensity Values over the Visible Spectrum for Average Noon Sunlight (Computed from Abbot's data)

Wave-Length $m\mu$	Relative Intensities		
	June 21	December 21	Mean
400	63.0	48.0	56.0
10	71.0	54.0	63.0
20	79.0	60.0	70.0
30	82.0	64.0	73.0
40	90.0	72.5	81.0
50	98.5	82.5	90.5
60	104.5	89.0	97.0
70	107.5	95.0	101.0
80	109.0	98.5	104.0
90	110.0	101.0	105.5
500	110.5	102.0	106.0
10	110.0	103.0	106.5
20	109.0	103.5	106.0
30	108.5	104.0	106.0
40	108.0	104.5	106.0
50	107.0	105.0	106.0
60	106.0	104.5	105.0
70	104.5	103.0	104.0
80	102.5	102.0	102.0
90	100.0	100.0	100.0
600	98.5	100.0	99.0
10	97.0	99.5	98.0
20	95.5	99.0	97.0
30	94.0	98.5	96.0
40	92.5	98.0	95.0
50	90.3	97.5	94.0
60	88.0	96.5	92.0
70	86.0	95.0	90.5
80	84.0	93.0	88.5
90	81.5	92.0	87.0
700	79.0	90.0	84.5
10	76.5	87.0	82.0
20	74.5	84.0	79.0

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at an efficiency of approximately 8 lumens per watt. This corresponds fairly closely with the color of an ordinary mazda B tungsten light burning at 1.25 watts per mean horizontal candle power. Other selectively absorbing glasses designed to make possible the production of artificial daylight include the Ives-Brady glass, a very satisfactory but rare product, the Luckiesh "Tru-tint" glass, and the Corning "Daylite" glass.

C. NORMAL GRAY LIGHT.—The conception of "white light" is one which is of fundamental importance to many of the purposes of colorimetrics, for example in colorimetry by the "monochromatic" method, in defining complementaries, etc. There are a vast number of characteristic intensity distributions of radiant energy which can be used with practical success to meet this need. Although the one most frequently employed is that of average noon sunlight, to be of the greatest theoretical as well as practical significance, the definition of "white light" should evidently determine a spectral distribution which will generate a pure gray by its action on the normal human visual apparatus in a normal condition. Since there are an infinity of conceivable distributions which would satisfy this requirement, it seems advisable to limit the general form of the distribution to a species of which only one member can excite a gray. Distributions of the Planckian type meet this requirement (32, 198–202), and are further to be recommended because of the approximate conformity of all natural and artificial radiant sources to the Planckian law, and the comparative ease with which distributions of this sort can be reproduced in the laboratory. Preliminary determinations by Priest, (82), using a system of nicol prisms and quartz plates as a filter to yield Planckian distributions representative of temperatures lying between 4200 and 6200°K, indicate 5200° as the value for the gray stimulus. This figure is regarded at present as highly tentative, on account of the small number of subjects tested and doubt as to the normality of certain of them. Criteria for the selection of the pure gray, other than that of the simple introspective judgments used by Priest, may also be advisable.

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TABLE 9

Color Temperatures of Common Illuminants[a]

Hefner amyl acetate lamp	1875
Pentane (10 c. p. standard)	1914
Candle (paraffin)	1920
Candle (sperm)	1925
Kerosene lamp (round wick)	1915
Kerosene lamp (flat wick)	2045
Acetylene, ordinary (as a whole)	2368
Acetylene, ordinary (central spot)	2448
Acetylene (Eastman Standard)	2360
4 W. P. M. H. C. Carbon (4.85 W. P. M. S. C.)	2070
3.1 W.P.M.H.C. Treated Carbon (3.73 W.P.M.S.C)	2153
2.5 W.P.M.H.C. Gem	2183
2 W.P.M.H.C. Osmium	2176
2 W.P.M.H.C. Tantalum	2249
1.25 W.P.M.H.C. Tungsten (Mazda B)	2385
.9 W.P.M.H.C. Tungsten	2543
Carbon arc (solid carbon)	3780
Carbon arc (cored carbon)	3420

[a] Hyde, E. P. and Forsythe, W. E., *Jour. Franklin Inst.*, **183**, p. 354; Forsythe, W. E., *Phys. Rev.* (2), **17**, p. 147, 1921; Priest, I.G., Color Temperature, Op. Soc. Am., Conv., Rochester, Oct., 1921, *J. Op. Soc. Am.*, Jan., 1922.

D. STANDARD ILLUMINANTS.—All common illuminants having a Planckian distribution are of course characterized by a temperature lower than that of the sun and of the black body which emits normal gray light, and hence evoke an unsaturated yellowish or orange color. The spectral distributions of these various illuminants can obviously be specified by a statement of their respective color temperatures. Table 9 lists these temperatures for a group of familiar light sources (28). The radiation from Welsbach gas mantles cannot be matched satisfactorily with that from a black body at any temperature, and varies quite widely in distribution with the proportions of ceria and thoria in the mantle as well as with the average degree of incandescence. Probably the illuminant whose characteristics are best established at the present time is the acetylene flame produced by a standard burner under specified conditions. The spectral distributions for vacuum tungsten electric lamps are determined by the efficiency, or lumens per watt, at which they are operated, and extensive measurements made at the Nela Research Laboratory (27), enable one to trans-

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late an efficiency value into a corresponding color temperature and hence to ascertain the distribution. Table 10 shows the relation between efficiency and color temperature. Gas filled and carbon filament lamps have been less accurately calibrated and are inherently more variable.

Table 10

Color Temperatures of Vacuum Tungsten Filaments at Various Efficiencies[a]

Lumens per Watt (Uncorrected)	Color Temperature (Uncorrected)	Lumens per Watt (Corrected)	Color Temperature (Corrected)
0.5	1644	0.58	1663
1.0	1777	1.14	1794
1.5	1866	1.70	1883
2.0	1939	2.26	1955
2.5	1998	2.82	2014
3.0	2050	3.37	2066
3.5	2096	3.93	2112
4.0	2138	4.48	2153
4.5	2175	5.02	2190
5.0	2208	5.57	2224
5.5	2241	6.12	2257
6.0	2269	6.66	2285
6.5	2299	7.21	2315
7.0	2327	7.76	2343
7.5	2354	8.30	2370
8.0	2380	8.85	2397
8.5	2406	9.39	2423
9.0	2431	9.94	2449

The first two columns show the lumens per watt and color temperatures as directly determined experimentally from a given lamp. The second two columns give these same quantities corrected for losses due to cooling effects of leading in and supporting wires and absorption of the lamp bulbs.

[a] Hyde, E. P., Cady, F. E. and Forsythe, W. E., Color Temperature Scales for Tungsten and Carbon, *Phys. Rev.*, (2), **10**, Table I, p. 401; 1917.

3. STANDARDS OF SPECTRAL TRANSMISSION

The characteristics of physical objects which determine their colors, when viewed by radiation from other sources, can be expressed almost completely by means of spectral reflection or transmission curves, representing as a function of wave-length or frequency, the fraction of the original radiation impinging upon the object, which finally leaves it as reflected or transmitted rays respectively. Such curves are most readily determined by means

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of a spectrophotometer. For the intercomparison of the color values of objects, without reference to the radiation by which they are viewed or the observer's visual system, reflection and transmission curves are of great utility in colorimetrics, although such curves represent properties of objects rather than of immediate stimuli to color. However, in view of Hering's principle of "the color-constancy of visual objects" (**24**), representing the tendency of the visual processes to compensate for variations in spectral constitution and intensity of the illuminating source, these curves attain some direct significance for consciousness. Although colors due to selective reflection are of more common occurrence than those due to selective transmission, the latter are of greater scientific importance because of the far higher degree of selectivity which is obtainable by transmission than by reflection.

The spectral transmission distributions for a number of technically important materials are considered below.

A. STANDARD THREE-COLOR ADDITIVE FILTERS.—There are several common applications of the principle of matching or of reproducing colors by the mixture of two or three stimuli, of constant relative spectral constitutions but varying proportions, which utilize selectively transmitting radiation "filters." Although their transmissions may be varied within certain limits without deleterious effects, it is desirable to specify the transmission curves of certain of these filters which have been found satisfactorily to fulfill their purposes.

(*a*) *Trichromatic Analyzer Filters.* Table 11 gives the transmissions of three filters employed in the Ives colorimeter (**29; 30**), which is employed for the designation of the colors of materials in terms of three mixed elementaries, determined by these filters.

(*b*) *Photographic Taking Filters.* Table 12 records the transmissions of four filters designed for making color separation photographic negatives on panchromatic emulsions. Rigid standardization of filters for use in this connection is not possible on account of the variations in sensitiveness to radiation of different wave-lengths exhibited by these emulsions, but the filters specified in the Table have been found to give fairly satis-

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factory results with emulsions of average character. Nos. 1, 2, and 3 constitute the tri-color set, while Nos. 1 and 4 can be used successfully for two-color taking. The choice of three-color taking filter transmissions should in general be such as to duplicate

TABLE 11

Transmissions for Various Wave-Lengths of Ives Colorimeter Filters[a]

mμ	Percentage Transmissions		
	Red	Green	Blue
400	....	....	26.0
410	....	....	26.0
420	....	....	25.9
430	....	....	25.9
440	....	....	24.9
450	....	....	22.0
460	....	....	17.5
470	....	....	11.0
480	....	.9	6.9
490	....	3.7	2.5
500	....	7.8	1.2
510	....	11.2	.3
520	....	13.0	....
530	....	12.9	....
540	....	11.1	....
550	....	8.2	....
560	....	4.8	....
570	....	2.2	....
580	....	.8	....
590	....	....	....
600	.8	....	....
610	4.5	....	....
620	18.0	....	....
630	45.0	....	....
640	64.7	....	....
650	72.4	....	....
660	76.9	....	....
670	79.4	....	....
680	81.3	....	....
690	82.3	....	....
700	82.9	....	....
710	83.0	....	....

[1] The values given in this table were supplied by Dr. H. E. Ives. Ives colorimeter filters measured at the Bureau of Standards depart appreciably from the above specifications, as do those examined by E. C. Bryant, *Astrophys. J.*, 55, p. 9, 1922.

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TABLE 12

Transmissions of Approved Photographic Taking Filters

λ = Wave-length in $m\mu$.

T = Fraction of incident radiation transmitted.

No. 1		No. 2		No. 3		No. 4	
λ	T	λ	T	λ	T	λ	T
700	.82	620	.01	400	.07	460	.02
680	.82	600	.03	410	.09	470	.09
660	.82	580	.18	420	.11	480	.19
640	.82	560	.30	430	.15	490	.35
620	.80	540	.46	440	.21	500	.49
610	.75	530	.53	450	.28	510	.58
600	.60	520	.57	460	.26	520	.59
590	.25	510	.52	470	.19	530	.55
585	.10	500	.35	480	.10	540	.49
580	.02	490	.12	490	.04	550	.38
....	....	480	.02	580	.01	560	.25
....	....	....	....	....	....	570	.74
....	....	....	....	....	....	580	.07
....	....	....	....	....	....	590	.02

photographically, as closely as possible, the mixing proportions of the corresponding reproducing filters which are required to match the colors of the photographed objects. The production of such photographic records of the values in question will evidently depend not only upon the transmissions of the filters but also upon the spectral distribution of sensitiveness of the given photographic emulsion. Similar general principles apply to the choice of two-color taking filters if the blue-violet values of the photographed objects are left out of consideration, since two-color taking filters are ordinarily selected so as to neglect those values of the scene which are poorly represented in yellowish, artificial illumination.

(*c*) *Photographic Reproducing Filters.* Filters necessary for three-color and two-color additive projection of photographic positives printed from color separation negatives, or for use in such instruments as the Ives Chromscope (54, 218) depend in quality upon the character of the light source which is employed. Since the sources vary widely, it is difficult to specify the transmissions of the filters very exactly. Filters for three-color addi-

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tive reproduction should be so selected that mixtures of the transmitted radiations are capable of matching a maximal number of colors. This means that the radiations in question should evoke hues approximating as nearly as possible to spectral saturation, and so distributed in the color triangle that the lines joining them lie maximally close to the spectral locus. Two-color reproducing filters in practice usually resemble closely the red and green members of a three-color set but may have a somewhat reduced saturation, and the dominant hue of the green is usually shifted somewhat more towards the blue than is that of the red number.

B. STANDARD SOLUTIONS.—The spectral transmissions of dyes and inorganic salt solutions of known purity and concentration are in course of determination at the Bureau of Standards, and some of the most important of these will be presented in later Reports by the present Committee.[17]

C. LOVIBOND AND OTHER COLORED GLASSES.—Similar statements apply to the Lovibond glasses, which are widely used as technical standards in the ranking of oils and other materials as regards color. There are a very large number of these glasses, and the accuracy with which they are reproduced in different sets is often relatively low (77). The spectral transmissions of a considerable number of precision-made glass plates of various colors have been determined at the Bureau of Standards, and these plates can be borrowed under proper restrictions by those desiring to check their spectrophotometric equipment.

4. STANDARDS OF SPECTRAL REFLECTION

A. SUBTRACTIVE PIGMENT ELEMENTARIES.—A committee of the American Institute of Graphic Arts is at the present time working on the colors of inks for three- and four-color printing processes. Pending their findings the present Committee will offer no recommendations on this matter. As previously noted, the pigments or dyes required for the satisfactory rendering, by the subtractive method, of photographic "color separations" are in general the *physical complementaries* of the corresponding additive

[17] Three notable atlases of absorption spectra are (39), (96), (58).

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reproducing filters, i.e., they transmit or reflect those portions of the spectrum which the latter absorb, and *vice versa*.

B. SYSTEMS OF PIGMENT STANDARDS.—There are now available several notable systems of reflection color standards (**6**). Those which are best known to American workers are the Munsell system (**61**) designed for use by artists and the Ridgway system (**87**) intended for the ornithologists. More recently Ostwald (**71; 72**) in Germany has published an elaborate scheme of this sort. Each of these systems comprises pigments of various selective and total reflection spaced fairly evenly over the total field of possible pigment colors.

(*a*) *The Munsell System* is based upon ten hues and nine degrees of "value," or of light reflecting power. Each of the hues is represented at each level of reflectivity by as many different saturation steps as is feasible. The system as published includes a text on "Color Notation,"(**61**) an atlas, (**62**) and a color sphere, as well as cards embodying the separate pigment values. Priest and his associates (**83**) have determined the spectral reflection curves of representative members of the pigment system, and have made recommendations for its improvement, although in general commending it as it stands. Evaluations of certain Munsell colors in terms of elementary sensations are given in the next Part of the present Report. For the spectral reflections reference should be had to Priest's original paper.

(*b*) *The Ridgway System* utilizes thirty-six hues, having approximately equal spacing on the spectral chroma scale, each of these being diluted with white in three degrees and with black in three degrees, making 1115 colors in all. Jones (**42**) has made careful monochromatic analyses of the undiluted colors, and has determined the separations of their dominant hues on the hue scale, the average for the spectral hues being 3.5 and for the purples 4.4 just noticeable steps. Considering the practical difficulties encountered in the preparation and reproduction of pigment samples the gradation of values exhibits excellent uniformity. The results of Jones' analyses are given in Table 13. Spectrophotometric measurements on the Ridgway pigments

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TABLE 13

Monochromatic Analyses of Certain Ridgway Colors[a]

1	2	3	4	5	6	7
No.	Name	λ(mμ)	Per cent. Hue	dλ	ds	ds
1	Spectrum Red....	633	55	....	....	....
2		616	31	17	5.0	5.0
3		610	34	6	2.0	3.0
4		605	34	5	1.5	2.0
5		....	....	....	2.0	2.5
6	Orange Chrome...	597	40	8	2.5	2.5
7		595	34	2	1.5	1.5
8		593	26	2	1.0	1.5
9		589	23	4	2.0	3.0
10		586	31	3	2.0	3.0
11		582	34	4	2.5	3.0
12	Lemon Yellow	579	29	3	3.0	3.0
13		577	34	2	2.0	2.0
14		574	39	3	2.0	2.0
15		569	42	5	3.0	3.0
16		566	42	3	2.0	2.0
17		548	47	18	6.0	8.0
18	Emerald Green....	521	63	17	9.0	9.0
19		518	66	3	2.0	2.0
20		510	63	8	3.0	3.5
21		495	55	15	10.0	10.0
22		490	54	5	4.0	5.5
23		486	51	4	3.0	3.0
24		484	73	2	1.0	1.5
25	Spectrum Blue....	479	70	5	2.0	2.5
26		476	57	3	2.0	2.0
27		470	62	6	3.0	3.0
28		460	62	10	3.0	4.0
29		445	58	15	5.0	8.0
30	Spectrum Violet...	425	60	20	7.0	10.0
Mean....	....	....	....	....	3.2	3.8
31	Purples..........	570	22	....	....	....
32		559	15	11	5	6
33		555	13	4	2	2
34		538	16	17	4	5
35		534	11	6	3	2
36		512	9.4	22	7	8
Mean....	....	....	....	....	4.2	4.6

[a] The numbers in column 1 show the spectral order of the thirty-six undiluted Ridgway chromas. Column 2 gives the names assigned to certain of the colors by Ridgway. Column 3 shows the wave-length of the dominant hue and column 4 the per cent. hue. Columns 5, 6, and 7 list the differences between succeeding members in wave-length, Steindler "hue scale," and Jones "hue scale" units respectively (See Jones, Ref. 42, pp. 73–77).

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are being made at the Bureau of Standards, and will be considered in later reports of the present Committee.

(c) *The Ostwald System*, besides having been elaborately described by its author in a number of publications, has been given two concrete exemplifications. The most elaborate one, "Der Farbenatlas," presents approximately twenty-five hundred colors which are systematically indexed in order of their color tone and content of black and white respectively. An abridged system, "Der Farbkörper," comprises seven hundred and sixty-eight of the principal colors used in the more comprehensive system. Ostwald's arrangement, like that of Munsell and Ridgway, is based upon psychological rather than physical criteria. Kohlrausch (44) has made a spectrophotometric analysis of sixty of Ostwald's colors taken from three of his color circles having different saturations or reflectivities. Kohlrausch has furthermore computed the excitation values of these colors on the basis of König and Dieterici's excitation curves. He has also made direct monochromatic analyses of the colors in question.

V. METHODS OF COLORIMETRY AND THEIR INTERRELATIONS

It is not our purpose in the present Report to consider in detail the various methods which are in use, or which have been proposed, for the practical measurement of color. This important task, which involves the description of instruments and the establishment of a terminology for each of the methods, is reserved for a later report. However, it is desirable here at least to catalogue the available systems of colorimetry, and to consider in a preliminary way some of the problems which arise in connection with them, especially that of the reduction of data obtained by the various methods to a common comparison basis.

1. RÉSUMÉ OF AVAILABLE METHODS

The color of an object, considered as an impression which the object produces on the observer, is determined by at least three general sets of factors: (1) the physical characteristics of the object, (2) the physical characteristics of the radiant energy falling upon or emitted by it, and (3) the nature and condition of the observer's visual apparatus. Our control over color in its

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technical applications is confined almost exclusively to the first two sets of factors, although the ultimate goal is always to be found in the consciousness of some observer. Practical colorimetry is therefore concerned with means for the unambiguous designation of those properties of objects and radiation which determine color perception. Most of the means actually employed, however, utilize the visual apparatus of an observer as an essential element—in determining an equation of colors—and hence the results are frequently not independent of the nature and special condition of this apparatus. For this reason it is necessary as in photometry, that the observers should be tested as average and normal.

A. SPECTROPHOTOMETRY.—A method of specifying the physical characteristics of objects and samples of radiation, for the purposes of colorimetrics, which is actually independent of the observer is found in various applications of spectrophotometry or spectroradiometry. These devices enable us to establish the spectral distributions of reflection or transmission for objects, or of energy for radiation, and thus to specify perfectly the essential factors in their values as color stimuli. On the side of the stimulus, pure and simple, spectrophotometry is the fundamental method of colorimetric specification. All other methods (except spectroradiometry) fail to give an equally complete account of the stimulus characteristics. The excuse for their use, however, lies in the fact that the detail of spectral distributions is not actually required if our concern is solely with final color result produced in the observer's consciousness, and methods which dispense with this detail have the advantage of increased simplicity, both in practical application and in expression of results. (Since there is a special Committee of the Optical Society on the subject of Spectrophotometry, it can not be a function of the present Committee to consider this topic in detail.)[18]

Complete data based upon spectrophotometry specify the colorimetric value of a stimulus in terms of the identical radiant power actually evoking the color, both in regard to total amount and

[18] On Spectrophotometry in general see (45).

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spectral distribution. Other methods accomplish this result in terms of a complex stimulus, having general characteristics varying with the method, but seldom spectrally identical with the stimulus to be measured, which nevertheless is found empirically

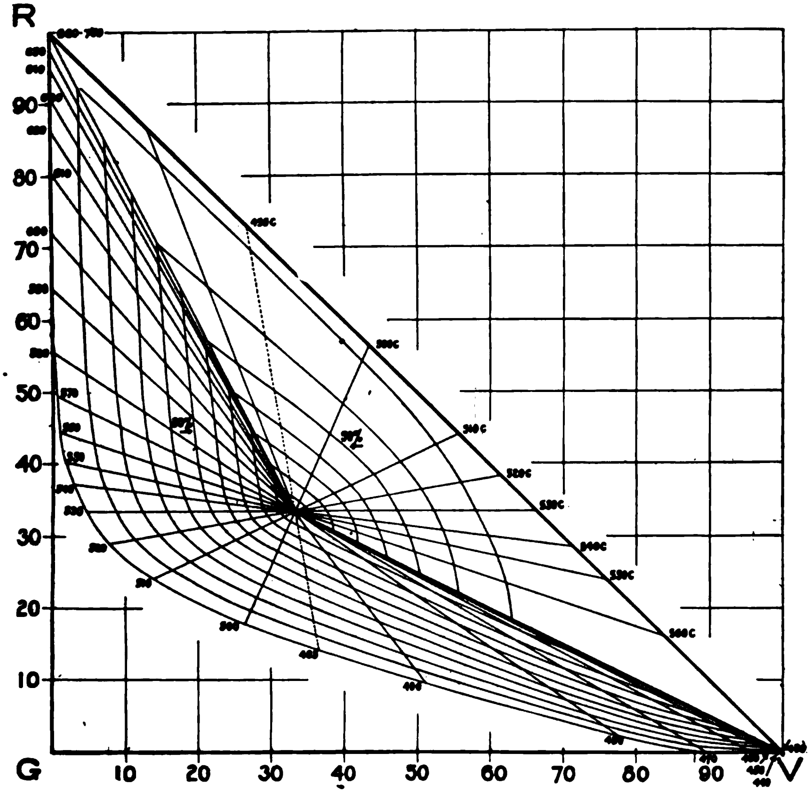

FIG. 9

Triangle, Showing Loci for Various Monochromatic Analyses

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

to evoke the same color as the latter. The most important of these methods which depend upon simple color matching are as follows.

B. MONOCHROMATIC ANALYSIS.—In this method the variable stimulus is composed of heterogeneous radiation, which by itself evokes white or gray, combined with homogeneous radiation of variable wave-length. The total intensity, ratio of homogeneous

to heterogeneous, and wave-length of the former, are adjusted until a match is obtained (66). The measured color is then designated in terms of (1) luminosity, (2) dominant wave-length, and (3) "per cent. hue." It is apparently not essential, although advisable, in this method that the heterogeneous radiation

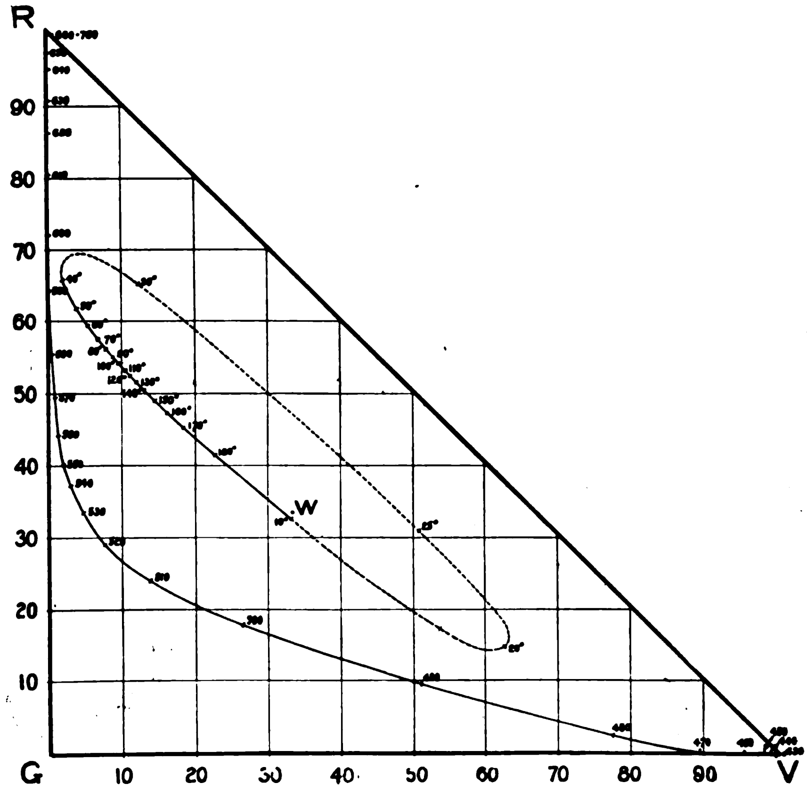

Fig. 10

Color Triangle, Showing Loci of Spectral and Certain Rotatory Disperson Excitations

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

should have an invariable constitution, e.g., that of "normal gray light," so long as it always produces an achromatic color with the given observer. Results obtained by this method are subject to serious errors with deviations, of common occurrence, of the observers' visual systems from the average normal. It is

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important, therefore, that the observers should be carefully selected so as to have standard visibility and color excitation curves. The direct application of this method is limited to colors possessing spectral hues. In order to specify purples, it is necessary to determine the wave-length of homogeneous radiation which when mixed with them matches the heterogeneous standard, i.e., which yields a gray of the given brilliance. It is clear, however, that the method is capable of dealing with colors of all degrees of saturation.

C. TRICHROMATIC ANALYSIS. (**29**; **30**.) In this method the variable stimulus is composed of three constituents having appropriate spectral constitutions, which yield, respectively, colors corresponding in hue to the two end, and the middle regions of the spectrum. The colors ordinarily chosen are red, green and blue, and to render the field of applicability of the system as wide as possible the three stimuli should each be maximally homogeneous. Relative spectral distributions within the components remaining constant, their proportions and total luminosity are adjusted until a match is obtained. The measured color can then be designated by three intensity or luminosity values, one for each of the three components. If the spectral distributions of the components are determined by fairly narrow-banded filters, this method is capable of dealing with practically all reflection colors, which are relatively unsaturated, but is not satisfactorily applicable in its simplest form to many saturated filter or spectral colors. In order to extend the method to deal with the latter it is necessary to add a variable quantity of "white" to the sample which is being measured. If this is done the colorimetric significance of the added white can be recorded in the resultant measurements by subtracting its amount from each of the three-color readings, yielding a specification in terms of red, green and blue, with one minus coefficient, a mode of expression which is in no way incompatible with the trichromatic principle. The trichromatic analyses are of fundamental interest on account of their maximally direct relation to the triadic response mechanism which apparently underlies all color vision,

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but the results which are obtained vary with the color excitation curves of the given observer.

A new and promising application of the trichromatic method of color analysis is to be found in Jones' subtractive colorimeter (**43**). Although the psychophysical principles involved in this instrument are substantially the same as in three-color analyzers based upon additive mixture, the physical operation of the instrument is quite different from that of a three-color additive color-matching device. White or tungsten light is passed in succession through three wedge filters which absorb respectively red, green and blue, and an adjustment of the three wedges can be found which yields a color-match with the sample which is under examination. A neutral wedge is also provided in the path of the beam, permitting a match to be obtained by the use of only two of the chromatic wedges in combination with the neutral one.

D. ROTATORY DISPERSION SYSTEMS.—Several partial systems of color specification have been based upon the rotatory dispersion of quartz, the degree of this dispersion being a function of the thickness of the quartz plate which is employed (**76; 81**). By placing such a plate properly between nicol prisms or other polarizing devices, variations in the relative angular positions of the latter with respect to their extinction positions may be employed to determine a wide variety of spectral transmissions, all of which, however, follow a definite law. Still greater flexibility is obtained by the use of two quartz plates and three nicol prisms. A radiation source of known energy distribution is ordinarily utilized. Instruments based upon this principle are the Arons chromoscope and the leucoscope, recently studied very thoroughly by Priest. The principle appears to have promise of very wide applicability, especially as a means of producing and of specifying a large variety of spectral distributions which it is difficult or impossible to obtain with filters or original sources of radiation. In general, however, the method depends upon the matching of apparent colors, rather than of identical energy distributions.

E. PLANCKIAN DISTRIBUTION ANALYSIS.—The series of spectral distributions determined by successive values of T in Planck's

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equation (see page 556) for the radiation emitted by a "black body" provide a system of stimuli, easily specifiable, and evoking a characteristic series of colors (including gray).[19] However, since the system is virtually unidimensional it can only be applied to a very limited range of colors, practically only to those due to stimuli whose relative spectral distributions fit the Planckian equation. The value of T required for the black body to produce the color match is used as an index of the color, being called the "color temperature."

F. COMPARATOR METHODS.—Methods of color specification based on color matching with arbitrary standards are at present of great technical importance. Such standards include selectively transmissive solutions of definite composition, as well as colored glasses—as in the Lovibond Tintometer—and variegated pigments—as in the Munsell, Ridgway, and Ostwald systems. The final results of measurement by means of one of these methods are expressed in terms of a number or numerical symbol, standing for the particular standard which most nearly approximates the sample in color. These devices are simple in their practical applications, but tend to be unreliable and inaccurate, while the results obtained by different systems are difficult of intercomparison.

2. THE INTERCONVERSION OF DIVERSE COLOR SPECIFICATIONS

One of the main interests of the present Committee is to provide means by which color specifications in terms of different systems can be reduced to a common denominator and, so far as possible, be interconverted (**36**). Spectrophotometric data are potentially convertible into the data of any other system whatsoever, but no specifications which are based upon simple color-matching can be reduced to spectrophotometric terms, without additional information. However, a satisfactory common denominator for all systems is apparently provided by the *elementary color excitations*. Values of these excitations can be found which will specify completely the color characteristics of any stimulus, and each

[19] Use of color temperature as a means of color specification has been developed extensively by E. P. Hyde and his collaborators (**28**).

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member or possible specification in every color system can be reduced to such excitation values, and hence can be assigned a certain position on the color-mixture triangle. In this way the data of separate systems can be definitely intercompared, and can be interconverted in so far as the representations of the several systems overlap; with the obvious restriction that peculiarities of the stimulus—such as spectrophotometric details—which determine no characteristic excitation values, are necessarily lost.

It would therefore appear that the first step in our task is to provide means for transforming the data of each colorimetric system into elementary excitation values, and where possible, means for the reverse transformation. When such transformations have been made, it will be easy to determine the equivalents of one system in terms of any other system. The general principles underlying these computations have already been outlined briefly in our presentation of the excitation curves (*vide supra*). The spectral energy distribution of a given standard stimulus is required if the latter is to be dealt with directly, but can be dispensed with as soon as its combination with the elementary excitation curves has provided a specification of the stimulus in terms of the elementaries.

A. SPECTROPHOTOMETRIC DATA TO EXCITATIONS.—Spectrophotometric data are usually given in the form of spectral transmission or reflection curves. Such curves require combination with a certain energy distribution—representative of the particular source by which the object is viewed—in order that they should become determinative of a definite color. The process of reducing any given spectrophotometric specification to excitation values is therefore as follows. (a) Multiply each of the ordinates of the transmission or reflection curve by the corresponding ordinates of the energy distribution curve of the source. (b) Multiply each of the ordinates of the resulting curve by the corresponding ordinates of each of the color excitation functions as given in Table 6 (under "Excitations"), this being a separate operation for each of the three excitations, yielding three separate curves which represent the respective excitation values for each wave-

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length of the given stimulus. (c) Determine separately the areas of the three curves thus found. This latter operation can be performed by applying a planimeter to a graph of the resultant curves or—with sufficient accuracy—by finding the sum of representative ordinates of each curve, taken separately at uniform, small, intervals—such as 10 $m\mu$—throughout the range of the curve in question. (4) Reduce the three areal values thus obtained to percentage form, so that their determined ratio remains unchanged but their sum becomes equal to 100. The color excitation values can now be expressed by means of two numbers, representing the red and violet excitation percentages, that for the green being obtainable by subtracting the sum of these two values from 100.

As already pointed out, it is in general impossible to reverse the above process, and to convert color excitation specifications into definite spectrophotometric form, because there are an infinite number of spectrophotometric conditions for the majority of color excitation ratios. However, it is possible by means of the color triangle to determine stimuli for given sets of excitation values. The most feasible method of procedure is to plot the position of the given color in the triangle and to note its relation to the locus of the spectral colors. If it lies outside of the area bounded—on two sides—by this locus it possesses no realizable stimulus. If it lies exactly on the locus, in a region of the latter which exhibits curvature, it possesses a unique condition, viz., the homogeneous spectral stimulus having a wave-length indicated by its position with respect to the wave-length scale plotted on the spectral locus. If it falls on a straight portion of the locus in question, it can be evoked by the homogeneous wave-length which immediately corresponds with its position, or by mixtures of any stimuli having wave-lengths represented on either side of it in the given straight portion of the locus, the proportions of these mixed stimuli being determined by the "center of gravity" principle (*vide supra*). If the point representing the given color lies *within* the area bounded by the spectral locus, the color can be produced by mixtures of spectral stimuli lying at the intersections, with the locus, of any straight line passing through the

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point in question, the proportions being determined again by the "center of gravity" principle, applied to the segments of the line thus established. There are obviously an infinite number of such mixtures, not only of two components but of any number of components.

It is one of the functions of the present Report to provide the excitation values of characteristic stimuli, computed by the method outlined above. Tables of such values, for black body colors, colors obtained by rotatory dispersion, Munsell colors, etc., will be found below in conjunction with the discussion of the excitation equivalents of these various standards.

B. MONOCHROMATIC ANALYSIS DATA TO EXCITATIONS.—The general principles underlying the reduction of monochromatic analysis specifications to color excitation values are similar to those outlined above, but with certain complexities which are introduced by the use of a photometric method for establishing and expressing the ratio between the amounts of "white" and monochromatic stimulus in any given case. It is of course natural in practice to specify this ratio in luminosity terms, but these terms play no part in determining the excitation values given in Table 6. Consequently, in order to effect the requisite transformation, it is necessary to make use of the luminosity valencies of the several excitations, which were discussed on page 551. To simplify computation, these valences have been expressed so as to represent the fractional contributions of the three excitations to the luminosity of a white, taken as unity, the values being: for the red 0.370, for the green 0.617, and for the blue 0.012.

The actual steps which are involved may be outlined as follows. (*a*) It is first necessary to know the spectral distribution of radiant intensities for the stimulus which is employed as a white in the given measurements. In case the distribution in question is that of average noon sunlight—or a distribution which color-matches this—it is only necessary to multiply each of the luminosity valences above considered, by the "per cent. white" of the specification. If, on the other hand, the "white" departs in effective character from average noon sunlight, excitation values must be computed for it by the method described under

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"A" above. Each of these values is then multiplied by the corresponding luminosity valence, and the products thus obtained are reduced, with ratios unchanged, so that their sum is equal to unity. These figures are now multiplied separately by the percentage measure specified for the "white." Either one of these operations—for average noon sunlight or the arbitrary "white"—yields a set of three figures, one for each of the excitations. (*b*) The next step is to treat the "per cent. hue" measure in a similar manner. The excitation values for the wave-length employed in the given match must first be looked up in Table 6. Each of the values thus found is next multiplied by the corresponding luminosity valence and the products are reduced so that their sum is equal to unity. Each of the resulting values must now be multiplied by the percentage measure of the monochromatic component in the original specification. (*c*) The corresponding members of the two sets of values, thus secured,—for the "white" and "hue" respectively—are now added. (*d*) The three resulting sums express the excitation values of the monochromatic specification in luminosity terms. In order to reconvert them into the color valence terms of Table 6, each of these sums must be divided by the corresponding luminosity valence, and the values thus obtained reduced to the usual percentage form.

In the case of specifications by monochromatic analysis, of colors possessing a purple hue, in terms of per cent. of the given color and the per cent. of its spectral complementary required to be mixed with it to match the standard white, the procedure for reduction of the data to excitation values differs from the above in the following way. (*a*) The "white" is treated exactly as described under "(*a*)" of the preceding paragraph, except that the final percentage employed as a multiplier is 100. (*b*) The complementary monochromatic stimulus is treated exactly as under "(*b*)" in the same paragraph, the final percentage multiplier representing the per cent. which this stimulus is of the mixture comprising it and the measured color. (*c*) The individual members of the set of values thus obtained for the complementary stimulus are then *subtracted* from the corresponding members of the set obtained for the white. (*d*) The three resulting differ-

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ences may now be reconverted into color valence terms, as directed under "(*d*)" of the preceding paragraph.

At the present time, owing to the unreliability of the magnitudes assigned to the luminosity valences for the three excitations, the conversion of data obtained by monochromatic analysis into excitation values, and thence into terms of other methods of color specification, cannot be accomplished with as great an accuracy as could be wished for. Present indications, moreover, are that these luminosity valences vary considerably among individual observers, without parallel variations in the color valences. Such variations evidently accompany deviations in the form of the observer's visibility curve from normal, and demand that special care be taken in the selection of observers for use of the monochromatic method. In general, this method would appear to be more sensitive to the personal equation than the trichromatic and certain other methods.

The reverse conversion, of color excitation values into monochromatic specifications, is theoretically possible without ambiguity for all sets of values represented in the color triangle by points lying within the area determined by the spectral locus (*vide supra*). The easiest means for accomplishing this conversion consists in the use of a color triangle having represented upon it not only the spectral locus, but also the loci of the spectral colors, and purples, mixed with various proportions of white. In the absence of such a diagram, which is provided by Fig. 9, a cumbersome "trial and error" method is necessitated. To determine the monochromatic equivalents of any color excitation specification, its position should be plotted on the color triangle of Fig. 9, and a straight line drawn through this point and the point representing the white (the white of the given monochromatic system). The intersection of this line with the spectral locus will indicate the dominant hue or wave-length,—direct or complementary, as the case may be—, and the relation of the color point on this line to its intersections with the loci for various percentages of admixed white will serve to determine the "per cent. white."

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Table 14 (A and B) gives the excitation values computed for a large number of representative monochromatic specifications, which were read off from Fig. 9. It should be borne in mind that all of these values are subject to correction with improved determinations of the luminosity valences.

C. TRICHROMATIC DATA TO EXCITATIONS.—Data obtained by the trichromatic method of analysis bear the most direct possible relation to color specification in terms of elementary excitations. However, no actual colorimeter based on this principle can duplicate in saturation the elementaries which were employed in computing the values of Table 6, at least in the case of the green excitation, and probably also in the cases of red and violet excitations. Extant three-color measuring systems naturally

TABLE 14B

Color Excitation Values for Representative Monochromatic Analyses; Purple Hues

Complementary Wave-Length		Per cent. Hue						
		10	20	30	40	50	60	70
495C	R	35.5	38.0	41.2	45.3	50.2	57.8	69.6
	V	32.9	32.5	32.0	31.3	30.5	29.2	27.3
500C	R	34.7	36.4	38.4	40.8	44.0	48.2	54.2
	V	33.8	34.6	35.5	36.6	38.0	39.8	42.5
510C	R	34.0	34.9	35.8	37.0	38.4	40.2	42.5
	V	34.7	36.6	38.6	41.0	44.0	47.8	52.6
520C	R	33.7	34.0	34.5	35.0	35.6	36.3	37.3
	V	35.0	37.2	39.8	42.6	46.2	50.5	56.0
530C	R	33.3	33.3	33.3	33.3	33.3	33.3	33.3
	V	35.3	37.7	40.5	43.6	47.4	52.3	58.1
540C	R	33.0	32.7	32.3	31.9	31.4	30.8	30.0
	V	35.5	38.0	41.0	44.2	48.3	53.5	59.9
550C	R	32.7	32.2	31.6	30.8	30.0	28.8	27.3
	V	35.6	38.2	41.3	44.7	48.8	54.2	61.0
560C	R	32.4	31.6	30.5	29.3	27.8	25.9	23.4
	V	35.8	38.4	41.7	45.3	49.6	54.9	62.2

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TABLE 14A

Percentage Color Excitation Values for Representative Monochromatic Analyses: Spectral Hues.

Wave Length of Dominant Hue	Per Cent. White																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	R	R	V	R	V	R	V	R	V	R	V
400	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
410	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
420	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
430	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
440	0	99.3	.3	98.7	.6	97.9	1.0	97.2	1.5	96.2	2.5	94.5	3.4	92.6	4.7	90.0	7.4	84.5	13.0	74.0
450	0	98.3	.3	97.8	.7	97.0	1.2	96.0	1.8	94.7	2.8	92.8	4.0	90.5	5.5	87.5	8.5	81.5	14.0	71.0
460	0	95.7	.5	94.9	1.0	94.0	1.7	92.5	2.6	90.9	3.9	88.4	5.5	85.5	7.3	82.0	10.8	75.5	17.6	63.0
470	0	89.6	.9	88.2	2.0	86.4	3.1	84.5	4.6	81.9	6.3	78.8	8.7	74.7	11.8	69.4	16.3	62.2	22.7	51.0
480	2.3	77.6	3.8	75.5	5.3	73.3	7.2	70.5	9.2	67.5	11.6	64.2	14.6	60.0	18.0	55.0	21.9	49.4	27.0	42.5
490	9.5	51.1	11.5	49.6	13.6	48.0	15.8	46.4	18.0	44.7	20.3	43.0	22.6	41.2	25.2	39.3	27.9	37.4	30.5	35.4
500	17.8	26.4	19.6	27.2	21.3	28.0	23.0	28.8	24.8	29.5	26.5	30.2	27.8	30.8	29.3	31.4	30.6	32.0	32.0	32.7
510	24.0	13.8	25.3	16.4	26.5	18.9	27.6	21.1	28.6	23.2	29.5	25.2	30.4	26.9	31.2	28.7	31.8	30.3	32.5	31.8
520	29.0	7.6	29.6	11.2	30.2	14.5	30.7	17.5	31.2	20.3	31.6	23.0	32.0	25.3	32.4	27.4	32.7	29.5	33.0	31.4
530	33.3	4.7	33.3	8.6	33.3	12.4	33.3	15.9	33.3	19.0	33.3	21.7	33.3	24.4	33.3	26.9	33.3	29.2	33.3	31.3
540	37.1	2.8	36.6	7.2	36.1	11.2	35.7	14.9	35.3	18.2	34.9	21.2	34.5	23.9	34.1	26.5	33.8	28.9	33.5	31.2

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TABLE 14A—*Continued*

Wave Length of Dominant Hue	Per Cent. White																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V
550	40.2	1.7	39.3	6.2	38.4	10.3	37.5	14.1	36.7	17.6	36.0	20.8	35.5	23.6	34.9	26.3	34.3	28.8	33.8	31.1
560	44.2	1.0	42.6	5.5	41.2	9.6	40.0	13.5	38.8	17.2	37.7	20.4	36.7	23.3	35.7	26.0	34.9	28.6	34.1	31.0
570	49.5	0.7	47.2	5.2	45.0	9.3	43.2	13.0	41.4	16.6	39.8	20.0	38.3	23.0	36.9	25.8	35.6	28.5	34.4	30.9
580	55.4	0.4	52.5	4.7	49.7	8.8	47.2	12.6	44.7	16.2	42.3	19.6	40.3	22.7	38.4	25.5	36.6	28.2	34.9	30.8
590	64.3	0.0	60.4	4.3	56.6	8.4	53.0	12.3	49.7	15.9	46.3	19.3	43.4	22.4	40.6	25.3	38.2	28.0	35.7	30.7
600	72.2	0.0	67.2	4.1	62.6	8.1	58.1	12.0	53.9	15.6	50.1	19.0	46.2	22.1	42.7	25.1	39.5	27.9	36.3	30.6
610	80.4	0.0	74.7	4.0	69.1	7.9	63.8	11.7	58.9	15.3	54.0	18.7	49.6	21.8	45.2	24.8	41.1	27.7	37.2	30.5
620	86.3	0.0	80.0	3.9	74.0	7.7	67.8	11.5	62.3	15.0	57.1	18.4	52.0	21.5	47.1	24.6	42.4	27.5	37.8	30.4
630	90.8	0.0	84.1	3.9	77.6	7.6	71.4	11.4	65.3	14.8	59.6	18.2	53.9	21.3	48.5	24.4	43.3	27.4	38.2	30.3
640	95.1	0.0	88.0	3.9	81.2	7.5	74.6	11.3	68.2	14.6	61.9	18.0	55.7	21.2	50.1	24.2	44.3	27.3	38.7	30.3
650	97.5	0.0	90.2	3.8	83.0	7.4	76.1	11.2	69.4	14.5	62.8	17.8	56.5	21.1	50.4	24.1	44.5	27.2	38.8	30.2
660-700	100.0	0.0	92.4	3.8	85.0	7.4	77.9	11.2	70.9	14.4	64.1	17.7	57.6	21.0	51.2	24.0	45.0	27.1	39.1	30.2

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differ also in the dominant hues of the three components which they employ. In order to convert the data obtained by the application of any such system to terms of our three elementaries, it is necessary to employ nine coefficients which represent the degree of participation of each of our elementaries in each of the components of the given system or vice versa. The reverse conversion involves nine reciprocal coefficients based upon the same relationship. The operations involved may be represented by the following equations:—

$$(1)\quad R = ar + bg + cv$$
$$(2)\quad G = dr + eg + fv$$
$$(3)\quad V = hr + ig + jv$$

where a, b, c, etc., are the coefficients in question, r, g, and v are the values of the given trichromatic measuring system, and R, G, and V are the desired excitation values.

TABLE 15

Coefficients for Interconverting Ives Colorimeter Data and Excitation Values

If r, g, and b are the components of a color according to the Ives colorimeter, and R, G, and V are its values in terms of the elementary excitations used in this Report;

$$R = 1870\,r + 2080\,g + 14\,b$$
$$G = 134\,r + 3710\,g + 124\,b$$
$$V = 506\,g + 3460\,b$$

Conversely:

$$r = 1275\,R - 719\,G + 21\,V$$
$$g = -46\,R + 646\,G - 23\,V$$
$$b = 7\,R - 95\,G + 665\,V$$

There is an arbitrary factor in all of the above coefficients, so that the results are significant only as proportionalities.

The only trichromatic additive system which is in any way well known at the present time is that employed in the Ives colorimeter. Table 15 provides values for the coefficients which must be employed to convert color specifications in terms of the Ives system into excitation terms by use of the formulae given above. These coefficients were obtained by applying the excitation data of Table 6 to the spectrophotometric curves for the filters and light source employed in the Ives colorimeter in accordance with the principles outlined under Section A of the present part of this report. The light source assumed was average noon sunlight.

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Table 15 also gives coefficients for the converse operations. Similar methods of calculation may be applied to any three-color system, such as, for example, that involved in the new subtractive colorimeter designed by Jones (43). In the special case of the subtractive colorimeter, however, it is not possible to assign fixed excitation values to the three primaries of the instrument, since the spectral distributions vary qualitatively for each individual setting of the instrument. The resulting necessity of applying the principles for the interconversion of colorimetric data to the spectrophotometric analyses of individual settings is a theoretical demerit of the subtractive as opposed to the additive trichromatic colorimeter.

It is clear that in order to compare the data obtained by trichromatic analysis with those due in any other system, such as the monochromatic, it is only necessary to convert both sets of data into color excitation values, in which condition they may be translated by a further operation into terms of any desired system.

D. ROTATORY DISPERSION DATA TO EXCITATIONS.—Colorimetric data based upon any rotatory dispersion system may be converted into color excitation terms by determining the spectrophotometric curves for the given dispersion stimuli and applying the methods outlined under A above. The reverse conversion is best accomplished by means of a plot in the color triangle of the loci for the various series of dispersion colors which are involved. Such reverse conversion is of course possible only when the given excitation values determine a point in the triangle which falls upon one of these loci, although the great flexibility of the rotatory dispersion method will permit the duplication of a wide variety of conditions of color excitation.

Table 16 gives the excitation values for the rotatory dispersion colors produced at various angles of the Nicol prisms for a quartz plate one mm in thickness. These values are plotted in the color triangle in Fig. 10. It will be seen that their locus is approximately elliptical in form and corresponds very closely with that of a certain range of black body colors (Cf. Fig. 3).

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TABLE 16

Excitation Values of Certain Rotatory Dispersion Colors

(Comparison source = acetylene color)
(Quartz thickness = 1 mm.)
(Φ = the angle between the Nicol prisms)

Φ	Per cent. R	Per cent. V	Nearest Black Body Color Temperature
0°	41.3	22.7	3494°
10°	32.6	33.1	5050°
20°	14.8	62.6	....
30°	65.1	12.3	....
40°	65.8	1.8	....
50°	61.7	3.7	1690°
60°	59.2	5.3	1840°
70°	57.5	6.8	1970°
80°	56.1	7.9	2070°
90°	55.0	8.9	2160°
100°	54.1	9.7	2227°
110°	53.2	10.5	2302°
120°	52.4	11.2	2370°
130°	51.5	12.1	2448°
140°	50.4	13.1	2543°
150°	49.0	14.4	2673°
160°	47.2	16.2	2850°
170°	45.1	18.4	3050°

TABLE 17

Percentage Excitation Values for Black Body Colors Computed by Means of the Planckian Formula (C_2 = 14,350)

Degrees Kelvin		0	100	200	300	400	500	600	700	800	900
1000	R	78.1	75.4	73.0	70.7	68.4	66.3	64.3	62.4	60.6	59.0
	V	0.6	0.8	1.2	1.7	2.3	3.0	3.8	4.7	5.6	6.5
2000	R	57.4	56.0	54.6	53.4	52.2	51.0	49.9	48.8	47.8	46.8
	V	7.5	8.5	9.6	10.6	11.7	12.8	13.8	14.9	16.0	17.0
3000	R	45.8	44.9	44.0	43.2	42.4	41.6	40.8	40.2	39.7	39.0
	V	18.1	19.2	20.2	21.2	22.2	23.1	24.0	24.8	25.7	26.5
4000	R	38.4	37.8	37.4	36.8	36.3	35.8	35.3	34.9	34.5	34.1
	V	27.2	27.9	28.6	29.3	30.0	30.7	31.3	31.9	32.4	33.1
5000	R	33.7	33.3	32.9	32.6	32.3	32.0	31.7	31.4	31.1	30.8
	V	33.8	34.3	34.8	35.3	35.8	36.2	36.7	37.2	37.6	38.1
6000	R	30.6	......	......	......	7000	28.4	......	......	......	......
	V	38.5	......	......	......	......	41.9	......	......	......	......

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E. PLANCKIAN DISTRIBUTION DATA TO EXCITATIONS.—The various black body colors can evidently be reduced to color excitation values by means of their spectrophotometric representations, utilizing the methods already outlined. The reverse conversion is best accomplished by means of a plot of the positions for these colors in the color triangle.[20]

Table 17 gives the color excitation values for a wide range of black body temperatures and these values are represented graphically in Fig. 3.

F. COMPARATOR DATA TO EXCITATIONS.—The conversion of color specification data, based upon color-matching with arbitrary standards, to color excitation terms will of course involve not only the spectral transmissions or reflections of the given standards but also the spectral distribution curve of the particular radiation source which is employed in making the given color-match. The reverse conversion is best accomplished by means of a representation in the color triangle of the values for the various arbitrary standards, and the possibility of such reverse conversion

TABLE 18

Percentage Excitation Values for Certain Munsell Colors[a]

D	E	D	E	D	E
R 7/5	39R 29V	R 5/5	44R 25V	R 3/2	40R 28V
Y 7/4	43R 19V	Y 5/5	47R 14V	Y 3/2	40R 23V
G 7/4	34R 29V	G 5/5	33R 26V		
B 7/4	30R 37V	B 5/5	26R 42V		
P 7/3	32R 36V	P 5/5	29R 43V		

[a] "D" is the designation of the given color in the Munsell System and "E" is the corresponding percentage excitation value.

[20] It is clear that since the black body colors form a single linear series the reverse conversion will seldom be possible.

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will of course depend upon the scope of the given comparator system and in general will be only approximate.

In Table 18 will be found color excitation values for various Munsell pigments as seen under average noon sunlight. The equivalents of certain Ridgway colors in terms of monochromatic analysis have been given in Table 13. As already noted, the excitation equivalents of many colors in Ostwald's pigment system have been computed very carefully by Kohlrausch in a recent article. In subsequent reports the committee will endeavor to provide values for further standards, both in these two and other systems, such as that utilized in the Lovibond tintometer.

VI. SUMMARY AND CONCLUSION

The above report, being a more or less pioneer effort of its kind, must naturally be regarded as incomplete and tentative. However, the purpose of the report is an earnest one and is directed towards at least four ends; (1) the clarification and standardization of color terminology, (2) the compilation of data which are fundamental to color science, (3) the specification of standard stimuli and conditions, for use in practical color work, and (4) the encouragement of discussion and research along these lines. An outline of the contents of the present report is given at the beginning in the form of a Table of Contents. It is hoped in later reports by the present Committee to deal more specifically with details in the terminology and application of the various methods for colorimetry as well as with the design of instruments for the utilization of these methods. Finally, the Committee desires to express once more its wish that workers in the field of color science communicate freely their criticisms and specific needs.

NOTE: No attempt is made here to summarize the progress in color science during the years, 1920–21, since this topic has been treated by the Chairman of the Committee in general summaries published in the *American Journal of Physiological Optics*, October 1921, pp. 316–391, and forthcoming.

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HETEROMORPHIES DUE TO THE VARIATION OF EFFECTIVE APERTURE AND VISUAL ACUITY[1]

BY
K. HOROVITZ

1. By the use of optical apparatus the image of the surrounding space sometimes undergoes a complete change, as in some kinds of prisms and mirrors. But also in other cases, in which the instrument produces a perfect image, apparent alterations of space are perceptible. These apparent defects of the visual space are called heteromorphies.

Expert microscopists are always surprised that beginners are unable at first to find the image, or, when drawing it, always make it too small. And to each of us it is well known that the drawing or photograph of a microscopical object seems much bigger than really shown by the microscope. Impressions of this kind are also perceptible in other cases than when using an optical instrument. If, for instance, instead of through the microscope we look through an empty tube with one eye and fix it on a distant object[2] while the other eye is directed to the same object in the ordinary manner, the image seen through the tube appears much smaller than to the naked eye.[3] Exactly the same thing can be observed by looking at a landscape through the finder of a camera or by holding a narrow stop before the eye. In all these cases the aperture of the rays of the optical system, consisting of the eye together with the effective stop, is changed.

The phenomena described can be observed by persons with normal vision (emmetropes), under certain conditions to be described later, by ametropes and, as already mentioned, in using optical instruments. The phenomena are therefore, as it seems, independent of the dioptrical properties of the system, but only

[1] Communicated by Dr. Ludwik Silberstein.

[2] It must be a distant object in order that the rays appear to come from infinity in the same manner as when using the microscope.

[3] Cf. St. Meyer, Phys. Zs. 21, p. 124, 1920; and K. Horovitz, Phys. Zs. 21, p. 499, 1920.

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so far as the dioptrical system remains unchanged and the image remains clear.

2. The effective aperture may influence the image in different ways. When the entrance-pupil is altered, the size of the image-forming diffraction-disks is also altered. The smaller the latter are, the sharper the images, but their brightness is reduced (case of a small diaphragm). A stop which reduces the aperture of the image-forming pencils without altering the entrance-pupil, changes the field of view. The image of the stop in the focal plane (in the case of the eye, on or before the retina) is the exit-window (case of the tube and the finder of a photographic camera).

An alteration of the entrance-pupil changes also the actual size of the image, for the circles of confusion become smaller. As the aberrations in the eye and the formation of the image by wide-angle pencils bring it about the size of the diffraction-disks depends not only on the size of the entrance pupil, this reduction of the image on the retina is difficult to perceive. Thus the diminution observed, when looking through a diaphragm, need not depend necessarily on the actual reduction of the image on the retina. When, the entrance-pupil remaining constant, the entrance-window or the stop of the field of view is changed, the structure of the image is only changed in those places where now, instead of the former images, the image of the exit-window is formed. If nevertheless the impressions brought about by the unchanged retinal images are changed, evidently this must be connected with the mutual influence of neighboring parts of the retina,[4] especially with the influence of the simultaneous contrast-sensibility which brings about a change of the adjustment of the eye. Experiments have shown, that the diminution is as much increased as the stop of the field of view decreases (that is to say, in the case of the tube it would be a longer one). A diminution of the field of view also occurs with the replacement of the binocular by monocular vision, which transition also is connected with an apparent diminution of size.

[4] This is Hering's induction.

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But an alteration of the aperture is also connected with an alteration of the distribution of light in the image. This is caused by a change of the circle of confusion by variation of the entrance-pupil. We have, further, to consider a change of the simultaneous contrast, when, by an alteration of the stop of the field of view, parts of the image are covered either by light or darkness. The former case occurs when an object is observed through a glass-tube, which, by total reflection, forms on the retina a brightly illuminated background; the latter when looking through a tube blackened on the inside, in which case the part of the field of view around the retinal image is darkened. But this affect's in a decisive way the acuteness of vision and this influence therefore must always be taken into account.

3. It is well known, that the acuteness of vision is diminished by a reduction of the intensity of illumination, by flooding the retina with useless light, by the darkening of simultaneous contrast and of course also by wrong adjustment.[5] All these produce a reduction of the relative differences in the sensations of the brightness of two points. An increase of the visual acuity is caused by: a diminution of the entrance-pupil, a moderate increase of the intensity of light (provided that the peripheral portions of the retina are not sensibly illuminated by stray light), and the illuminating simultaneous contrast. Each change of the incident light alters the pupillary diameter leading to disturbing secondary phenomena (the acuteness of vision may be changed thereby and also focusing movements be liberated).[6] Therefore, it was necessary to make experiments in which the influence of the size of the pupil was eliminated. For this purpose investigations were made on persons, whose pupils had lost the reaction to light. *The result was, that a reduction of the acuteness of vision is always followed by an apparent diminution in size and, at the same time, the removing and bringing nearer together of the objects observed.* On the other hand a sudden increase of the visual acuteness produces

[5] In the latter case also dioptrics may be of importance as mentioned before.

[6] Cf. E. Hummelsheim, Arch. f. Ophth. vol. 45, 1898, p. 357 and K. Horovitz, Ber. d. deutschen physik. Ges. 1921, 2, pp. 9–11 and Sitz. Ak. Wiss. Wien, vol. 1130, 1921, pp. 405–421.

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an increase of size. With atropinisized ametropes it could be observed that a small (stenopaïcal) stop which in the case of normal sight would cause a diminution, improves the acuteness of vision so much that an increase may be observed. Beside this *new* effect of the alteration of size in the field of vision, the known influence of the visual acuity must also be considered and also the influence of the aperture on the depth of focus of the image-space. The importance of the acuteness of vision for the resolving power of the microscope was pointed out by F. E. Wright.[7] (Here the illuminating effect of the simultaneous contrast comes into play.) In the following I intend to deal with a series of contrivances, for which the above mentioned effects are of importance, and then set forth the theory of the phenomena.

4. For the series of readings a simple lens is used, before which a diaphragm is placed, to increase the distinctness. With a very small diaphragm (e.g., of 0.1 mm. diameter) the usual magnification cannot be observed. I made this observation myself with an Elster-Geitel-electroscope and found it confirmed by other observers: the scale and the leaves of the electroscope seem to be in one plane and far more distant than without a diaphragm. As these phenomena are not always perceptible with both eyes with the same intensity they are also of importance when first one eye is used and then the other in observing and above all in comparing the images with one eye aided and the other naked. This is the case with the well-known examination of the magnifying power of a telescope which consists in comparing the lines of a distantly suspended scale with one eye naked and the other aided by the telescope. If then the exit-pupil of the telescope is much smaller than the pupil of the eye, the phenomenon mentioned above is observed. Thus sometimes it is not sufficient to pull out the tube to diminish the parallax between the two images. A similar examination of the magnifying power of a microscope is open to the same difficulties. To avoid such faults, fix close to the instrument, whose magnifying power is to be examined, a diaphragm for the naked eye of the size of the exit-pupil of the

[7] F. E. Wright, *Jour. of the Opt. Soc.* 2-3, p. 101, 1919.

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instrument. The right perception of the increase, when using night-glasses depends also on the correct proportion between the exit-pupil of the instrument and the eye-pupil. A. Gehlhoff[8] pointed out that the size of the field of view is also of psychological importance for the resolving powers of these instruments. By employing drawing apparatus, as for instance the well-known camera lucida, the influence of the opening of the stop is remarkable. On using this instrument the scenery appears smaller on the drawing paper than to the naked eye. *If the stop is made gradually smaller* (as far as it is possible without making the illumination of the image so faint, that it is no longer perceptible) *the image becomes smaller as well.* Qualitative tests, made with regard to this, have shown that the variation in size by stopping down the diaphragm from 3 mm to 0.1 mm is about 15%.

It is natural to take into account the variation of visual acuity by dazzling. Therefore every physicist or astronomer uses in exact measurements a dark eye-shield to cover the non-observing eye. Also blinding of this eye, as is easily proved, diminishes the acuteness of vision and for this reason apparently the magnification and the perception of depth for the observing eye. Therefore it seems necessary, not to change the conditions for the formation of image in the eye, in applying optical apparatus for subjective use.

5. According to the doctrines of physiological optics, the defective aperture of the rays and the acuteness of vision do not immediately determine the perception of size. But these factors do determine the depth of focus which depends on $\frac{p}{\sigma}$, where p denotes the diameter of the pupil and σ is the angular measurement of the visual acuity.[9] If $\frac{p}{\sigma}$ diminishes, the depth of focus increases, and *vice versa*. The distinctness of the perception of depth is inversely as the optically defined range of distinct vision (depth of focus). On the other hand the sensation of size is con-

[8] A. Gehlhoff, Zs. f. techn. Phys. 66, p. 477 *et seq.*

[9] Cf. Rohr, Brit. Jour. of Phot. 48, p. 454, 1901, and S. Czapski, Grundzüge der Theorie der optischen Instrumente, p. 256, 267, 1904.

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nected with the accommodation of the eye. Any object seems smaller in proportion as the accommodation is greater or even when the sense of greater accommodation is excited, and this is the case although the image on the retina is unchanged in size. We will assume now, that if anywhere a variation of the range of distinct vision takes place, the accommodation or the innervation of accommodation increases as far as possible without perceiving the image less sharply (*Principle of maximum accommodation*).[10] If the range of distinct vision increases, a point is approached at which the depth begins to be practically infinite and it is, therefore, useful to focus at a nearer point. This does not mean that these focusing impulses are always connected with a real accommodation, for we would then see the nearer point as sharply as the point at which we focused previously. On the contrary, impulses to relax the accommodation take place if the depth of focus decreases again because points at a greater distance are now also distinctly visible and thus the depth of the visual space is enlarged. By this conception, all the experiments mentioned above are intelligible: whether the pupil p or the angular size of visual acuity σ are changed,[11] the innervation to focus begins and together with it a variation of the impression of size and depth.[12]

[10] J. K. Horovitz, Sitz. Akad. Wiss. Wien. vol. 130, 1921; Beiträge zur Theorie des Sehraums.

[11] It is necessary to mention, that a change of the pupil also entails a variation of visual acuity (but not *vice versa*).

[12] It is partly due to the influence of limiting the field of view, that the diminution of the object, when seen through concave spectacles, is much more intensely felt (as shown by Isakovitz), than the diminution of the image on the retina alone could bring about. Therefore, the variation of the depth of focus also must be considered: the entire depth being greater than without spectacles, because $\frac{p}{\sigma}$ which determines the distinctness is only $\frac{pB^2}{\sigma}$ (B < 1).—It may be mentioned here, that also other dysmegalopsies, micropsy after the injection of atropin, macropsy after the use of pilocarpin (eserin) are intelligible if we assume the principle of maximum accommodation. In the first case the innervation is unlimited, the apparatus of accommodation being paralysed: hence micropsy. In the other case the external eye is in a spasm of accommodation, the impulses are arrested: hence macropsy.—For these and also other cases of physiological interest see K. Horovitz, *loc. cit.* and further an article appearing in "Pflüger's Archiv" (Grössenwahrnehmung und Sehraumrelief).

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6. To show how these changes of our optical sensations are connected with normal sight, we proceed to consider the conception that the space as seen by us, is an optical transformation (in the sense of Maxwell and Abbe) of the physical space.

In this transformation any point of the object-space has a one to one correspondence with a point of the visual space and the lines of sight remain invariant, while the points at infinity are transformed into a plane at finite distance perpendicular to the axis of vision. Mathematically formulated these conditions give, instead of the usual optical transformation

$$x' = \frac{a_1x+a_2y+a_3z+a_4}{ax+by+cz+d}, y' = \frac{b_1x+b_2y+b_3z+b_4}{ax+by+cz+d}, z' = \frac{c_1x+c_2y+c_3z+c_4}{ax+by+cz+d}$$

the equations:

$$x_v = \frac{a_1x}{ax+d}, y_v = \frac{b_2y}{ax+d}, z_v = \frac{c_3z}{ax+d}, \text{ with the condition}^{13}\ b_2 = c_3 = d.$$

These are well known formulae of projective geometry and give analytically the geometry of a relief-perspective for the general case. The origin of the coördinates lies in the first eye-point.[14] If $d = a_1$, the relief is the image of the real space which the *quiescent* eye sees, if the eye is contemplating the point at which it is focused (the point of view is identical with the first eye-point).[15] If the eye is focused on a point, which is nearer than that which is contemplated, then it is necessary for the restoration of the previous conditions to displace forward the eye until the point of view will be so near to the contemplated point, that the latter again is a focused one. But if the eye remains in the place of the first eye point we must say that the point of view is shifted forward only *virtually:* the psychical adjustment of the eye and the innervation corresponds to a smaller distance. Then we have

[13] As postulated by the invariance of the line of sight. It is not necessary for the visual space to be always symmetrical around the x axis, as in the case of astigmatism.

[14] Cf. Burmester, Grundzüge der Reliefperspektive.

[15] The case $d = a$ was developed by H. Witte, proceeding from experimental facts, without connection with the relief-formulae explained above. Physik. Zs. vol. 19, vol. 20, several articles "Über den Sehraum."

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$d < a_1$ $\left(\frac{a_1 - d}{a}\right.$ being the range of the virtual displacement of the point of view) and the objects seem to be smaller and at a greater distance. Thus the conditions of variation of the visual space can be conceived by the rules given for normal vision.

For an exact theory it would be necessary to take into account these transformations only as a first approximation and to find the infinitesimal transformation which takes into account the apparent changes induced by the moving eye and the curvature on the margin of the field of vision.[16]

SUMMARY

The influence of the effective aperture and the acuteness of vision have been dealt with in reference to well-known facts regarding the eye in connection with an optical instrument. It has been shown, that a reduction of the visual acuity entails a diminution of the apparent size. The importance of these facts for some optical observations and measurements is explained. This explanation is based on the assumption that maximum impulses to accommodate are liberated in such a manner that the distinctness of the image is not blurred when the conditions of the formation of the image are altered. The optical transformation of the object-space into the visual space for a quiescent eye is defined by simple postulates of invariance and is proved to be a relief-perspective. Usually the point of view coincides with the first eye-point. The heteromorphies considered above may be constructed as a virtual shifting forward of the point of view.

FIRST PHYSICAL INSTITUTE OF THE UNIVERSITY,
VIENNA, FEBRUARY 15, 1922.

[16] The present writer is pursuing these investigations in connection with Riemannian geometry.—The observations stated here are qualitative and it would be most interesting to obtain some exact investigations on these points, which it was impossible to undertake here.

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THE ABSORPTION OF THE EYE FOR ULTRA-VIOLET RADIATION

BY

WINIFRED P. GRAHAM

The eye, as the most delicate sense organ, has always been of great interest, and any investigation adding anything to the sum total of knowledge concerning it, would seem to be worth while. Several investigations have been carried on to determine the relative sensibility of the eye for light of different wave-lengths, but data concerning the limits of the visible spectrum, and the absorption of the tissues of the eye, is conflicting and lacking in information regarding the source and condition of the material. It is, of course, a well-known fact that the eye is sensitive to only a very small portion of the total radiation. The question naturally arises as to whether the limits of our vision are determined by the wave-lengths that are able to affect the retina, or whether the fluids and tissues of the eye actually do not transmit these certain wave-lengths. Nutting says[1] that the retina is most sensitive to radiation in the blue green between wave-lengths .50 and .55μ; that good seeing requires radiation between wave-lengths .41 and .75μ; and if the source is sufficiently intense radiation as far out as wave-length .321 (ultra-violet) or 1.0μ (infra-red) may be perceived. F. W. Edridge-Green states[2] that the limits of the visible spectrum are practically the lines A and H, A wave-length .764μ for the red and H .3968μ for the violet. He also states that the wave-lengths vary with different persons. It may be seen that there is quite a discrepancy between these two writers.

The production of fluorescence in the eye is an important consideration. Wrong conclusions are likely to be drawn as to the limits of the visible spectrum from such experiments as those of Helmholtz and others. When a portion of the spectrum is exposed and produces merely the sensation of light, there is no

[1] Outlines of Applied Optics, p. 120.

[2] The Physiology of Vision, p. 136.

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definite proof that the short wave-lengths actually reach the retina, as they might be transformed into long wave-lengths by the tissues of the eye. This would be equivalent to a source very close to the retina. On the other hand if a slit were used with some characteristic shape such as an arrow, for example, if a distinct image were formed with the slit illuminated with ultra-violet, it would seem to prove that we can see by means of ultra-violet light. Nutting's observation is borne out by a member of the Physics Department of this University who claims to have seen the doublet of the mercury spectrum ($.3132\mu$ and $.3126\mu$). It might be possible that about the same wave-lengths get through the average eye but that the sensitiveness to the ones near the absorption band vary with the individual.

Dr. Fritz Schanz studied[3] the effect of the ultra-violet light on the eye and the absorption of the different parts. He used a quartz spectral photometer and a Nernst lamp. The corneas of three people of different ages were investigated. He found that the cornea begins to absorb at $.360\mu$, and at $.310\ \mu$ the rays of the Nernst lamp were absorbed completely. He studied the lenses of three different people of ages 40, and 28 years, and a child's. He states that the lenses begin to absorb in the blue and absorb extensively in the ultra-violet. Using a quartz spectograph he took photographs of the absorption of the cornea and the lens. The latter was pressed between two quartz plates to a thickness of 3 mm. Since the lens in the region of the pupil is considerably thicker than 3 mm, this would only give the partial absorption. His plates show that the cornea begins to absorb at $.360\mu$ and absorbs completely at $.300\mu$ and the lens absorbs completely at about $.350\mu$ on a forty-second exposure. This exposure varied from twenty to forty seconds.

In a second article[4] in 1920, he apparently refutes the data given in the former discussion. This time he used a quartz spectograph. He states that nearly all the ultra-violet is absorbed but although

[3] Über die Veränderungen und Schädegungen der Augen durch die nicht direkt suchtbaren Lichtstrahlen. Archiv. für Ophthalmologie, **86**, p. 549; 1913.

[4] Der Gehalt des Lichts an Ultraviolett. Archiv. für Ophthalmologie **102**. 103 p. 158; 1920.

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that which is left reaches the retina, it does not produce the sensation of light but of fluorescence. By using an intense source he says it is possible to perceive light up to $.392\mu$, and if there is light beyond this it is caused by the fluorescence of the retina.

Dr. W. E. Burge gives[5] an absorption spectrum of the cornea of a rabbit which transmits wave-lengths as short as $.297\mu$.

The object of this experiment was to determine what wave-lengths in the ultra-violet region get through the tissues and liquids of the eye, or in other words, what light, in the ultra-violet region actually reaches the retina. It was also determined to compare the absorption of the various parts of the eye and to point out, as much as the material which could be procured would permit, the change in the absorption due to disease, solution in formaldehyde, etc.

The measurements are not as numerous as was wished on account of the difficulty of obtaining material. Human eyes were desired immediately after death and before embalming, or soon after removal in the case of enucleation operations. The specimens procured by the latter method can usually only be used in part because of some pathological condition.

APPARTUS AND MATERIALS

A quartz-prism spectroscope was used, with a dispersion of about 4 inches. A cadmium spark furnished the source of radiation. A transformer connected with nine storage cells gave a good spark across a gap of about a centimeter. The electrodes were sticks of cadmium formed by drawing the molten metal up into small glass tubes. These were stuck through a hollow cork and connected to some Leyden jars and thence to transformer.

A holder was made for the corneas examined out of thin copper sheeting. Two pieces were cut approximately two inches square and an oblong hole cut in each $\frac{1}{2}$ cm$\times 1\frac{1}{2}$ cm. A fold of a half centimeter was then made along the edge of one and the other trimmed off an equal amount. Thus one slipped into the fold and the holes coincided. The cornea to be examined was cut off where it joined the sclerotic coat and placed between the two

[5] "The Production of Cataract." Archives of Opthalmology, 47, p. 12; 1918.

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plates, covering the openings. The edges opposite the fold were then placed in a clamp.

Since the crystalline lens in a fresh condition is fairly soft, stiff paper was found very satisfactory as a holder. A cardboard screen was used about 8 by 10 inches with a circular hole of an inch diameter cut in it. The screen was then covered with black paper on each side. A round hole, slightly smaller than the lens to be examined was cut in either side of the black paper, concentric with the hole in the cardboard. This served as a very convenient socket for the lens.

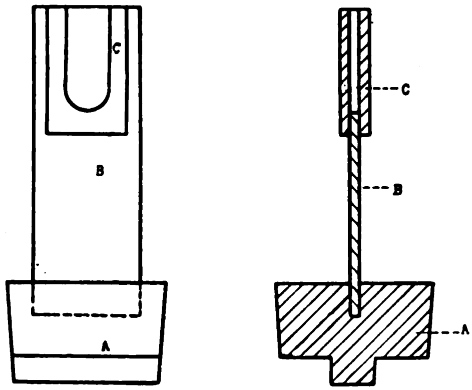

FIG. 1

The liquids of the eyes were put in a small quartz cell for examination. An oblong cut with a semicircular bottom was made in one end of a cover glass, see Fig. 1. Quartz plates of 2½ mm thickness were fastened on either side of this with sealing wax. A small container was thus formed with 1½ mm thickness, 1 cm width, 2½ cm depth. This was very satisfactory as four or five drops could be examined.

The animal eyes used (specimen numbers 1–6) were those of cows, pigs, and sheep. These were mostly secured while they were yet warm, dissected and used the same day. Some were put on ice and dissected and used some four or five days later.

Specimen No. 7 of the human material was secured from an enucleation operation. It was the eye of a man aged twenty

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years who had been struck in the eye two years previously with a pen point. The puncture seemed to be completely healed as there was no mark on the cornea which appeared to be normal. The aqueous humor also seemed normal. On dissecting the eye the lens was found to be completely calcified. The choroid had been nearly all absorbed and in its place was a layer of salt. The vitreous humor was very thin and resembled bloody water.

Specimen No. 8 was the lens taken out in a cataract operation. The cataract was mature.

Specimen No. 9 was a lens obtained from a cataract operation. This cataract was not so fully developed. Both these lenses were intact when received and both of the patients had the perception of light and darkness. The lenses appeared opaque in the center but seemed to transmit quite a good deal of light around the edges.

Specimen No. 10 was the lens of an infant of 4 months. The eye had been infected at birth. The lens only was used and it appeared perfectly normal. This view was corroborated by the physician and oculist in charge.

PROCEDURE

The method used in determining the amount of absorption was one of comparison. Plates were taken of the sparks of cadmium, zinc and tin. The wave-lengths on these were marked, see Fig. 2, by comparison with the photographs and data given by Eder and Valenta.[6] Cadmium was found to be most satisfactory for this purpose, as has been mentioned before, because the groupings of the lines are very characteristic and the spark can be easily maintained for a comparatively long time. Spectral photographs were then taken with the material in front of the slit and compared with the original photograph. In some cases the material only covered part of the slit so that a comparison spectra was given on the same plate.

The plates were examined with a small lens and the last line toward the ultra-violet end of the spectrum was taken as the last

[6] Atlas Typischer Spektren, J. M. Eder and E. Valenta.

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wave-length to be transmitted, or the beginning of the absorption band.

A cut was made in the edge of the cornea and the aqueous humor was removed. The cornea was then cut off and the lens taken out. Then the posterior of the eye was cut into and a small amount of the clear vitreous removed.

The cornea was put in the holder mentioned above and placed before the slit of the spectroscope. Exposures of varying time were made in order to determine the amount of absorption.

The lens was placed in the cardboard screen and the light from the spark focused by means of it on the slit.

The humors were placed in the quartz cell and put directly in front of the slit and very close to it.

DATA AND RESULTS

The data obtained are given in Tables 1 to 4.

TABLE 1
Cornea

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
1	Cow	13	1½ min.	In formalin	3251
1	"	14	3 "	" "	3251
2	"	20	1 "	Fresh	3134
5	Pig	29	1 "	4 days on ice	3251
7	Human	36	30 seconds	Fresh	3066
7	"	37	45 "	"	2981
7	"	38	60 "	"	3066
7	"	39	1½ min.	"	2981
7	"	40	2 "	"	2981
7	"	41	25 "	"	2981

Seed dry plates number 26 were used except for spectrograms 15 and 16 for which Seed Process plates were employed. The lines were often very faint and at times it was hard to tell just exactly where the last line was located. The last wave-length toward the ultra-violet as given in the last column of the data is only a close approximation since when partial absorption has set in only the strong lines of the source show. Also it might be possible for the transmission to continue for several wave-lengths further and not

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TABLE 2
Crystalline Lens

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
1	Cow	15	3 min.	In formalin	3613
1	"	16	3 min.	" "	3613
2	"	19	15 sec.	Fresh	3251
2	"	18	30 sec.	"	3251
3	sheep	21	45 sec.	Kept 3 hours	3251 (very faint)
3	"	22	60 sec.	Fresh	3134
3	"	23	18 min.	"	3134
3	"	24	45 min.	"	3134
10	Human infant	50	1 min.	Fresh	3134
10	"	49	20 "	"	3134
8	Human	44	25 "	"	4415
9	"	45	25 "	"	3404
6	Pig	33	45 sec.	5 days on ice	3251 Not very definite
6	"	34	45 "	"	3134
6	"	35	20 min.	"	3134

TABLE 3
Aqueous Humor

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
4	Sheep	27	15 sec.	Fresh—kept 3 hours	2145 (very faint)
4	"	26	30 "	"	2145
4	"	25	45 "	"	2145
4	"	28	60 "	"	2145
7	Human	42	20 "		2573
7	"	43	40 "		2573

TABLE 4
Vitreous Humor

No. of Specimen	Kind	No. of Plate	Time of Exposure	Condition of Specimen	Last Wave-length
5	Pig	31	15 sec.	4 days on ice	2265
5	"	30	30 "	"	2313
5	"	32	1 min.	"	2265

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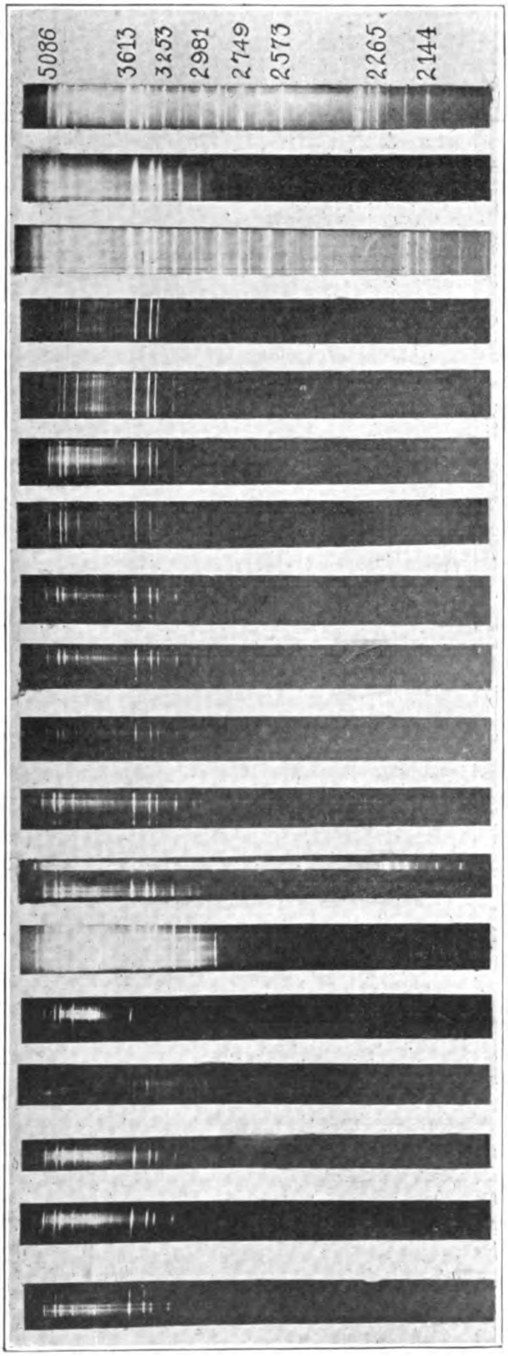

1. Cadmium

2. Cd. Cover glass

3. Cd. Water

13. Cd. Cornea (cow) 1.5 min. in formalin

14. Cd. Cornea (cow) 3 min. in formalin

20. Cd. Cornea (cow) 1 min. Fresh

29. Cd. Cornea (pig) 1 min. 4 days on ice.

36. Cd. Cornea (human) 30 sec. Fresh

37. Cd. Cornea (human) 45 sec. Fresh

38. Cd. Cornea (human) 60 sec. Fresh

39. Cd. Cornea (human) 1.5 min. Fresh

40. Cd. Cornea (human) 2 min. Fresh

41. Cd. Cornea (human) 25 min. Fresh

15. Cd. Lens (cow) 3 min. in formalin

16. Cd. Lens (cow) 3 min. in formalin

19. Cd. Lens (cow) 15 sec. Fresh

18. Cd. Lens (cow) 30 sec. Fresh

21. Cd. Lens (sheep) 45 sec. kept 3 hours

FIG. 2

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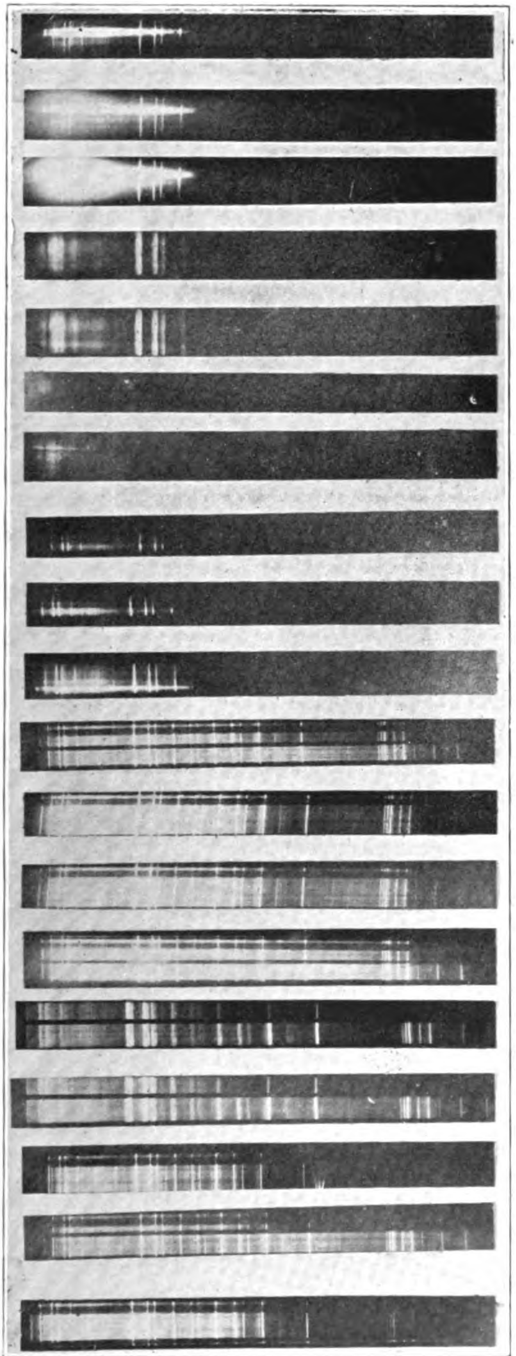

22. Cd. Lens (sheep) 1 min. Fresh

23. Cd. Lens (sheep) 18 min. Fresh

24. Cd. Lens (sheep) 45 min. Fresh

50. Cd. Lens (human infant) 1 min. Fresh

49. Cd. Lens (human infant) 20 min. Fresh

44. Cd. Lens (human) 25 min.

45. Cd. Lens (human) 25 min.

33. Cd. Lens (pig) 45 sec. Kept 5 days on ice.

34. Cd. Lens (pig) 45 sec. Kept 5 days on ice.

35. Cd. Lens (pig) 20 min. Kept 5 days on ice.

27. Cd. Aqueous Humor (sheep) 15 sec.

26. Cd. Aqueous Humor (sheep) 30 sec.

25. Cd. Aqueous Humor (sheep) 45 sec.

28. Cd. Aqueous Humor (sheep) 1 min.

42. Cd. Aqueous Humor (human) 20 sec. Fresh

43. Cd. Aqueous Humor (human) 40 sec. Fresh

31. Cd. Vitreous Humor (pig) 15 sec. Kept 4 days on ice.

30. Cd. Vitreous Humor (pig) 30 sec. Kept 4 days on ice.

32. Cd. Vitreous Humor (pig) 1 min. Kept 4 days on ice.

FIG. 3

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show on the plate if there was no line in the cadmium spectrum where the absorption began.

It can be seen from the data that most of the absorption bands begin rather abruptly, that is the region of partial absorption is not very great, as long exposures did not bring out many additional lines.

Although the aqueous humor of specimen 7 seemed to be normal, it is apparent from Table 3 that it was not. It seems very probable to suppose that it had absorbed additional salt as well as the posterior part of the eye and the lens. The cornea of specimen 7 (Table 1) seems to have been normal, or at least nearly so.

The following conclusions may be drawn from the foregoing data:

1. The combined tissues of the eye absorb the ultra-violet radiations up the neighborhood of $.3134\mu$.
2. The lens has the largest region of absorption.
3. Formalin changes the absorption.
4. Any injury or disease tending to increase the salt content in the eye radically changes the absorption.

Further, it might seem reasonable to suppose, although the data here given are scarcely definite enough to say conclusively, that the absorption in the animals' eyes does not differ radically from that in the human.

In conclusion, I wish to thank Dr. R. S. Minor for his help; Drs. L. D. and A. S. Green for kindly supplying me with material; and all others who made this work possible.

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INSTRUMENT SECTION

A NEW PRINCIPLE AND ITS APPLICATION TO THE LUMMER-BRODHUN PHOTOMETER

BY

E. P. HYDE and F. E. CADY

With the development of different types of photometers from the 18th Century "shadow" form of Rumford through the Ritchie "wedge" and the Leeson-disk improvement of the Bunsen "grease spot," photometric accuracy increased until it reached what appeared to be the maximum in Lummer and Brodhun's contrast type of cube. This was some thirty years ago and the instrument is today quite generally accepted as the one capable of giving the best results where the highest accuracy is desired with the possible exception of those cases involving decided color differences. The secret of the sensitivity lies in the ability of the eye to detect small differences in contrast. While the principle and design of the instrument are well known to those engaged in regular photometric work it will be described for the benefit of others and in order to make the modifications more clear.

The cube, Fig. 1, is made of two triangular glass prisms *ABC* and *ADC* cemented together at their hypotenuse faces *AC* with canada balsam. At *Aa*, *bc*, *de*, the surface of one cube is cut away so that light from the right is reflected in the direction of sight, while light from the left is reflected out of the field of view. As a consequence the observer on looking in the eyepiece of the instrument sees a figure made up of two adjacent semicircles each containing a trapezoid.[1] In the position of balance or when the illuminations on the two sides of the photometer disk are equal, the two trapezoids stand out with equal sharpness from their surrounding backgrounds. If the photometer head is moved and thus thrown out of balance one of the trapezoids will tend to get darker and the other will tend to approach in brightness that of the surrounding background. A movement of the photometer

[1] Instruments made in this country use a segment of a circle in the place of the trapezoid.

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head in the opposite direction will reverse this condition and make the other trapezoid less distinct. In the position sought for, the contrast between each trapezoid and the surrounding background is the same. This contrast results from the presence at *EA* and *FD* of two plain pieces of clear glass which by reason of surface reflection decrease the transmitted light by about 8 per cent.

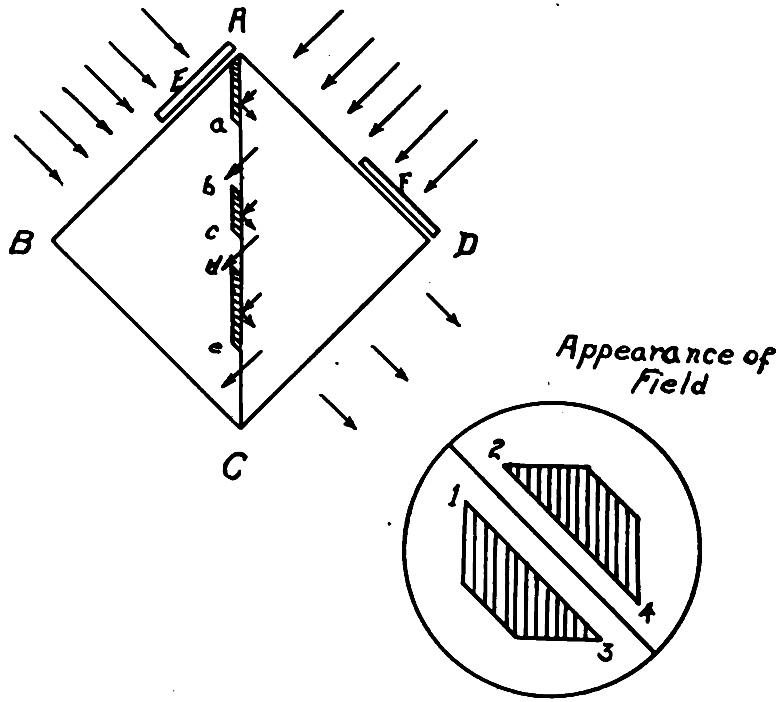

FIG. 1. Schematic diagram of Lummer and Brodhun Photometric Cube, Contrast Type.

These pieces may be removed if it is desired to dispense with the contrast principle and use the cube for a simple equality-of-brightness or match photometer.

It was while studying the Brace spectrophotometer that there occurred to one of the authors the possibility of increasing the accuracy of photometric settings by the introduction of a new principle. It may be recalled that in one form of the Brace spectrophotometer the photometric field is made up of three parts,

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Fig. 2, the center receiving light from one source while the upper and lower portions receive it from the other source. In the condition of balance the field "*B*," which receives its light from the source *S* is of the same apparent brightness as the two fields "*A*" and "*C*" receiving light from the source *R* and which are of the same actual brightness if the instrument is in adjustment. If it is assumed that the eye can just detect a difference in brightness of 1% in a field of view of this kind, it is evident that there would be an apparent balance for all values of brightness of field "*B*" lying within $\pm$ 1% of the brightness of "*A*" and "*C*." That is, the range of a setting would be $\pm$1%, or, in total, 2%.

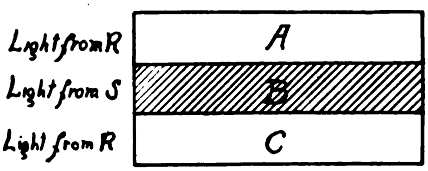

FIG. 2. Field of view in Brace Spectrophotometer

Suppose, now, that by some suitable optical means the brightness of field "*C*" were made definitely 1% less than the brightness of field "*A*." In a condition of balance the brightness of "*B*" obviously would now lie between the brightness of "*A*" and the brightness of "*C*" which has been made 1% less than the brightness of "*A*." For when "*B*" is in actual match with "*A*" it is, by hypothesis, 1% brighter than "*C*," and the difference in brightness between "*B*" and "*C*" would be detected. Similarly, where "*B*" is in actual match with the modified "*C*" it is 1% less bright than "*A*," and again the difference would be detected. Hence, with this modified field the range of error would be only 1% instead of 2% as in the original arrangement.

It is true that the center of balance is shifted from the brightness of "*A*" to a value ½% less, but since for other obvious reasons the substitution method of measurement would always be employed this shift of ½% offers no difficulty. It should be noted that this principle was subsequently used and described by

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Pfund[2] in an article on a new photometer involving the use of mirrors.

Although this principle was thought of in connection with the Brace spectrophotometer it is of general application, and was first actually tried out in connection with the Lummer-Brodhun contrast photometer. Since with this instrument the condition of balance consists not in brightness equality but in equal brightness contrasts the application of the principle involves the establishment of unequal contrasts in different parts of the field. Thus if the contrast of the field 1–3 (Fig. 1) is made to decrease from 1 to 3, and that of the field 2–4 to increase from 2 to 4, by some chosen small amount, a resultant total field of view is obtained such that the range of balance is greatly reduced, as is apparent by a similar process of reasoning to that presented in the first case.

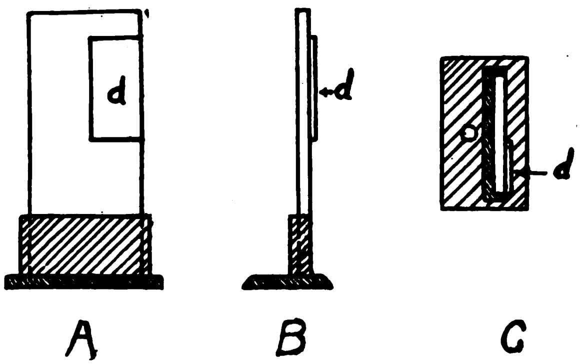

FIG. 3

A, B, C front, side and top view of enlarged strips showing attachment of small absorption piece, *d*

The practical difficulty of applying this principle to obtain a graduated contrast lies in the fact that the glass strips "*E*" and "*F*" absorb by reflection and any variation in thickness produces only negligible changes in the absorption and therefore in the contrast. This difficulty was overcome by enlarging the clear glass strips to cover the entire faces of the cube, and then attaching to these by canada balsam other strips of the same size as the original strips "*E*" and "*F*," but made of smoked glass, which would absorb in proportion to their thickness, see Fig. 3. By

[2] *Phys. Review 4*, p. 477, 1914.

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making these strips wedge-shaped, a graded absorption and therefore a graded contrast could be obtained.

Moreover, by accomplishing the contrast in this way it is possible to make the average absorption, and, therefore, the average contrast any amount that might be desired, instead of having it determined by the reflection of clear glass strips which give a fixed absorption of about 8%. This led to another increase in sensibility. Lummer and Brodhun had found by experiment that 3½% contrast gave the most sensitive photometer, but they apparently thought of no easy way of accomplishing this, and satisfied themselves with the 8% contrast obtained by the clear glass strips. With the introduction of the scheme described above for getting graded contrast it was a simple matter to choose a density and thickness of smoked glass that would yield an average absorption of 3½% with the added feature of a properly chosen graded contrast.

Messrs. Franz Schmidt and Haensch of Berlin undertook to construct for us a photometer involving these two features, and delivered the instrument to us over ten years ago. Just what gradation in contrast was aimed at the authors have now forgotten. At that time some measurements were made using a photometric method to determine whether any gradation was present, but none was found. However, recent measurements using a much more sensitive method gave a difference in one direction of 0.7 per cent in one piece and in the opposite direction of 0.2 per cent in the other piece indicating an effort to have them comply with the principle. At the time of the latter measurements, readings were taken on six other pairs of low contrast strips, but in all these cases whatever variation was found was in the same direction from top to bottom or bottom to top on both strips and so would not show the application of the principle referred to. The absorption of these strips was such as to give a contrast of about 5 per cent. In spite of the lack of opposed variation in absorption, the instruments using these strips have shown a very much greater sensitivity than the ordinary type, doubtless due to the decreased contrast.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
APRIL, 1922.

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AN ELECTROMAGNETIC METHOD OF DETECTING MINUTE IRREGULARITIES IN CURVATURE OF SPHERES AND CYLINDERS AND OF CONTROLLING THE OSCILLATIONS OF A MASS OF METAL SUSPENDED BY MEANS OF A TORSION FIBRE

BY
ALEXANDER MARCUS

It is well known that a single unidirectional alternating magnetic field is equivalent to two constant fields revolving with equal angular velocities in opposite directions and it is commonly believed that a coil of wire or a mass of metal placed in a uniform alternating field so that it can rotate will remain at rest if it happens to be at rest. The reason for this is that the two oppositely revolving fields which may be considered equivalent to the single impressed alternating field, induce two[1] symmetrical polyphase systems of currents of the same frequency and amplitude. Therefore, the mechanical reactions between these currents and their corresponding fields are equal and opposite and the body experiences no resultant torque. If the body be given a start in either direction the opposite torques will no longer balance. For, suppose the body be given an angular velocity ω, and the angular velocities of the fields are p, and $-p$ respectively, then relative to the body one of the fields

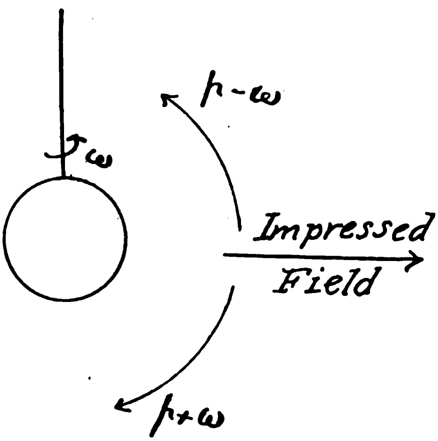

FIG. 1

[1] Steinmetz: A. C. Phenomena, 4th Ed., pp. 639–642.

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will revolve with angular velocity p-ω and the other with angular velocity $p+\omega$. The two systems of induced currents now have frequencies proportional to $p-\omega$ and $p+\omega$ respectively and have different amplitudes. Therefore the mechanical reactions between the currents and the fields inducing them will no longer be equal and opposite. This is in essence the theory of the single-phase induction motor and explains why such a motor is not self-starting.

However, the writer has recently found in the course of an experiment with a copper ball suspended by means of a torsion fibre perpendicular to a uniform alternating magnetic field, that the ball will start to rotate even if it be perfectly stationary when the field is thrown on. It seemed to act like a self-starting induction motor, and inasmuch as according to well established electrodynamic theory such a motor cannot be self-starting, it became a matter of importance from theoretical as well as practical considerations to explain the existence of the extra torque. It was apparent that the starting torque was not an effect of the free electromagnetic oscillations produced by the sudden application of the field, for, as long as steady conditions in the field were maintained, the ball continued to oscillate about a definite new zero position. On repeating the experiment with both solid and hollow spheres and cylinders of different non-magnetic materials, it was found that the extra torque varied directly as the intensity of the "skin effect" for the different samples. Owing to the fact that all the spheres and cylinders had highly uniform curvature, it seemed difficult to attribute the effect to irregularities in curvature. Nevertheless this was assumed as a working hypothesis and two identical hollow circular cylinders were suspended alternately in the field. The observed deflections from the normal zero position were nearly equal for the two samples. When one of the samples was compressed so as to have an approximately elliptic cross-section, with the long axis about twice as long as the short one it gave a deflection many times larger than before distortion. Furthermore, the torque was always in such a direction as to tend to bring the long axis into coincidence with that of the field. A bronze ball-bearing was then suspended in the field and no starting torque was detected.

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On linking together the two facts, first, that the torque on the asymmetric body increased with the "skin effect" and, second, that the torque always tended to make the body set its long axis in the direction of the uniform field the writer came to the conclusion that his discovery was only a new illustration of a fact already established but not generally known, namely:—

A conductor placed in a variable magnetic field tends to behave like a diamagnetic substance both in the way it disturbs the field and in the way it tends to move under the influence of that field.[2] The reason for this is that as the frequency of the impressed field rises the induced currents and magnetic fields tend to become concentrated in a thin layer near the surface and consequently the space within the body is traversed by a weaker field than before the introduction of the conductor. The higher the frequency the weaker will be the internal field. Indeed even in the case of iron the penetration of the field and currents may be so small at high frequency that the total flux through the space occupied by the body may be less than what it would be, were the iron removed. Dr. Louis Cohen[3] has proved that in an iron cylinder of one centimeter radius subjected to an alternating magnetizing force having a frequency of a million cycles per second the total flux through the iron is only seven-tenths as much as the flux through that space before the introduction of the iron. It should also be remembered that whenever any body of a given permeability is immersed in a medium of a different permeability it will tend to move toward the region of lowest potential.[4] For this reason a diamagnetic substance in a uniform magnetic field tends to turn its longest dimension in the direction of the field in contradiction to the statement frequently made that a diamagnetic substance turns its longest dimension across the field.

This diamagnetic behavior of a conductor in an alternating field has two practical applications. First, it affords a very sensitive method of detecting irregularities in curvature in metal spheres and cylinders and secondly, may serve as a means of

[2] J. J. Thomson: Recent Researches in Electricity & Magnetism, pp. 556–7.
[3] Calculation of Alternating Current Problems, p. 38.
[4] Poynting & Thomson: Electricity & Magnetism, pp. 257–259.

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starting, stopping, and of controlling the oscillations of a mass of metal in experiments involving torsional oscillations of such a mass. In both of these applications a pair of Helmholtz coils will be found convenient for the production of a uniform field. The ball should be suspended midway between the coils. The mean distance between the centers of the latter should be about equal to the radius of the coils. If it be desired to test the ball for irregularities in curvature suspend it like a galvanometer coil with a small mirror attached to the suspension fibre for the purpose of observing the deflections of the ball. Before applying the field let the ball come to perfect rest and note the position on a scale of the reflected beam of light. Throw on a field of ordinary frequency and determine the new zero about which the oscillations occur. The sensitivity of the method depends upon the magnitude of the "skin effect" for different materials and hence varies directly as the square root of the frequency of the impressed field and inversely as the square root of the resistivity of the material. In this case as with galvanometers the sensitivity depends also upon the stiffness of the suspension fibre and upon the distance of the scale from the mirror. Since the torque on the ball depends upon the reaction between the impressed field and the currents induced by that field, it will vary directly as the square of the amplitude of the field.

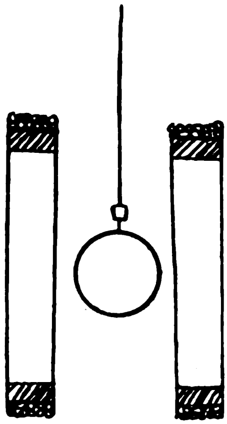

FIG. 2

The writer had a mass of pure copper cast into the shape of a ball about two inches in diameter. It was then turned down and polished to be as nearly spherical as possible. Measurements with a vernier caliper showed deviations of any single diameter measurement from the mean, of about one-half of one per cent. On suspending the ball in a field alternating with a frequency of sixty cycles per second and having an effective intensity of about thirty gausses the torque due to the diamagnetic behavior of the

ball gave a deflection of several centimeters on a scale about a meter from the mirror on the suspension.

The second practical application of the diamagnetic behavior of a conductor in an alternating field is to the control of the oscillations of a torsion pendulum. There are some electrodynamic experiments and others of a purely mechanical nature based on the use of the torsion pendulum and require that the motion shall be purely torsional. For example, from the decrement of the oscillations of a mass of metal vibrating as a torsion pendulum in a constant field, it is easy to compute the resistivity of the material. Or one may wish to determine the coefficients of viscosity and of rigidity of a substance in the form of a fibre by the method of the torsion pendulum. In such experiments, mechanical methods of starting the oscillations are quite apt to produce a variety of undesirable modes of vibration. Moreover, in order to avoid troubles due to tremors of the building, it is often necessary to mount the apparatus in places not readily accessible. The magnetic control of the oscillations makes it possible to start the motion and to establish any amplitude by the mere closing of a switch. The only extra apparatus required is a pair of field coils, the metal being placed midway between them. The increase or decrease in amplitude is produced by closing the switch when the motion is either in the same direction as the torque due to the field or in the opposite direction.

The writer wishes to acknowledge his indebtedness to Mr. Robert Dressler, Mechanician of the Department of Physics of the College of the City of New York for his valuable assistance in winding the field coils and in preparing some samples of metal for the experiments.

COLLEGE OF THE CITY OF NEW YORK

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AN IMPROVED FORM OF NICHOLS RADIOMETER

BY

B. J. Spence

Abstract. **The improvement over the customary forms of the Nichols radiometer consists in the reduction of the moment of inertia of the rotating system to such a value that the system has a period of eight seconds and a sensitivity equal to that of the thermopile or bolometer under their best working conditions. A comparison of the thermopile and the radiometer is given.**

Numerous articles have been written dealing with the descriptions and relative merits of the radiometer, thermopile, and bolometer. Investigators who have used these instruments are familiar with the difficulties in the way of zero drift and fluctuations when the bolometer or thermopile is used in conjunction with the sensitive Thomson astatic galvanometer. The system of the astatic galvanometer is of small mass and moment of inertia. It is suspended by a fine quartz fibre in a relatively strong non uniform control field. Slight building vibrations or tremors cause a displacement of the system to other positions in the field of different intensity and directions giving rise to an amplification of these disturbances.

In the course of a study of some infra-red absorption spectra, using a grating of relatively large dispersion and resolving power and small energy of the radiating source, it was found necessary to abandon the thermopile and bolometer and attempt to develop a radiometer of the Nichols type which was free from the usual objection of the long period possessed by such an instrument.

The Nichols[1] radiometer as customarily constructed consists of a pair of mica vanes of comparatively large dimensions and mass fastened by fine glass rods to a glass staff with a mirror attached for purposes of observation. The system is suspended by a fine quartz fibre in a container pumped out to a pressure of approximately .02 mm Hg. Such a system has a large moment of inertia and when suspended by a sufficiently fine fibre to give a sensitivity of the order of magnitude similar to that of the bolometer,

[1] **Ann. der Physik, *60*, p. 402; 1897.**

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the period in some cases is 60 seconds or more. This constitutes the main objection to its use in spite of the remarkable stability and freedom from zero drift.

Accordingly, it seemed feasible to attempt to reduce the moment of inertia of the system and thus its period. To this end two strips of phosphor bronze ribbon .13 mm wide, .018 mm thick and 15 mm long were used for vanes. These strips were laid parallel about 4 mm apart and between them a finely drawn glass staff 40 mm long. The strips and glass staff were held together by finely drawn glass cross pieces by means of the smallest amount of shellac. To one end, which we shall designate as the tower end, was fastened a plane mirror of about 1 mm^2. One face of each strip was blackened with lamp black and alcohol containing a trace of shellac to cause the lamp black to adhere to the strips. The system was suspended from its upper end by means of a quartz fibre in a thick walled iron chamber cylindrical in shape and then pumped out to a pressure of approximately .02 mm Hg. The iron chamber, Fig. 1, mounted on a base with leveling screws had a bore of 25 mm, with 10 mm wall thickness and a length of 20 cm. Near the bottom and opposite each other were cut two windows. The one covered with quartz or rock salt was 3 mm wide and 20 mm long, the other for observation purposes and covered with glass was 5 mm wide and 35 mm long. The top of the chamber was provided with a ground iron plug and mercury seal. The plug served to suspend the system and by rotation to adjust the zero of the instrument. Near the top projected a tube carrying a carefully ground glass stop cock for evacuation purposes. The thick walled iron chamber was used to smooth out possible temperature fluctuations. As an extra precaution against temperature fluctuations the chamber was covered with a layer of felt 10 mm thick. The system thus protected was free from zero drift. It could be operated for a period of hours without a drift of more than 1 mm with a scale at 2.5 meters distant. In addition, it was free from fluctuations due to building disturbances.

It is difficult to make a rigorous comparison of the sensitivity of the radiometer with the bolometer or thermopile. It is cus-

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tomary to state the sensitivity of those instruments in terms of the deflection produced by a candle at a meter distance and scale 1 meter distant. Incorrect conclusions may be drawn from such a rating. It does not indicate clearly the working sensitivity. Drift and fluctuations are not usually considered. The thing most desired in infra-red spectroscopy along with sensitivity is reliability of deflection.

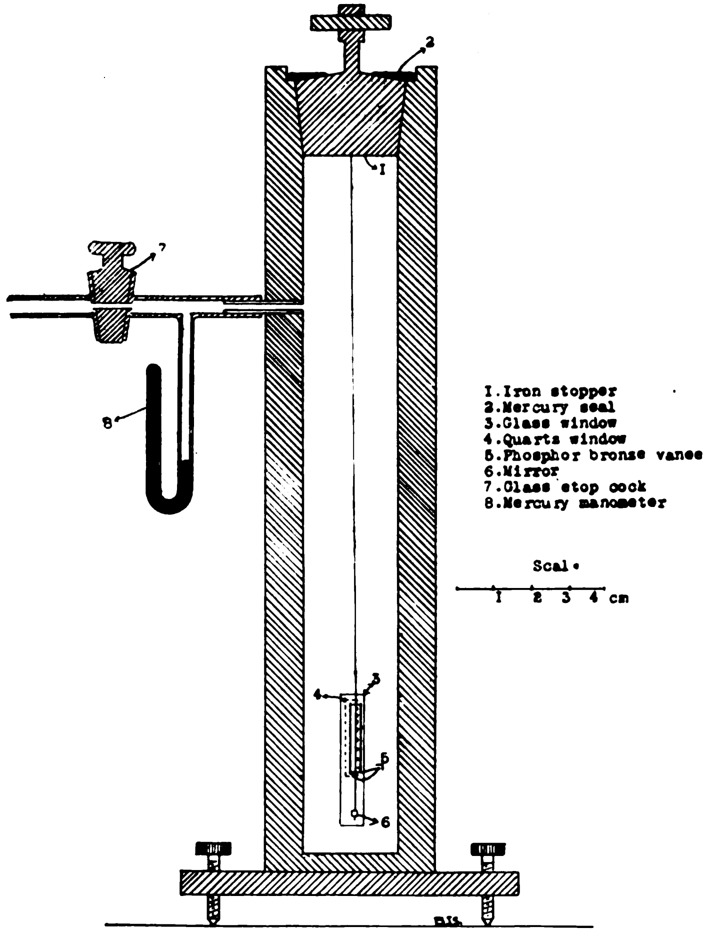

FIG. 1

A comparison was made of a bismuth-antimony bismuth-tin thermopile of 20 receiving junctions and 6 mm^2 receiving surface,

in series with a low resistance astatic galvanometer of 3×10^{-10} amperes sensibility, with a salt window radiometer of 2 mm^2 receiving surface and a period of six seconds. The comparison was made by subjecting the instruments in turn to the region of the spectrum near the sodium lines. The spectrum was produced by passing the radiation from a Nernst glower through an infra-red spectrometer. Deflections of approximately 50 mm were obtained in both cases. The radiometer deflection was free from zero drift and fluctuation.

In studying transmission spectra, it is customary to return to the zero reading after each deflection but with no zero drift this procedure is not necessary and the objection to the period of eight seconds as compared to a period of 1.5 seconds for the thermopile galvanometer combination is not serious.

In conclusion it may be stated that a radiometer of sensitivity equal to that of the thermopile or bolometer and a period of eight seconds has been developed. The instrument is free from zero drift and fluctuations that would ordinarily prohibit the use of the bolometer or thermopile. It is recognized that such an instrument has not been developed to obtain quantitative measurement of the small amounts of energy such as may be done with the bolometer.

The radiometer was developed in the course of an investigation for which funds were appropriated by the Rumford Committee of the American Academy of Arts and Sciences. Acknowledgment is hereby made of the courtesy of the Rumford Committee for the appropriation.

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A PORTABLE SEISMOMETER

BY

P. G. Nutting

Vibrations in buildings due to nearby machinery or traffic constitute today a serious engineering problem. Troublesome noises are usually readily traceable to their sources through their pitch. Jars and vibrations are far more difficult to trace. Laboratory forms of seismographs, used in recording earthquake tremors, are of little service because of lack of portability. Stethoscopes and other forms of delicate sound detectors are ineffective in observing vibrations of slow period. The portable seismometer here described was designed for engineering purposes. It weighs but a few pounds and can safely withstand rather rough handling, yet is of ample sensibility for detecting and measuring all ordinary tremors.

The amplitudes to be measured lie in the range from 0.001 mm to 0.1 mm, the former being just perceptible to the hand, the latter being decidedly annoying. Preliminary determinations of amplitudes were made with a very simple form of seismometer assembled in a few minutes from two paper weights, two pieces of drill rod, a galvanometer mirror and a pocket flash lamp. Two strips of plate glass about 2 x 6 inches would serve quite as well as the two paper weights. The lower plate is placed upon the table or desk whose vibration is to be observed. Upon this plate are laid the two pieces of drill rod, 2 or 3 inches long, parallel with each other; then the upper plate placed face down on these as shown in Fig. 1.

A half inch galvanometer mirror is attached to the projecting end of one piece of rod, throwing on a wall the image of the filament of a flash lamp. The two rollers of drill rod or wire must of course be straight, round, and free from burrs. About 1 mm is a suitable diameter. Soft wire may be used if carefully rolled straight. Some drill rod requires grinding to give it a sufficiently circular section.

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In operation the upper plate remains stationary while the lower moves with its support. The angular displacement of the reflected light beam is of course twice that of the roller carrying the mirror. Hence the relative linear displacement of light and test plate is in the same ratio as distance of light spot to radius of roller. A vibration of 0.005 mm (easily perceptible to the hand) will give an angular displacement of 0.01 to a roller of 0.5 mm radius and therefore will cause a displacement of 0.2 foot in a spot of light reflected 10 feet away. With rollers 0.2 mm in diameter the sensibility is 5 times as great. Frequencies at least as high as 100 per second are readily observed. Three or four components in a vibration may easily be detected.

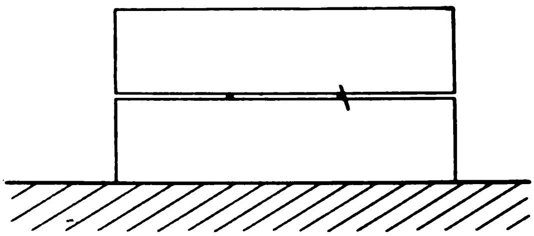

Fig. 1

Such a simple form of seismometer as that just described, although quite serviceable in horizontal positions, is quite useless for determining vertical components. The form of portable instrument finally developed is of very different design although essentially the same in principle (Fig. 2). A mass of lead supported by a diaphragm forms the stationary member while the case moves with the object with which it is in contact. Both rollers project and carry mirrors reflecting light from a miniature flash lamp upon an attached scale.

Fig. 2 shows the construction of the instrument in sectional diagram. The base is a hollow circular box 6 inches in diameter and 1 inch thick of light stiff material such as bakelite micarta. The interior is ½ by 5 inches diameter. Between the halves of this box is clamped a circular diaphragm of similar material about 1/32 inch in thickness, just sufficient to support the lead weight in zero position yet permit the housing to vibrate independently.

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The lead inertia weight, 20 x 35 mm in section and 6 inches long, is attached at one end by a thin nut to the center of this diaphragm by means of a brass rod passing longitudinally through it. At the head of the lead weight is a brass block of the same section milled out on either side to carry the glass plates forming the bearing surfaces for the two rollers. On either side of the lead weight are heavy L shaped uprights of heavy strap brass forming the stem of the seismometer. These are firmly screwed to the top of the circular base and are also recessed near the top for bearing surfaces of plate glass at the top; each is screwed to a milled top plate of

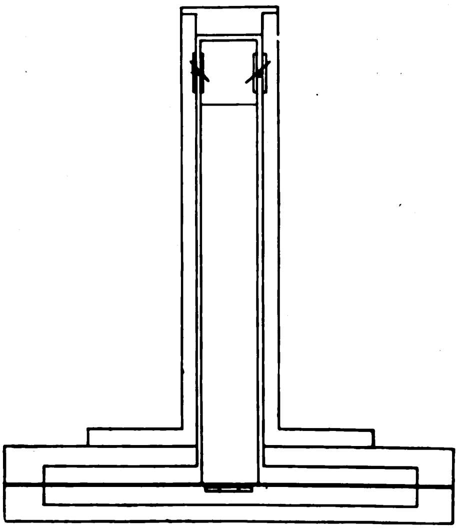

FIG. 2

heavy brass. An axial screw (not shown) through this top plate bearing against the top of the inertia bar serves to clamp its vibrations when the instrument is not in use.

The plate glass bearing surfaces must of course be set accurately parallel but if mounted in a fusible cement such as de Khotinsky's, their setting (before assembly) presents no great difficulty. Final adjustment of separation to the rollers is made by means of shims under the top ends of the brass side plates. When the adjustment is correct the rollers can just be slid into position with the fingers and will remain in place. The rollers may be quickly ad-

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justed to parallel horizontal position by means of a steel scale or try square.

Hard steel rod such as drill rod is preferable for this instrument. Such stock is usually not quite round and must be ground between hard flat surfaces in a fixed position. A pair of fine carborundum stones clamped together with a wedge shaped opening between serves very well. A rod diameter of about 1 mm is preferable. The mirrors used are carefully selected half inch plane galvanometer mirrors attached by means of soft wax (to avoid distortion) to a thin light metal plate which is in turn cemented to the roller with fusible cement.

The optical system (not shown in Fig. 2) is simple and readily attached to the top plate. The illuminator is a small pocket flash lamp having the bulls eye lens replaced by a corrected lens forming an image of the lamp filament about 6 inches distant on a printed scale attached at the rear of the top of the instrument. The upward beam after reflection from the second roller mirror is reflected horizontally across the top of the instrument by means of a fixed mirror mounted at an angle of 45°. The beam being twice reflected by rotating mirrors, the sensibility of the portable type of seismometer is of course double that of the simple form shown in Fig. 1.

This instrument registers deflections of from 2 to 40 mm for vibrations ordinarily met with. It is readily portable and has been extensively used by laymen without being damaged. Used on a vertical wall (held in the hand by the stem) it reads as well as on a horizontal table surface, the shift of zero position being of no consequence in reading amplitudes. A simple form of photographic registering attachment has been designed, but this and other refinements have been found superfluous in ordinary engineering observations.

This instrument was designed at the request of the Westinghouse Electric and Manufacturing Co., East Pittsburgh, for their own use and was developed in the Research Laboratory of that company during the winter of 1920–21.

Schenectady, N. Y.

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AN INSTRUMENT FOR THE GAMMA RAY MEASUREMENT OF THE RADIUM CONTENT OF WEAKLY ACTIVE MATERIALS

BY
N. Ernest Dorsey

For the determination of the radium content of a material of unknown composition the emanation method is the only one that can be considered as thoroughly satisfactory. This method, however, requires careful preliminary chemical treatment of the material, and consequently, is quite time consuming. On the contrary, a gamma ray measurement requires no chemical treatment whatever, is expeditious, and when satisfactory precision can be obtained, answers every purpose in all those cases in which it is known from other evidence that the material contains no radio-active material other than those belonging to the uranium-radium family.

The main difficulties encountered in the application of the gamma ray method to weakly active materials are (1) the securing of satisfactory sensitivity, (2) the determination of the correction for the volume distribution of the material, and (3) the determination of the correction for the absorption of the radiation by the material itself. If unlimited amounts of material are available, the first of these difficulties is not serious, and several modus operandi may be devised that will permit of a satisfactory determination of the two corrections mentioned. But the necessity of handling large amounts of material is in itself a disadvantage, and frequently only a small amount of material is available.

It was to meet these conditions that Walter Bothe[1] devised the double cylinder apparatus shown in Fig. 1. The material under test is placed in a thin walled test tube 2.6 cm in diameter, the latter is hermetically sealed and is placed in the cylinder B. The test tube is in all cases filled to a height of 12 cm. The cylinder B is of 3 mm brass covered externally with lead ½ mm thick.

[1] Physik, Zeits., 16 p. 33–36, 1915.

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It is 2.8 cm in internal diameter and is 16 cm long; it is carried by the leaf support, and the whole is insulated by the sulphur ball, S. In the figure, the radium standard is shown suspended in the center of B. The correction for the volume distribution is determined by a direct survey, a small tube of radium—essentially a point source of radiation—being placed successively at different points in B, and the corresponding rates of drift being determined. By a similar survey in which the radium tube is

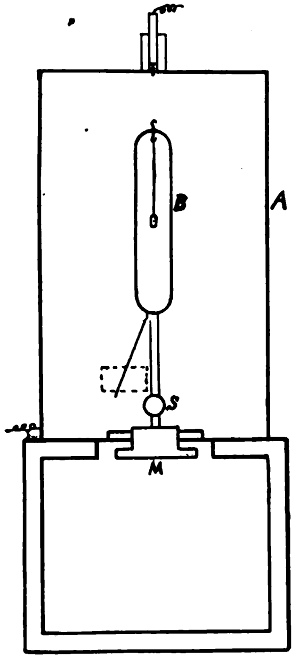

FIG. 1. Bothe's electroscope

buried at various points in fine sand contained in a test tube placed in B, the ratio of the resultant absorption of the radiation by the specimen to the absorption corresponding to the central point of B was determined. This, combined with the observed absorption by the material under test of the radiation from the small tube of radium buried in it so as to lie at the center of B, gives the correction for the absorption of the radiation by the material itself.

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For an instrument of this type, Bothe's instrument can scarcely be improved upon from an electrical standpoint, and in the hands of a skilled experimenter should give excellent results. But it has two annoying disadvantages that make it unsuitable for routine work. (1) The opening of the electroscope for every change in the specimen, with its attendant disturbance of the leaf, change in the air in the electroscope, and changes in the convection currents arising from slight changes in the temperature of *B*, keeps the observer doubtful of the confidence that can be placed in the results, and necessitates numerous repetitions. (2) Any slight escape of emanation from the specimen, either from an imperfect sealing of the tube or from a soiling of its exterior with the material, will produce a marked effect upon the observed rate of drift. Consequently in the determination of the correction for absorption, the tube has to be carefully sealed and cleaned at every step. Furthermore, throughout these manipulations care must be taken to prevent the introduction of radium emanation into the air of the room containing the instrument.

It was to obviate these disadvantages that the instrument shown schematically in Fig. 2 was devised. It may be described as a triple cylinder, or reentrant, ionization chamber attached to an electroscope. As in Bothe's instrument, the material under test is sealed in a test tube and placed in the cylinder *B*. The specimen can be introduced and removed readily without disturbing in any way either the insulated system or the air contained in the instrument. If the joints of the instrument are reasonably tight, a small amount of emanation in the air of the room will produce but a minimum effect, and this effect will change in a fairly regular manner and can be readily eliminated from the observations. Consequently, if the material is first aerated in another room, the tube containing it may remain open during the determination of the correction for the absorption of the radiation by the material itself. This greatly facilitates the work. If the instrument were made strictly air tight, it would be unaffected by the presence of emanation in the air, and all the manipulations of the material could be carried out in the same room. These advantages have been secured at the expense of the sensitivity, the electrostatic

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capacity of this instrument being much greater than that of Bothe's. But experiments with a tentative instrument of this type show that even with its reduced sensitivity it has a wide field of practical use.

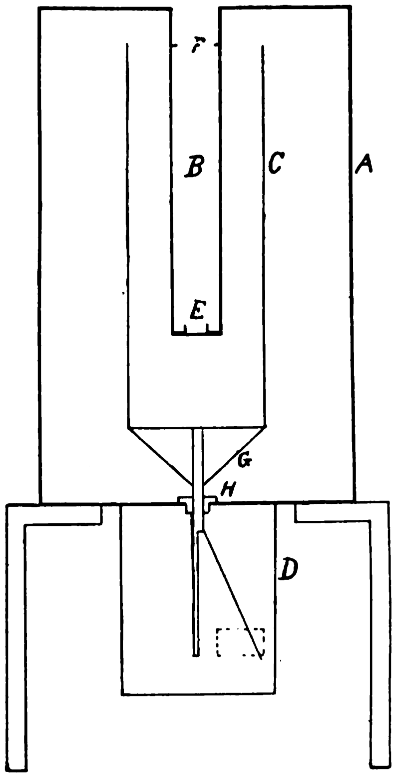

Fig. 2. Triple cylinder electroscope

In the tentative instrument, the cylinder A (Fig. 2) is 33 cm long and 21 cm in diameter; it is lined with lead 3 mm (1/8 in.) thick; lead of the same thickness is used in the construction of B, which is 21.5 cm long and 3.2 cm (1¼ inch) in internal diameter. *C* is a cylinder of aluminum 0.3 mm thick, it is 25.4 cm long, 9 cm in diameter, and is open at both ends; it is supported below by a spider which is carried by a brass rod passing through and supported by the sulphur casting, *H*. Guy wires, *G*, give the spider

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the necessary stiffness. The electroscope case, *D*, is of thin metal and is covered with felt; a heavy lead covered case would be preferable. The insulated system is charged by means of an insulated bell crank switch passing through the walls of *D* and not shown in the figure. In the bottom of *B* and coaxial with it is a pasteboard cylinder *E* about a centimeter long, and near the top of *B* is a pasteboard ring, *F*, 1 inch in internal diameter. These serve to center the tube of material and to lift it slightly above the bottom of *B*. The lead walls of *B* exert an excessive absorption upon the radiation from material situated extremely near them; consequently it is undesirable for the test tube containing the material to fit *B* snugly or to rest upon its bottom.

Corrections for the volume distribution of the material and for the absorption of the radiation by the material itself were determined in the general manner described by Bothe, except that the absorption surveys were made in the materials under test. As was to be expected, the transverse surveys show that while the curve showing the relative change in the effective absorption as the radium tube is moved along a radius is essentially the same for all sections throughout a relatively long central section of the column, it becomes much flatter as the ends of the column are approached. For these surveys, a small glass tube containing about 0.1 mg of radium was used.

The sensitivity of the instrument as used was about 2.8 divisions per second per milligram of radium. Specimens of radium concentrates (crude sulphates and carbonates) containing from 7 to 60 micrograms and small preparations containing as much as 300 micrograms of radium have been satisfactorily measured with it to a precision of at least one percent; and with considerable difficulty, duplicate specimens containing only 0.7 microgram were measured with a concordance of 2% and with a departure from the results of an emanation measurement of less than 10%. In one case, 4 samples of the same material dried to different extents were measured. One sample consisted of the material as received and contained over 50% of moisture; two contained about 7%; and one was dry. The actual radium contents of the specimens varied from 9 to 13 micrograms. The four determina-

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tions thus made of the radium content per kilo of the dry material covered an extreme range of only 0.7%.

Selected one-inch test tubes were used and these were always filled to a height of 6 inches; this amount of material weighed between 40 and 110 g, depending upon its nature. After the desired amount of material had been placed in a tube, a thin cork disc to which was attached a wire hook was pushed down firmly against the material and was covered with a thick layer of melted sealing wax. After the tube has been sealed, it may be put aside until the radiation has reached its maximum value before a gamma ray measurement is made, or the maximum value of the radiation may be determined, by the well known method of extrapolation, from a series of measurements made at suitable intervals. In the latter case, the graphical method[2] described by the author some years ago is very convenient.

Assuming that 70 g. of material is used, a 5 microgram specimen will correspond to material containing about 7×10^{-8} gm Ra per g of material. The tentative instrument can therefore be satisfactorily used for materials containing between 7×10^{-8} and 4×10^{-6} g Ra per g, and can, though with difficulty, be used with a precision of at least 10% for material containing only 7×10^{-9} g Ra per g. The last is a little less than the radium content of ore containing 3% U_3O_8.

Though the instrument is well suited to the determination of the radium content of concentrates, it is not suitable for work with low grade ores. For such work a more sensitive instrument, or one utilizing a larger amount of material is required.

WASHINGTON, D. C.

[2] Phys. Rev., (2), 14, p. 173, 1919.

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THE EFFECT OF A PHOTO-ELECTRIC MATERIAL ON THE THERMO-ELECTRIC CURRENT IN HIGH VACUUM AUDION BULBS

BY
THEODORE W. CASE

The author in his work on barium and strontium photo-electric reactions, which were first noted in Audion bulbs of oxide-coated filament type,[1] has observed peculiar effects of these light reactive substances on the thermo-electric current when the active coating is somewhere between the plate and filament. The active material can be used as a light reactive grid and made to control the thermo-electric current flowing between filament and plate. The outstanding feature of placing an electro-positive photo-electric material near the plate is to very greatly reduce the thermo-electric current to the plate when the photo-electric material is in comparative darkness.

In this latter condition it requires considerable voltage to get any thermo-electric current to the plate. If light is allowed to fall on the photo-electric material, the effect is to instantly allow the thermo-electric current to pass to the plate, and consequently large changes in the thermo-electric current may be easily induced by moderate changes in light intensity. The observed increase of thermo-electric current for low light intensities is not of the trigger action variety, but the maximum current obtainable is limited by the possible thermionic current which would obtain if the light active substance were not present in the bulb; therefore the current obtainable cannot be directly proportional to all light intensities.

It is quite easy to construct cells of this type by including an oxide-coated filament opposite a plate in a vacuum bottle and introducing potassium distilled from a glass side tube upon a grid, or upon the glass wall of the bulb if it be made tubular between the filament and plate. In operation, the voltage between the

[1] Paper before A. E. S., April 21, 1921.

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heated filament and plate (which latter is made positive) should be so adjusted that very little thermo-electric current flows when no light is on the potassium. Upon illuminating the potassium the increase of the thermo-electric current will be observed.

The author believes that this type of action warrants the trial of many substances, either conductors or non-conductors near the plate in the form of a grid, which may be thus studied for reaction to different types of radiation, including X-rays, with possibly very interesting results. Electro-negative elements should also be tried with a decrease of thermo-electric current looked for upon illumination.

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DIRECT CAPACITY MEASUREMENT

BY

GEORGE A. CAMPBELL

Synopsis: Direct capacity, direct admittance and direct impedance are defined as the branch constants of the particular direct network which is equivalent to any given electrical system. Typical methods of measuring these direct constants are described with especial reference to direct admittance; the substitution alternating current bridge method, due to Colpitts, is the preferred method, and for this suitable variable capacities and conductances are described, and shielding is recommended. Proposed methods are also described involving the introduction of electron tubes into the measuring set, which will reduce the measurement to a single setting or deflection. This gives an alternating current method which is comparable with Maxwell's single null-setting cyclical charge and discharge method. Special attention is drawn to Maxwell's remarkable method which is entirely ignored by at least most of the modern textbooks and handbooks.

The object of this paper is to emphasize the importance of direct capacity networks; to explain various methods of measuring direct capacities; and to advocate the use of the Colpitts substitution method which has been found preëminently satisfactory under the wide range of conditions arising in the communication field.

About thirty years ago telephone engineers substituted the so-called "mutual capacity" measurement for the established "grounded capacity" measurement; this was a distinct advance, since the transmission efficiency is more closely connected with mutual capacity than with grounded capacity. Mutual capacity, however, can give no information respecting crosstalk and accordingly, about twenty years ago, I introduced the measurement of "direct capacity" which enabled us to control crosstalk and to determine more completely how telephone circuits will behave under all possible connections.

For making these direct capacity measurements alternating currents of telephone frequencies were introduced so as to determine more exactly the effective value of the capacity in telephonic transmission, and to include the determination of the associated effective direct conductances which immediately assumed great importance upon the introduction of loading.

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Telephone cables and other parts of the telephone plant present the problem of measuring capacities which are quite impossible to isolate, but which must be measured, just as they occur, in association with other capacities; and these associated capacities may be much larger than the particular direct capacity which it is necessary to accurately measure, and have admittances overwhelmingly larger than the direct conductance, which is often the most important quantity. This is the interesting problem of direct capacity measurement, and distinguishes it from ordinary capacity measurements where isolation of the capacity is secured, or at least assumed.

The substitution alternating current bridge method, suggested to me in 1902 by Mr. E. H. Colpitts as a modification of the potentiometer method, has been in general use by us ever since in all cases where accuracy and ease of manipulation are essential.

After first defining direct capacities and describing various methods for measuring them, this paper will explain how this may all be generalized so as to include both the capacity and conductance components of direct admittances, and the inductance and resistance components of direct impedances.[1]

DEFINITION OF DIRECT CAPACITY

It is a familiar fact that two condensers of capacities C_1, C_2, when in parallel or in series, are equivalent to a single capacity (C_1+C_2) or $C_1C_2/(C_1+C_2)$, respectively, directly connecting the two terminals. These equivalent capacities it is proposed to call direct capacities. The rules for determining them may be stated in a form having general applicability, as follows:

Rule 1. The direct capacity which is equivalent to capacities in parallel is equal to their sum.

Rule 2. The direct capacity between two terminals, which is equivalent to two capacities connecting these terminals to a concealed branch-point, is equal to the product of the two capacities divided by the total capacity terminating at the concealed branch-point, i.e., its grounded capacity.

[1] Proofs of the mathematical results included in the present paper will be supplied in an appendix which will be added to the reprint of the paper which is to appear in an early number of the "Bell System Technical Journal."

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These rules may be used to determine the direct capacities of any network of condensers, with any number of accessible terminals and any number of concealed branch-points. Thus, all concealed branch-points may be initially considered to be accessible, and they are then eliminated one after another by applying these two rules; the final result is independent of the order in which the points are taken; all may, in fact, be eliminated simul-

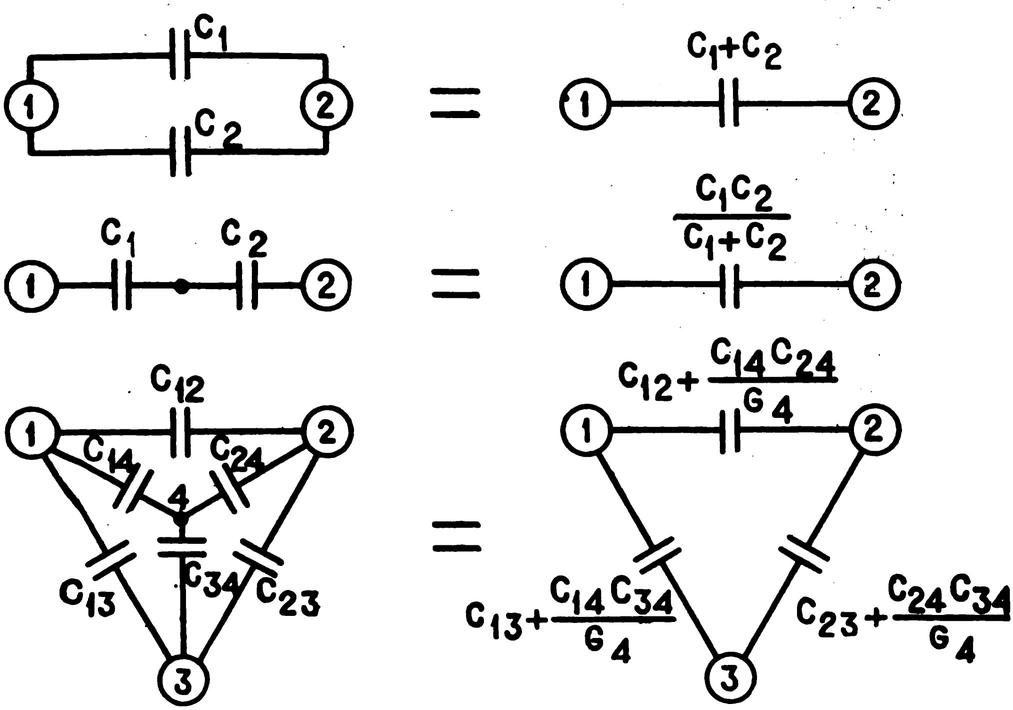

FIG. 1. Equivalent direct capacities. $G_4 = C_{14}+C_{24}+C_{34}$ = grounded capacity of branch-point 4

taneously by means of determinants;—a network of capacities, directly connecting the accessible terminals, without concealed branch-points or capacities in parallel, is the final result. Fig. 1 shows the two elementary cases of direct capacities and also, as an illustration of a more complicated system, the bridge circuit, with three corners 1, 2, 3 assumed to be accessible, and the fourth

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inaccessible, or concealed. Generalizing, we have the following definition:

The direct capacities of an electrical system with n given accessible terminals are defined as the $n(n-1)/2$ capacities which, connected between each pair of terminals, will be the exact equivalent of the system in its external reaction upon any other electrical system with which it is associated only by conductive connections through the accessible terminals. The total direct capacity between any group of the terminals and all of the remaining accessible terminals, connected together, is called the grounded capacity of the group.

This definition of direct capacity presents the complete set of direct capacities as constituting an exact, symmetrical, realizable physical substitute for the given electrical system for all purposes, including practical applications. Direct capacities are Maxwell's "coefficients of mutual induction," but with the sign reversed, their number being increased so as to include a direct capacity between each pair of terminals.

In considering direct capacities we exclude any direct coupling, either magnetic or electric, from without with the interior of the electrical system, since we have no concern with its internal structure; we are restricted to its accessible, peripheral points or terminals; some care has been taken to emphasize this in the wording of the definition.

ADDITIVE PROPERTY OF DIRECT CAPACITIES

Connecting a capacity between two terminals adds that capacity to the direct capacity between these terminals, and leaves all other direct capacities unchanged. Connecting the terminals of two distinct electrical systems, in pairs, gives a system in which each direct capacity is the sum of the corresponding two direct capacities in the individual systems. Joining two terminals of a single electrical system to form a single terminal adds together the two direct capacities from the two merged terminals to any third terminal, and leaves all other direct capacities unchanged with the exception of the direct capacity between the two merged terminals, which becomes a short circuit. Combining the termi-

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nals into any number of merged groups leaves the total direct capacity between any pair of groups unchanged, and short-circuits all direct capacities within each group.

These several statements of the additive property of direct capacities show the simple manner in which direct capacities are altered under some of the most important external operations which can be made with an electrical network, and explain, in part, the preëminent convenience of direct capacity networks.

Since the additive property of direct capacities is sufficient for explaining the different methods of measuring direct capacities we may now, without further general discussion of direct capacities, proceed to the description of the more important methods of measurement.

COLPITTS SUBSTITUTION BRIDGE METHOD, FIG. 2

The unknown direct capacity is shifted from one side of the bridge to the other, and the balance is restored by adjusting the capacity standard so as to shift back an equal amount of direct

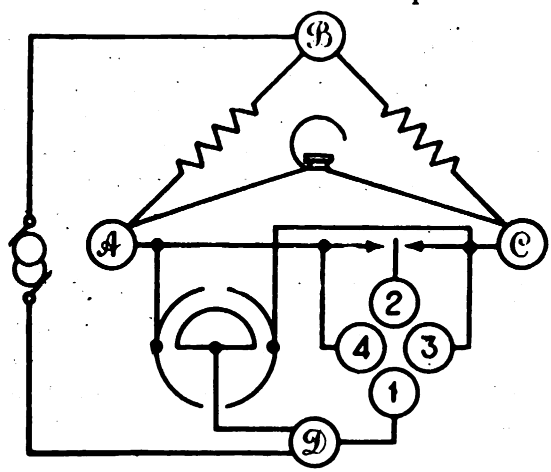

FIG. 2. Colpitts substitution bridge method for direct capacity

capacity. The method is therefore a substitution method, and the value of the bridge ratio is not involved. Both the standard and the unknown remain in the bridge for both settings, so that the method involves transposition rather than simple, ordinary substitution.

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Details of the method as shown by Fig. 2 are as follows: To measure the direct capacity C_{12} between terminals 1 and 2 connect one terminal (1) to corner D of the bridge, and adjust for a balance with the other terminal (2) on corner A and then on C, while each and every one of the remaining accessible terminals (3, 4, ...) of the electrical system is permanently connected during the two adjustments to either corner A or C. If the direct capacities in the standard condenser between corners A and D are C', C'' in the two balances,

$$C_{12}=C''-C',$$

and if the bridge ratio is unity,

$$C_{13}-C_{14}=C'+C''-2C^{\circ}$$

where C° is the standard condenser reading when the bridge alone is balanced.

Two settings are required by this method for an individual direct capacity measurement, but in the systematic measurement of all the direct capacities in a system the total number of settings tends to equal the total number of capacities, when this number becomes large. The number of settings may always be kept equal to the number of capacities by employing an equality bridge ratio, and using the expression for the direct capacity difference given above. The same remarks also hold for the group of direct capacities connecting any one terminal with all the other terminals.

In general, ground is placed upon corner C of the bridge, but is transferred to corner D, if it is connected to one terminal of the required direct capacity. The arbitrary distribution of the other terminals between corners A and C may be used to somewhat control the amount of standard capacity required; or it may be helpful in reducing interference from outside sources, when tests are made upon extended circuits. The grounded capacity of a terminal, or group of terminals, is measured by connecting the group to C, and all of the remaining terminals together to D.

The excess of one direct capacity C_{12} over another C_{56} is readily determined by connecting terminals 1 and 5 to corner D, terminals 3, 4, 7, 8, . . . to corner C or A, and then balance with

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terminals 2 and 6 on A and C, respectively, and repeat, with their connections reversed.

The required direct capacity C_{12} is balanced against one of its associated direct capacities, augmented by a standard direct capacity C', and the measurement is repeated with the required

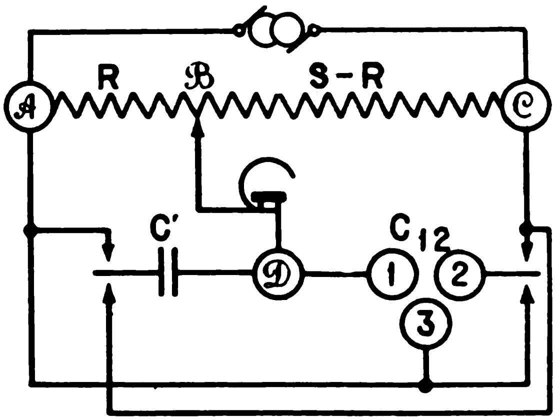

FIG. 3. Potentiometer method for direct capacity

POTENTIOMETER METHOD, FIG. 3

direct capacity and standard interchanged. Let R', R'' be the resistance required in arm AB of the bridge for the first and second balance, then, S being the total slide wire resistance and G_1 the grounded capacity of terminal 1:

$$C_{12} = \frac{R'}{R''},\ C'$$

$$G_1 = \frac{S-R''}{R''}\ C'$$

This ratio method requires for the bridge a variable or slide wire resistance and a constant condenser, and it may be employed as an improvised bridge, when sufficient variable capacity is not available for the Colpitts method. Not being a substitution method, however, greater precautions are necessary for accurate results. There must be no initial direct capacity in arm CD, or a correction will be required. Possibly variable capacity ratio arms would be preferable to resistances.

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NULL-IMPEDANCE BRIDGE METHOD FOR DIRECT CAPACITY, FIG. 4

Assuming that the electron tube supplies the means of obtaining an invariable true negative resistance, Fig. 4 shows a method which determines any individual direct capacity from a single bridge setting. The bridge arms are replaced by a Y network made up of two resistances R, R and a negative resistance $-R/2$; the Y has then a null-impedance between corner B and corners A, C connected together. The three terminals 1, 2, 3 of the network to be measured are connected to corners D, C, B and a balance obtained by adjusting the variable standard condenser C'.

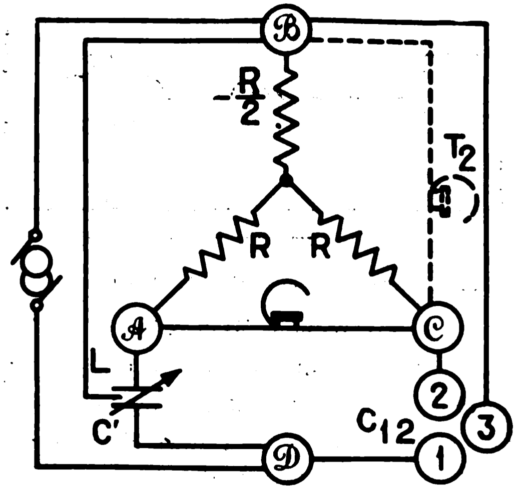

FIG. 4. Null-impedance bridge method for direct capacity

Then $C_{12}=C'$ regardless of the direct capacities associated with C_{12} and C', since these capacities either are short-circuited between corners B, A or B, C or are between corners B, D and thus outside of the bridge.

Correct adjustment of the negative resistance may be checked by observing whether there is silence in telephone T_2 after the balance has been obtained. Assuming invariable negative resistance, this test need be made only when the bridge is set up, or there is a change in frequency. The bridge may be given any ratio Z_1/Z_2 by employing a Y made up of impedances Z_1, Z_2, and $-Z_1Z_2/(Z_1+Z_2)$.

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MAXWELL DISCHARGE METHOD,[2] FIG. 5

Connect the terminals between which the direct capacity C_{12} is required, to A, B and the remaining accessible terminals of this electrical system to D. The adjustable standard capacity is C' and any associated direct capacities in this standard are shown as C'', C'''. If C_{12} is a direct capacity to ground, interchange C' and C_{12}. Balancing involves the following repeated cycle of operations:

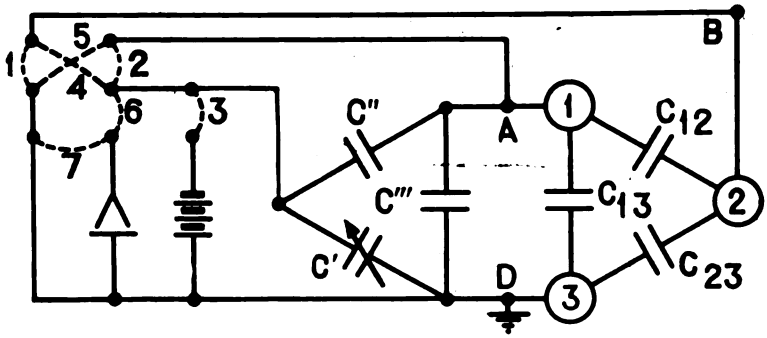

FIG. 5. Maxwell discharge method for direct capacity

1. Make connections 1, 2, 3 and 7 for an instant (thus charging C_{12}, C_{13}, C''', C' and discharging the electrometer).
2. Make connections 4, 5 and then 6 (to discharge condensers C_{13}, C''', mix charges of C_{12}, C' with polarities opposed and connect electrometer).
3. Adjust C' to reduce the electrometer deflection when the cycle is again repeated.

When a null deflection is obtained $C_{12}=C'$; the required direct capacity is equal to the standard direct capacity irrespective of the magnitudes of the four associated direct capacities. If all capacities are free of leakage and absorption, this remarkable method accurately compares two direct capacities by means of a single null setting, and it requires the irreducible minimum amount of apparatus.

[2] Electricity and Magnetism, v. 1, p. 350, (ed. 1892).

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BALANCED-TERMINAL CAPACITY MEASUREMENT, FIG. 6

This is defined as the direct capacity between two given terminals with all other terminals left floating and ignored, after a hypothetical redistribution of the total direct capacity from the given pair of terminals to every third terminal which balances the two sides of the pair. The balanced-terminal capacity, as thus defined, is equal to the direct capacity between the pair augmented by one-quarter of the grounded capacity of the pair, neither of which is changed by the assumed method of balancing.

As illustrated in Fig. 6, terminals 1, 2 are the given pair and terminal 3 includes all others, assumed to be connected together. A bridge ratio of unity is employed, and the entire bridge is

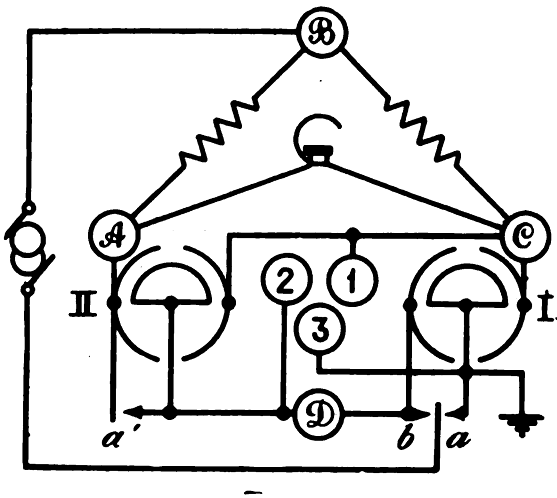

FIG. 6. Bridge for determining hypothetical capacity between two terminals with other terminals balanced and ignored

shielded from ground with the exception of corners C, D which are initially balanced to ground within the range of variable condenser I. The following two successive balances are made:

(1) With contacts a, a' closed and b open, balance is secured by varying condenser I (the total capacity of which is constant) giving the reading C' for its direct capacity in parallel with terminals 1, 3.

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(2) With contacts a, a' open and b closed, balance is obtained by varying condenser II, obtaining the reading C'' for its direct capacity in AD.

If C_o', C_o'' are the corresponding readings without the network, the balanced-terminal capacity C_b and the grounded capacity unbalance of the given pair of terminals are:

$$C_b = 2(C'' - C_o'')$$
$$G_2 - G_1 = 2(C' - C_o')$$

Any failure to adjust condenser I to perfectly balance the given pair of terminals, will decrease the measured capacity C_b. This fact may be utilized to measure the capacity with the second bridge arrangement alone, (contacts a, a' open and b closed) by adjusting condenser I so as to make the reading C'' of condenser II a maximum. This procedure presents no difficulty, since the correct setting for condenser I lies midway between its two possible settings for a balance with any given setting of condenser II; furthermore, C'' is not sensitive to small deviations from a true balance in C'.

Balanced-terminal capacity is of practical importance as a measure of the transmission efficiency to be expected from a metallic circuit, if it is subsequently transposed so as to balance it to every other conductor. In practice, when the unbalance of the section of open wire or cable pair, which is being measured, is relatively small, it is sufficient to set condenser I, once for all, to balance the bridge itself and ignore the unbalance of the pair. This favors an unbalanced pair, however, by the amount $(G_2 - G_1)^2/4(G_{12} + G_{CD})$ where $G_{12} + G_{CD}$ is the grounded capacity of the pair augmented by that of the bridge. For rapid working, condenser II is graduated to read $2C''$ and by auxiliary adjustment C_o'' is made zero, so that the required capacity is read directly from the balance.

ADDITIONAL METHODS OF MEASURING DIRECT CAPACITY

Measurement of the capacity between the terminals, taken in pairs with all the remaining terminals left insulated or floating, gives $n(n-1)/2$ independent results, from which all the direct capacities may be derived by calculation of certain determinants.

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Practically, however, we are in general interested in determining individual direct capacities from the smallest possible number of measurements, and the first step is naturally to connect all of the remaining conductors together, so as to reduce the system to two direct capacities in addition to the one the value of which is required. Three measurements are then the maximum number required, and we know that two, or even one, is sufficient if particular devices are employed.

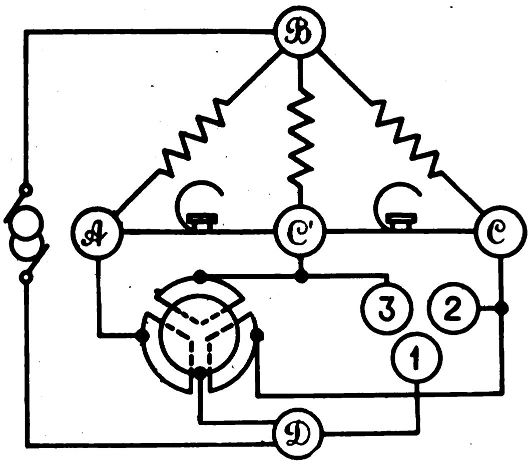

FIG. 7. Double bridge for direct capacity

The three measurement method of determining direct capacities from the grounded capacities of the two terminals taken separately G_1, G_2, and together G_{12}, is given by Maxwell.[3]

$$\begin{aligned} C_{12} &= \tfrac{1}{2}(G_1+G_2-G_{12}) \\ &= \tfrac{1}{2}C''' \end{aligned}$$

if $G_1=C'$, $G_{12}=C'+C''$, and $G_2=C''+C'''$; which indicates a method by which large grounded capacities can be balanced against three variable capacities, only one of which need be calibrated, and that one need be no larger than the required direct capacity.

Two-setting methods, as illustrated by the Colpitts and potentiometer methods, rest upon the possibility of connecting one

[3] Ibid., p. 110.

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of the associated direct capacities between opposite corners of the bridge where it is without influence on the balance, and not altering any associated direct capacity introduced into the working arms of the bridge. Numerous variations of these methods have been considered which may present advantages under special circumstances. Thus, if conductors 1, 2, 3 of Fig. 7 are in commercial operation, and it is not permissible to directly connect two of them together, a double bridge might be employed with a testing frequency differing from that of operation. A telephone is shown for each ear, and a constant total direct capacity is divided between the three branches in the proportion required to silence both telephones.

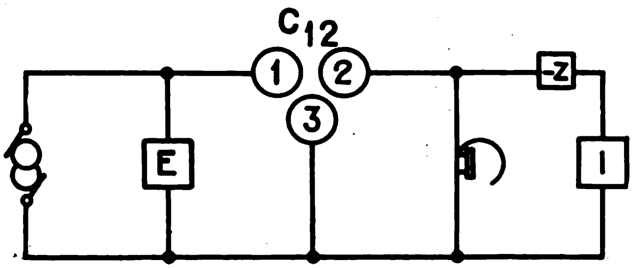

FIG. 8. Ammeter circuit for determining direct capacity

One-setting methods attained ideal simplicity in the Maxwell discharge method, but we found it necessary to use alternating current methods, and here negative resistances make a one-setting method at least theoretically possible, as explained above. Of possible variations it will be sufficient to refer to the ammeter method Fig. 8. Terminals 1 and 2 of the required direct capacity C_{12} are connected to the voltmeter and ammeter terminals, respectively, and all other terminals go to the junction point at 3. Then

$$C_{12}=\frac{I}{2\pi fE}$$

provided the ammeter actually has negligible impedance. The method is well adapted for rapid commercial testing. The ammeter impedance may be reduced to zero by a variable negative

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impedance device $(-Z)$, adjusted to reduce the shunted telephone to silence.

SHIELDING

In the discussion of the bridge, it has been assumed that the several pieces of apparatus forming the six branches of the bridge have no mutual electrical or magnetic reaction upon each other, except as indicated. In general, however, a balance will be upset by changes in position of the pieces of apparatus, or even by movements of the observer himself, whereas these motions cannot affect any of the mutual reactions which have been explicitly considered. The skillful experimenter, understanding how these variations are produced by the extended electric and magnetic fields, will anticipate this trouble and take the necessary precautions, possibly without slowing down his rate of progress.

Where hundreds of thousands of measurements are to be made, however, substantial savings are effected by arranging the bridge so that reliable measurements can be made by unskilled observers, and here it is necessary to shield the bridge so that any possible movements of the observer and of the apparatus will not affect the results. Magnetic fields of transformers are minimized by using toroidal coils with iron cases. Electrostatic fields are shielded by copper cases; the principles of shielding were explained in an earlier paper,[4] Fig. 13 of this paper showing the complete shielding of the balance as constructed for the measurement of direct capacity by the Colpitts method. Over five million capacity and conductance measurements have been made with the shielded capacity and conductance bridge and in a forthcoming paper Mr. G. A. Anderegg will give details of actual construction of apparatus and of methods of operation as well as some actual representative results.

DIRECT ADMITTANCE MEASUREMENTS

For simplicity, the preceding definitions and methods of measurement have been described in terms of capacity, but everything may be generalized, with minor changes only, for the definition and measurement of direct admittances with their capacity and

[4] The Shielded Balance, El. W., 43, 1904, (647–649).

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conductance components. The essential apparatus change is the addition, in parallel with the variable capacity standards employed, of a variable conductance standard, which shifts direct conductance from one side of the bridge to the other, without changing the total reactance and conductance in the two sides of the bridge. This may be practically realized in a great variety of ways as regards details, which it will suffice to illustrate by Fig. 9, where C', C'', C''', G', G'', indicate the continuously variable capacity and conductance standards with enough step-by-step extensions to secure any desired range.

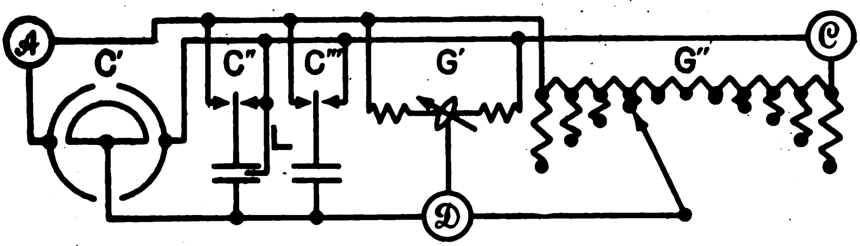

FIG. 9. Variable direct conductance and capacity standard for direct admittance bridge

For the continuously variable conductance standard a slide wire is represented, with a slider made up of two hyperbolic arcs so proportioned that, as the slider is moved uniformly in a given oblique direction, conductance is added uniformly on the left and just enough of the wire is short-circuited to produce an equal conductance decrease on the other side. The arcs are portions of the hyperbola $xy=(L^2-S^2)/4$, where L, S are the total length of the wire and of the portion to be traversed by the slider, and the coordinate axes are the slide wire and the direction of the motion of the slider as oblique asymptotic axes. $L=GS/g=4G/\rho(G^2-g^2)$, where G is the total conductance and $(G\pm g)/2$ the limiting direct conductances on either side.

If an ordinary slider replaces the hyperbolic arc slider, and the scale reading is made non-uniform so as to give one-half of the difference between the direct conductances A to D and C to D, the conductance standard will still give absolutely correct results

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with the Colpitts method, provided the bridge ratio is unity. This simplification in connection with the balancing capacity I of Fig. 6 would, however, not be strictly allowable. For improvised testing we have found it sufficient to use two equal resistances (R) with a dial resistance (r) in series with one of them, and take the defect of conductance introduced by the dial resistance as equal to r/R^2 or to $10^{-2}r$, $10^{-1}r$, r, micromho according as R was made 10 000, 3 162, or 1 000 ohms.

For a step-by-step conductance standard, Fig. 9 shows a set of ten equal resistances, connected in series between corners A, C, to the junction points of which there is connected a parabolic fringe of resistances, the largest of which is 2.5 times each of the ten resistances. With this arrangement the direct conductance in AD may be adjusted by ten equal steps, beginning with zero, while the conductance in CD is decreased by equal amounts to zero. The total resistance required for this conductance standard is only 21/25 of the resistance required to make a single isolated conductance equal to one of the ten conductance steps; the ratio may be reduced to ½ by doubling the number of contacts, and using one fringe resistance for all positions. Resistance may be still further economized by using as high a total conductance as is permissible in the bridge, and securing the required shift in conductance from a small central portion of the parabolic fringe.

Fig. 9 shows the variable capacity standards as well as the variable conductance standards and a few practical points connected with the capacity standards may be mentioned here.

The revolving air condenser standard has two fixed plates connected to A and C, so that the capacity will increase as rapidly on one side as it decreases on the other side. Since perfect constancy of the total capacity is not to be expected, on account of lack of perfect mechanical uniformity, the revolving condenser should be calibrated to read one-half of the difference between the capacities on the two sides, as explained above in connection with conductance. The capacity sections employed to extend the range of the revolving condenser include both air condensers

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C'' and mica condensers C''', the latter being calibrated by means of the air condensers and the conductance standard.

A novel feature of our standard air condensers is a third terminal called the leakage terminal, and indicated at L in Figs. 4, 9. Attached to it are plates so arranged that all leakages either over, or through, the dielectric supports from either of the two main terminals, must pass to the leakage terminal. There can be no leakage directly from one of the main terminals to the other. There is thus no phase angle defect in the standard direct capacity due to leakage, and that due to dielectric hysteresis in the insulating material is reduced to a negligible amount by extending the leakage plates beyond the dielectric, so as to intercept practically all lines of induction passing through any support. This leakage terminal is connected to corner C of the bridge; in the revolving condensers, it is one of the fixed plates.

DIRECT IMPEDANCE MEASUREMENTS

The reciprocal of a direct admittance is naturally termed a direct impedance; substituting impedance for capacity, the definition of direct capacity, given above, becomes the definition of direct impedance. The complete set of direct impedances constitute an exact, symmetrical, physical substitute for any given electrical system. Direct impedances are often, in whole or in part, the most convenient constants since many electrical networks are made up of, or approximate to, directly connected resistances and inductances. To make direct impedance measurements which will not involve the calculation of reciprocals, we naturally employ inductance and resistance standards in series, the associated direct impedances being eliminated as with direct capacities.

CONCLUSION

It has been necessary to preface the description of methods of measuring direct capacities by definitions and a brief discussion, since direct capacities receive but scant attention in textbooks and handbooks. By presenting direct capacities, direct admittances, and direct impedances as alternative methods of stating the constants of the same direct network, employed as an equiva-

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lent substitute for any given electrical system, it is believed the discussion and measurement of networks has been simplified. In another paper the terminology for admittances and impedances will be still further considered, together with their analytical correlation.

DEPARTMENT OF DEVELOPMENT & RESEARCH,
AMERICAN TELEPHONE & TELEGRAPH COMPANY,
JUNE 10, 1922.

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NOTICES

Preliminary Report of Committee of the Optical Society of America on the Proposed English Translation of Helmholtz's Physiological Optics

Columbia University, New York, N. Y., June 1, 1922

To the President of the Optical Society of America:

Dear Sir: The Committee appointed by the Council of the Optical Society of America to make arrangements for bringing out an English translation of Helmholtz's "Handbuch der physiologischen Optik" begs to submit the following preliminary report showing the progress of this project up to the present time.

After an extensive canvass of the situation, partly by advertisements in various journals but chiefly by direct correspondence with a large number of prominent scholars and scientists, for example, members of the Optical Society and of the Physical Society, besides psychologists, physiologists, ophthalmologists, oculists, etc., who from different angles and in different degrees might be presumed to be interested in the publication of an English version of this great treatise on physiological optics, the Committee finds that there is a very decided consensus of opinion favorable to going ahead with this enterprise. It is true that a few persons have expressed some doubt as to whether this book which admittedly ranks as one of the classics in science is sufficiently modern to be worth while; but the answer to that objection seemed to be that the third edition at any rate had been brought completely up to date a little more than a decade ago and that it was hardly to be supposed that the permanent value of this authoritative work had been seriously impaired by recent advancements in science, important as these may have been in some instances. However, for this and other reasons the opinion was practically unanimous that the third and, so to speak, definitive edition, edited by Gullstrand, von Kries, and Nagel, and published by Leopold Voss in Leipzig in 1910, was the only one to be considered for reproduction in English.

Accordingly, the Committee proceeded immediately to negotiate with Herr Leopold Voss about obtaining the rights of translation of the third edition in English. He responded to these overtures in a most generous and cordial spirit; and the outcome was that the Committee purchased these rights for the Optical Society at an extremely reasonable price. Herr Voss has also offered to sell the electrotype plates for the engravings and illustrations at a fair figure. The Committee has this offer still under consideration.

Estimates have been obtained from the printers of the cost of manufacturing one thousand copies of the third edition, perhaps in two volumes instead of in three as in the German edition. With additional incidental and unavoidable expenses the total cost will probably amount to ten thousand dollars. The Committee is reliably informed that it is not unreasonable to suppose that about five hundred copies can be disposed of to libraries and institutions in this country and in Great Britain and her dominions. It is fair to assume that an equal number of copies will be sold to individuals, chiefly scholars and professional men, both here and abroad. The retail price of the complete work will be at least $15 per copy. The Committee has received already a little more

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than $1000 in advance subscriptions and contributions; and there is at least a possibility of one or two large contributions which may aggregate as much as $1000 more.

As a purely business proposition it might be easily possible to get a reliable firm of publishers to take the work off our hands. Some tentative proposals of this kind have been under consideration. However, the Committee is disposed to think that it will be better for the Optical Society to undertake this business alone and to retain complete control.

In brief the Committee is persuaded that this task can and ought to be brought now to a successful completion, and they have every intention of doing so. Several competent scholars have offered to aid in the work of translation and this work has now been actually begun; but additional helpers are needed, and above all additional contributions are needed. The responsible editors should be chosen without delay. Neither the editors nor their assistants in the work of translation will receive any pecuniary compensation for their labors.

Appended to this report is a complete list of subscribers at the present time. One or two of these subscriptions were for fifty dollars but most of them averaged about fifteen dollars. Anyone subscribing as much as fifteen dollars will be entitled to receive a complete copy of the English edition when published but it is not necessary to send the money in advance. Subscriptions or orders should be sent to Adolph Lomb, Esq., 635 Saint Paul Street, Rochester, N. Y.

The list below contains also the "starred" names of those who are helping to prepare the English translation. The chairman of the Committee would be glad to know of other competent persons who might be willing to contribute in this most serviceable way.

Respectfully,

ADOLPH LOMB,
JAMES P. C. SOUTHALL, *Chairman*,
LEONARD T. TROLAND.

LIST

The following is a complete list of actual subscribers up to June 1, 1922, most of whom have already paid their subscriptions. "Starred" names indicate those who have offered to help with the work of translation.

C. G. Abbot
American Philosophical Society
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Wm. L. Benedict
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Alexander Ziwet
Max Zwillinger

CORRECTIONS

The following changes should be made on page 360 of this volume.

Line Two:

In the expression $\Phi\ (1-\Phi)\ F$ delete the first Φ

Equation 34:

Delete the Φ in the denominator

Equation 36:

Last term should read 19.8 instead of 21.

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OPTICAL SOCIETY OF AMERICA

EXHIBIT OF OPTICAL INSTRUMENTS AND APPARATUS
NATIONAL BUREAU OF STANDARDS

Oct. 26–28, 1922

Arrangements are now being completed for the exhibit of optical instruments and apparatus to be held at the National Bureau of Standards, Washington, in connection with the annual meeting of the Optical Society of America, Oct. 26–28, 1922.

The leading manufacturers of optical equipment have already signified their intention of participating. However, the exhibit will not be limited to standard commercial types. Individuals and research laboratories are also invited to exhibit special research apparatus. Brief descriptions of instruments and their purposes supplied by the exhibitors will be printed in the program and published later in the minutes of the meeting in the JOURNAL OF THE OPTICAL SOCIETY. The exhibit of new apparatus will thus constitute just as definite a contribution to science as a paper communicated to the meeting. The authors of papers communicated at this meeting are urged to supplement their papers by an exhibit of apparatus in case such an exhibit is suitable and practicable.

Exhibitors are urged to prepare their exhibits and descriptions so as to give them the maximum educational value.

Exhibits must be listed with the committee at the Bureau of Standards not later than Sept. 20, 1922. Blank entry forms for this purpose may be obtained from Prof. C. A. Skinner, Chairman, Exhibit Committee, O. S. A., Bureau of Standards, Washington, D. C. Exhibits may be installed Oct. 24–25 and installation should be completed not later than noon, Oct. 28.

IRWIN G. PRIEST, *Secretary.*

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Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Vol. VI · SEPTEMBER, 1922 · Number 7

MAGNETIC ROTATION IN VARIOUS LIQUIDS IN THE SHORT INFRA-RED SPECTRUM

By L. R. Ingersoll

Introduction. This paper describes the results of measurements of magnetic rotation on some forty different liquids in the spectral region between the wave-lengths $.56\mu$ and 2.3μ. Approximately half the substances worked with have been aqueous solutions of various salts—mostly of the ferromagnetic metals—and the others pure liquids. Also, mainly to assist in checking the foregoing results with theory, refractive index determinations have been made on the same materials and over the same wave-length range.

While a great deal of work has been done in the field of magnetic rotation in liquids most of the measurements have been confined to the visible spectrum, and in some cases,—e.g. practically all the extensive work of Perkin[1]—to a single wave-length. No attempt will be made at a comprehensive survey of previous results[2] but the work of Quincke,[3] Becquerel,[4] Castleman and Hul-

[1] W. H. Perkin, Jour. Chem. Soc. *36*, p. 330, 1882, and numerous succeeding papers.

[2] For extensive list of references see W. Voigt, Art. "Magneto-optik" in Graetz, "Handb. d. Elektr. u. Magn. vol. 4. Leipzig; 1915.

[3] G. Quincke, Wied. Ann. *24*, p. 606; 1885.

[4] H. Becquerel, C. R., *90* p. 1407, 1880; *100*, p. 1374, 1885; *125*, p. 683, 1897. Ann. Chim. Phys., (5) *12*, p. 42; 1877.

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burt,[5] and particularly of Siertsema[6] must be mentioned. Each of these investigators studied a variety of materials, including in some cases negatively rotating liquids such as titaniumtetrachloride and solutions of ferric chloride and potassium ferricyanide. The conclusion was reached that in this latter class of materials the rotation dispersion is much greater than that given by the approximate Verdet law of the inverse square of the wavelength (which holds for the vast majority of substances) and is indeed more nearly proportional to the inverse fourth power.

In the matter of correlation of experiment with magneto-optic theory in this field, Siertsema, among others, has made some interesting progress. Voigt's theory[7] of magnetic rotation on the basis of the inverse Zeeman effect can be shown, on certain assumptions to yield in a region of small absorption the formula

$$\phi=\frac{eHl\lambda}{2cm}\cdot\frac{dn}{d\lambda}$$

ϕ being the rotation in radians, n the refractive index, c the velocity of light and e the electronic charge in e.m.u. Lorentz[8] deduces a similar formula. This is of the same type as the Becquerel formula

$$\phi=\kappa\lambda\frac{dn}{d\lambda}$$

With this Siertsema[9] has calculated values of e/m from measurements of magnetic rotation (sodium light) and ordinary dispersion, for a variety of substances. They vary from 1.77×10^7 (approximately the value now accepted on the basis of other determinations) in the case of hydrogen, to somewhat less than half of this for carbon bisulphide. Similar calculations with the aid of an extension of the Lorentz theory have been made by Castleman and Hulburt.[10]

[5] R. A. Castleman, Jr. and E. O. Hulburt, Astrophys. Jour. *54*, p. 45; 1921.

[6] L. H. Siertsema, Leiden Comm. No. 62, 80, 82, 90, 91, and Supp. No. 1. Amst. Proc., *4*, p. 339, 1901–2; *5*, pp. 243, 413, 1902–3; *6*, p. 760, 1903–4; *18*, pp. 101, 925, 1916. Arch. Neer., (2) *5*, p. 447, 1900; *6*, p. 825, 1901.

[7] W. Voigt, *loc. cit.* p. 572.

[8] H. A. Lorentz, "Magneto-optische Phänomene," Ency. d. Math. Wiss. vol. V_2, p. 251.

[9] Leiden Comm., No. 82, p. 5.

[10] Astrophys. Jour., *54*, p. 63; 1921.

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The present investigation was undertaken for several different reasons. The extension of our knowledge of rotation dispersion over the increased wave-length range afforded by the spectrobolometric method of measurement hardly needs justification. The question of the effect of infra-red absorption bands is one whose answer should be looked for with considerable interest. Also, as indicated by the above formulas, magneto-optic theory connects in intimate fashion magnetic rotation with other optical properties, e.g., ordinary dispersion. These two quantities have as a rule been determined at different times and by different observers with some consequent difficulty and uncertainty in correlating results. It would seem highly desirable, then, to carry out two such series of measurements on the same specimens and over this relatively wide spectral range.

This near infra-red spectral region is also worthy of study in this connection for reasons other than the mere fact that we are thereby enabled to check theory over a wider extent of wave-length than visual observations alone could accomplish. As is well recognized, optical phenomena in the ultra-violet and possibly throughout the visible spectrum are, in general, conditioned by electronic vibrations. For wave-lengths longer than about 3μ, on the other hand, we have ample evidence—e.g., molecular rotation spectra and certain absorption investigations—that the vibrators may be of atomic or molecular dimensions. The short infra-red may then be regarded as a sort of transition region in which we must look to experiment to indicate something as to the coupling of the short wave-length and long wave-length theory—a region accordingly in which investigation is particularly desirable.

One way in which we may seek to throw light on this question of size of vibrator is to calculate e/m for as wide a frequency range as possible by substituting observed values of rotation and dispersion in the above formula. Such a series has been calculated out to 2μ and will appear in the results.[11]

[11] Too much must not be expected from such calculation. As Professor Lorentz remarked to the writer in a recent conversation on this subject, no very definite conclusions can be based on the Becquerel type of formula when it breaks down. It is of interest, however, to note the general trend of results for the large variety of liquids tested, and one or two inferences may be safely drawn.

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Experimental Details. A full account of the spectrobolometric method of measuring magnetic rotation has been given in previous papers by the writer[12] and need not be repeated here. The apparatus was practically the same as that formerly used, with the exception of the substitution of a small eight coil Thomson galvanometer (made in this laboratory) for the former four coil. It was carried on a special pier built on a massive concrete block cushioned on five sides with sawdust. The liquids were contained in a cell 9.54 mm thick with thin strain-free plate glass ends, whose rotation was, of course, measured and allowed for. This was located between the poles of the large electromagnet in a field of approximately 12050 gausses. A tungsten strip filament lamp (from the Nela Laboratory) served as source and gave sufficient energy to allow rotation measurements on the more transparent liquids for the region between .56μ and 2.3μ.

Satisfactory temperature control proved difficult and was not, indeed, entirely attained. The best results were obtained by water-jacketing the magnet pole pieces and by blowing a stream of chilled air at the cell. In this way the temperature could usually be kept within a degree of 23°C, which was the mean aimed at throughout this work. Tests indicated that the errors due to uncertainty in temperature were smaller than the unavoidable ones from other causes.

Visual measurements, by way of check, were made with a Lippich tri-field polariscope, using sodium light. In practically every case the point determined in this way fitted very closely indeed on the bolometric rotation dispersion curve.

The refractive index measurements were made by the method of double dispersion, using a small spectrometer and hollow prism, in connection with the large spectrometer—minus, of course, its polarization auxiliaries. Large scale plots were made of these indices and the dispersion was determined by drawing tangents to the curves.

Accuracy. In this matter the rotation measurements were an

[12] L. R. Ingersoll, Phil. Mag. (6) *11*, p. 41, 1906; *18*, p. 74, 1909. Phys. Rev. *23*, p. 489, 1906; (2) *9*, p. 257, 1917.

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improvement over any previously made with this apparatus. One or two small and hitherto unsuspected sources of error were traced out and corrected. The runs were almost all made at night to secure the freedom from vibration so necessary in bolometric work. The results were plotted on large size section paper and the measurements listed here were taken from the smooth curve. As nearly as can be estimated the resultant accuracy of the rotation measurements is of the order of 1% through the middle of the spectral range, with a somewhat larger probable error at the ends where the available energy is small. Considering the difficulties involved this is considered quite satisfactory.

In view of the fact that the refractive index measurements were a somewhat secondary consideration the highest order of accuracy was not aimed at in them. In the few cases possible the values determined were compared with the infra-red indices determined by other observers. While the accordance in absolute values was not in all cases such as might be desired, the *slopes* of the curves, giving the dispersion, were in good agreement, and it is to be noted that for the purposes of this research the absolute value is secondary to the dispersion, which is the quantity entering into formulas of the type above noted. Moreover as regards formulas into which n enters directly the situation is equally good; for the probable error of n, measured in per cent would in any case be almost negligibly small.

Materials. As already noted these were divided into two classes—pure liquids and aqueous solutions of metallic salts. The liquids studied were chosen to give as great a variety as possible. They ranged from heptane with a density of only .68 and correspondingly low refractive index, to methylene iodide, the heaviest organic liquid known, with a density of 3.31 and refractive index over 1.7; from the chemically simple carbon bisulphide to α-monobromnapthalene. In most cases they were purified by redistillation, or otherwise, shortly before use, and for this service I am greatly indebted to several members of the Chemistry Department of this University, particularly to Professor F. Daniels.

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The salts of the ferromagnetic metals used in the aqueous solutions were as pure as could be readily obtained. The solutions were in general very nearly saturated.

RESULTS

The results a e shown in the accompanying tables and curves. The tables give for a selected series of wave-lengths Verdet's constant R (rotation in minutes of arc for unit length in unit field), the refractive index n, and (Table 1 only) values of e/m calculated with the formula given above, in the form

$$e/m=\frac{6\times 10^{10}\pi R}{180\times 60}/\lambda\frac{dn}{d\lambda}$$

The complete series of values of rotation measurements can be taken from the curves—which, to make evident the law of dispersion are plotted with inverse squares of the wave-lengths as abscissas—, and the dispersion for the five wave-lengths listed may readily be found, if desired, by calculating backwards from R and e/m. Hence material is here available for a very extensive test of magnetic rotation theories.

The densities of all the liquids, with a few exceptions, were carefully measured at the chosen working temperature, viz., 23°C, with a calibrated specific gravity bottle. Exceptions are ethyl iodide and tin and titanium tetrachlorides, the densities of which were taken from tables. The percentages of salt in the solutions are by weight, the figures being taken from tables, with the aid of the density determinations. They checked satisfactorily with calculations based on weights. In the few cases where density determinations are lacking the solutions were what would ordinarily be called "saturated."

It will be found that, with one or two exceptions, the values of density, and of the rotation and refractive index at $.6\mu$ are in satisfactory agreement with such as can be drawn (by interpolation or extrapolation) from standard tables. Xylene does not show as good agreement as most of the other liquids in this particular. It was not of the highest purity, being mainly ortho- but with more than a trace of para- and meta-xylene.

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DISCUSSION OF RESULTS

DISPERSION OF ROTATION IN PURE LIQUIDS

The first thing which strikes the attention in glancing at the curves of Figs. 1 to 3 is the rather remarkable way in which the rotation dispersion follows the simple law of proportionality to

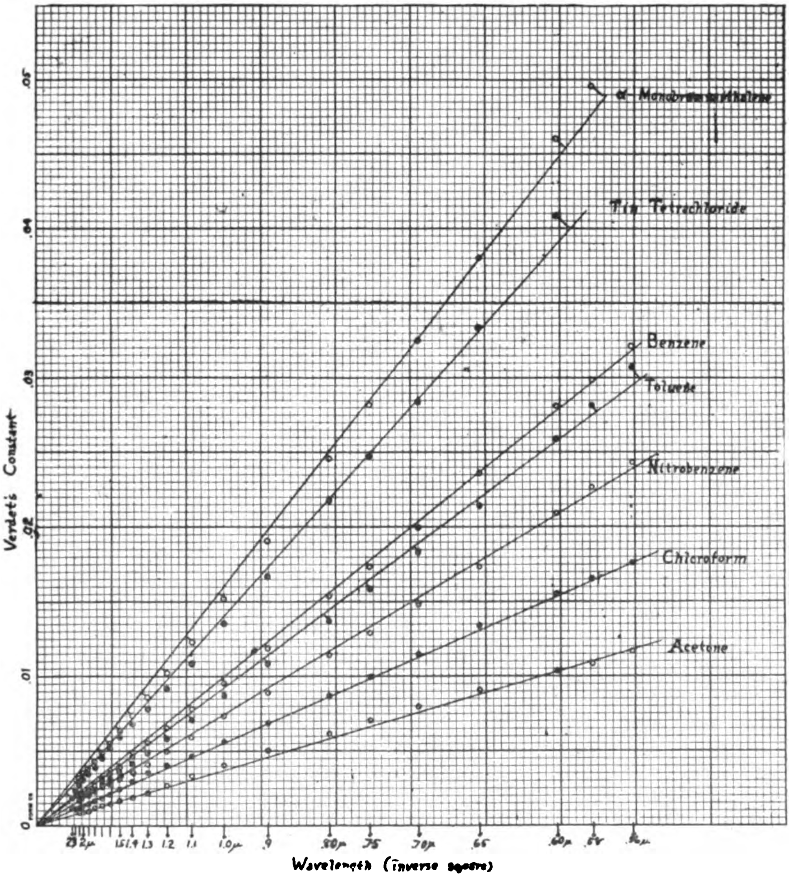

FIG. 1. Curves plotted with magnetic rotations as ordinates and inverse squares of the wave-lengths as abscissas. For convenience the actual wave-lengths in microns are indicated

the inverse square of the wave-length. For the alcohols, ether, chloroform and heptane the departure of the points from a straight line is not much greater than the probable experimental error. The other curves, however, with the exception of acetone,

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TABLE 1.—*Summary of data for pure liquids at 23°C*

Liquid		Carbon Tetrachloride $C\,Cl_4$	Tin Tetrachloride $Sn\,Cl_4$	Titanium Tetra-chloride $Ti\,Cl_4$	Ether $C_4H_{10}O$	Chloroform $C\,H\,Cl_3$	*n*-Heptane $C_7\,H_{16}$	Acetone C_3H_6O	Carbon Bisulphide CS_2	α-monobrom-napthalene $C_{10}H_7Br$	Water H_2O
Density		1.583	2.22	1.74	.713	1.475	.681	.791	1.260	1.486	.997
$\lambda = .6\mu$	R	.0161	.0408	−.0131	.0102	.0155	.0119	.0103	.0394	.0460	.0126
	n	1.4599	1.5079	1.5951	1.3505	1.4425	1.3850	1.3569	1.6227	1.6558	1.3324
	$\frac{e}{m}\cdot 10^7$	.99	1.46	....	.95	1.08	1.03	1.12	.72	.79	1.25
$\lambda = .8\mu$	R	.0089	.0217	−.0050	.0058	.0086	.0066	.0061	.0214	.0245	.0070
	n	1.4539	1.4993	1.5764	1.3465	1.4370	1.3811	1.3527	1.6031	1.6375	1.3280
	$\frac{e}{m}\cdot 10^7$	.96	1.79	....	.88	1.01	1.13	.88	.80	.99	.80
$\lambda = 1.0\mu$	R	.0057	.0135	−.0026	.0036	.0056	.0043	.0040	.0135	.0152	.0044
	n	1.4509	1.4952	1.5689	1.3444	1.4343	1.3791	1.3506	1.5947	1.6296	1.3247
	$\frac{e}{m}\cdot 10^7$	.87	1.70	....	.86	.94	1.03	.73	.75	.95	.50
$\lambda = 1.5\mu$	R	.0025	.0060	−.0010	.0016	.0024	.0018	.0017	.0058	.0063	*.0029
	n	1.4479	1.4911	1.5613	1.3419	1.4314	1.3768	1.3479	1.5858	1.6222	*1.3210
	$\frac{e}{m}\cdot 10^7$	.76	1.16	....	.36	.93	.56	.50	.65	.94	*.29
$\lambda = 2.0\mu$	R	.0013	.0031	−.0005	.0008	.0013	.0009	.0009	.0031	.0035	
	n	1.4463	1.4895	1.5585	1.3402	1.4301	1.3752	1.3464	1.5819	1.6192	* $\lambda = 1.25\,\mu$
	$\frac{e}{m}\cdot 10^7$	.38	1.48	....	.11	.44	.15	.20	.39	.48	

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TABLE 1 (continued)

Liquid		Methyl Alcohol CH_4O	Ethyl Alcohol C_2H_6O	*n*-Butyl Alcohol $C_4H_{10}O$	Benzene C_6H_6	Nitrobenzene $C_6H_5NO_2$	Toluene C_7H_8	Xylene C_8H_{10}	Ethyl Iodide C_2H_5I	Methyl Iodide CH_3I	Methylene Iodide CH_2I_2
Density		.790	.788	.807	.874	1.198	.863	.861	1.93	2.263	3.310
$\lambda = .6\mu$	R	.0093	.0111	.0120	.0281	.0209	.0258	.0232	.0279	.0318	.0476
	n	1.3285	1.3609	1.3992	1.4970	......	1.4932	1.4933	1.5085	1.5262	1.7376
	$\frac{e}{m} \cdot 10^7$	.88	1.22	1.05	.99	......	.97	1.09	1.13	1.15	.73
$\lambda = .8\mu$	R	.0051	.0060	.0067	.0153	.0114	.0137	.0128	.0151	.0178	.0268
	n	1.3245	1.3566	1.3950	1.4869	1.5355	1.4839	1.4842	1.4994	1.5157	1.7164
	$\frac{e}{m} \cdot 10^7$	.80	.85	.94	1.02	.59	.97	.87	1.15	1.12	.99
$\lambda = 1.0\mu$	R	.0032	.0038	.0043	.0095	.0073	.0087	.0080	.0097	.0112	.0169
	n	1.3222	1.3542	1.3928	1.4823	1.5295	1.4796	1.4799	1.4947	1.5108	1.7073
	$\frac{e}{m} \cdot 10^7$	.56	.70	.90	1.00	.60	1.17	.97	.95	1.13	.95
$\lambda = 1.5\mu$	R	.0019	.0019	.0018	.0039	.0031	.0035	.0035	.0041	.0048	.0073
	n	1.3180	1.3506	1.3896	1.4774	1.5234	1.4751	1.4757	1.4901	1.5064	1.6987
	$\frac{e}{m} \cdot 10^7$	.25	.29	.32	.88	.50	.67	.71	.84	1.10	.88
$\lambda = 2.0\mu$	R	.0013	.0010	.0014	.0022	.0018	.0020	.0019	.0024	.0027	.0040
	n	1.3135	1.3470	....	1.4756	1.5204	1.4734	1.4737	1.4880	1.5043	1.6956
	$\frac{e}{m} \cdot 10^7$	.12	.12	....	.43	.30	.29	.35	.35	.49	.73

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show a distinct tendency to fall below the straight line for the middle part of the spectral region investigated and to rise at the short wave-length end. Toluene and carbon bisulphide are good examples.

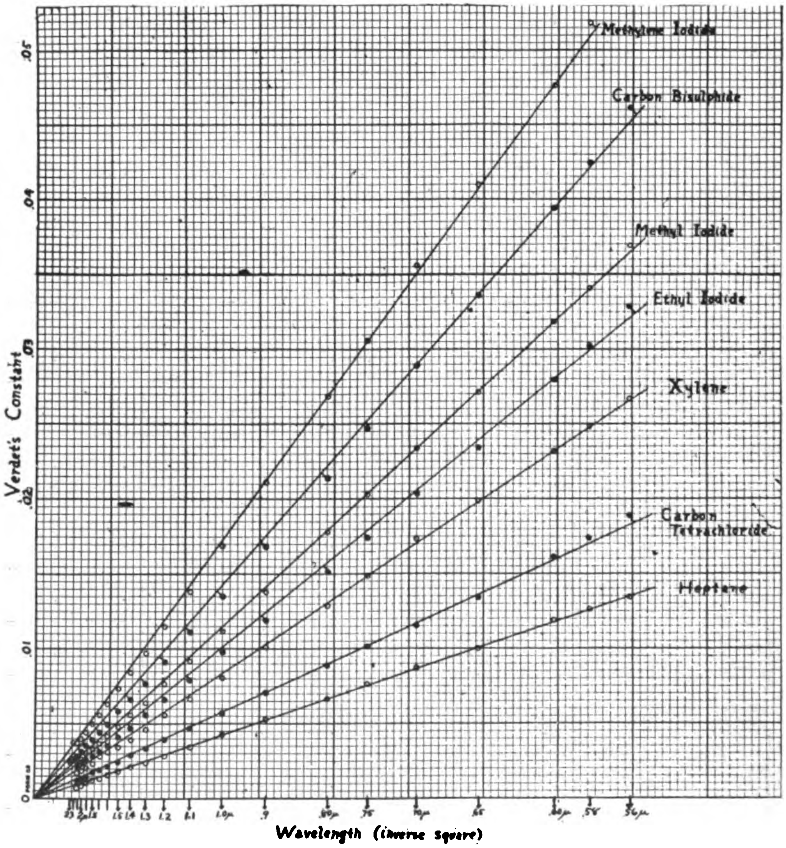

FIG. 2. For explanation see Fig. 1

Rotation Formulas. Of the many rotation dispersion formulas we shall discuss in connection with these curves only three, viz.,

$$\phi = n\left\{\frac{a}{\lambda^2} + \frac{b}{\lambda^2 - \lambda_1^2} + \frac{c}{\lambda^2 - \lambda_2^2}\right\} \tag{1}$$

$$\phi = \frac{1}{n}\left\{\frac{a'}{\lambda^2} + \frac{b'\lambda^2}{(\lambda^2 - \lambda_1^2)^2} + \frac{c'\lambda^2}{(\lambda^2 - \lambda_2^2)^2}\right\} \tag{2}$$

$$\phi = \kappa\lambda dn/d\lambda \tag{3}$$

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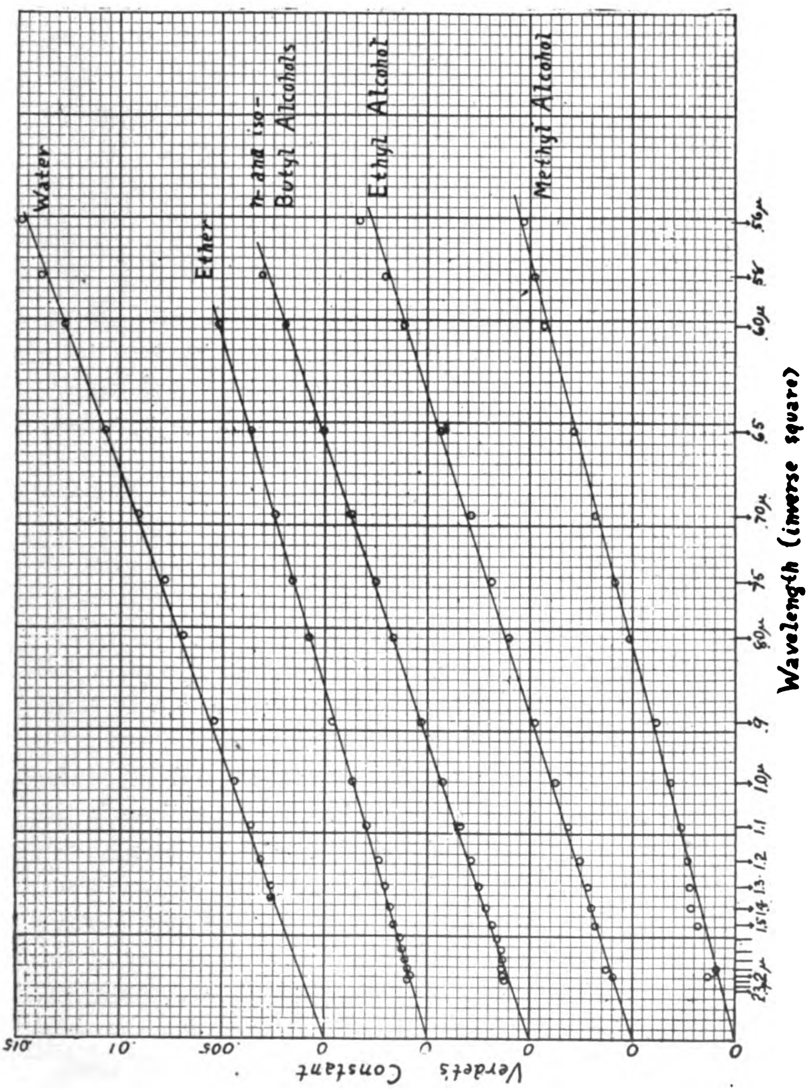

FIG. 3. For explanation see Fig. 1

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The first two are based on what might be termed the older magneto-optic theory and are simple extensions—by the addition of a third term—of those derived by Drude (Lehrbuch d. Optik) on the hypothesis of molecular currents, and of the Hall effect, respectively. λ_1 is the wave-length of an ultra-violet absorption line and λ_2 of an infra-red.

Now a little study of these formulas will make it apparent that, except for the long wave-length end, the three constants (five if we do not specify λ_1, and λ_2) could be chosen so that either formula would fit the observed curves—save for acetone—very well. This is not true, however, for the longer wave-lengths. As we approach, from the short wave-length side, an infra-red absorption band, that is, one—if there be such—which affects the magnetic rotation, formula (1) calls for a *decrease* of rotation. Similarly the Hall effect formula predicts an *increase*—it being assumed in both cases that the constants c and c' are positive. While in most cases the rotation dispersion shows a slight upward tendency on the long wave-length end, calling for the second formula, there are one or two liquids, notably tin tetrachloride, where the reverse is true.

It may be remarked, however, that it is not safe to place too much dependence on these measurements for the two or three longest wave-lengths. While a great deal of effort was expended to make them as accurate as possible the experimental difficulties were very great. The same may be said of determinations in the absorption region in the neighborhood of 1.7μ shown by many of the hydrocarbons. The conclusion we can safely draw—based on the fact that the departure from the straight line (inverse square law) on the long wave-length end is of the order of the experimental error—is that, in the region investigated the effect, on the rotation, of infra-red bands is *small*. There is no observable effect traceable to the 1.7μ absorption region in any case.

Becquerel Formula. Calculation of e/m. As already mentioned the more modern point of view in magneto-optic rotation theory leads to a formula of the type of (3) and this has been applied to the substances listed in Figs. 1–3. The results are given in

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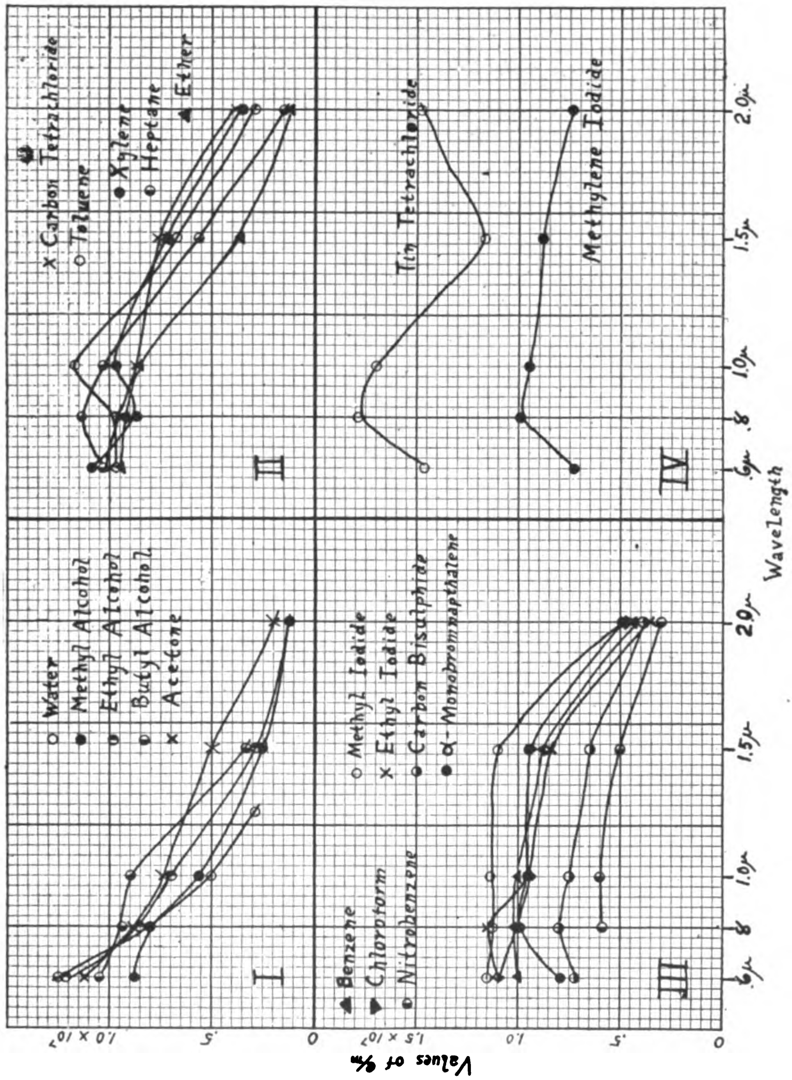
× Carbon Tetrachloride
○ Toluene
● Xylene
○ Heptane
▲ Ether
II
○ Water
● Methyl Alcohol
Ethyl Alcohol
Butyl Alcohol
× Acetone
I
Tin Tetrachloride
Methylene Iodide
IV
○ Methyl Iodide
× Ethyl Iodide
Carbon Bisulphide
● α-Monobromnapthalene
▲ Benzene
▼ Chloroform
Nitrobenzene
III
Wavelength
Values of ω/n
1.0 × 10⁷
1.5 × 10⁷

FIG. 4.

Table 1 and Fig. 4 not, however, as a comparison of observed and computed rotations, but by reversing the formula as previously explained and calculating e/m. In this way we are led—subject to the reservations already mentioned in the footnote—to some interesting conclusions as to the size of vibrator. When e/m shows a reasonably constant value (i.e. the Becquerel formula is obeyed) of the same order of magnitude as that found in other ways, as shown by tin tetrachloride and methylene iodide, we may assume that the vibrators are electronic in size. When, however, as in water and the alcohols, e/m decreases rapidly with longer wave-length we infer that the proportion of atomic vibrators is making itself felt. A *discontinuous* change from electronic e/m to atomic e/m is, of course, not to be expected.

It may be remarked that, in view of the fact that these liquids all obey substantially the same law of rotation dispersion the marked changes in the e/m curves between the four types are largely due to differences in the ordinary dispersion curves. This means that certain infra-red bands which are influencing the refractive index curves *have no effect* on the magnetic rotation.

Solutions. The results on solutions are shown in Fig. 5 and Table 2. It will be seen that the inverse square law also holds here in a general way, but not as well as for pure liquids. The spectral range is also considerably shortened by the fact that the absorption of water prevents measurement for wave-lengths longer than about 1.35μ. Considerable effort was spent in calculating corrections for the effect of the solvent (water) in an attempt to secure results characteristic of the salts alone. The curves obtained were in many cases most irregular and hardly justify reproduction. In general it may be said, however, that where the salt solution shows a smaller rotation than that of water (curve below the dash line) the salt itself may be considered to show a negative rotation, but there may be exceptions to this statement.

Thoulet's solution $(2KI+HgI_2+aq)$ is of interest as showing one of the highest known rotations. As used it had a density of only 2.45 and this could be increased by concentration with doubtless a corresponding increase in its Verdet constant.

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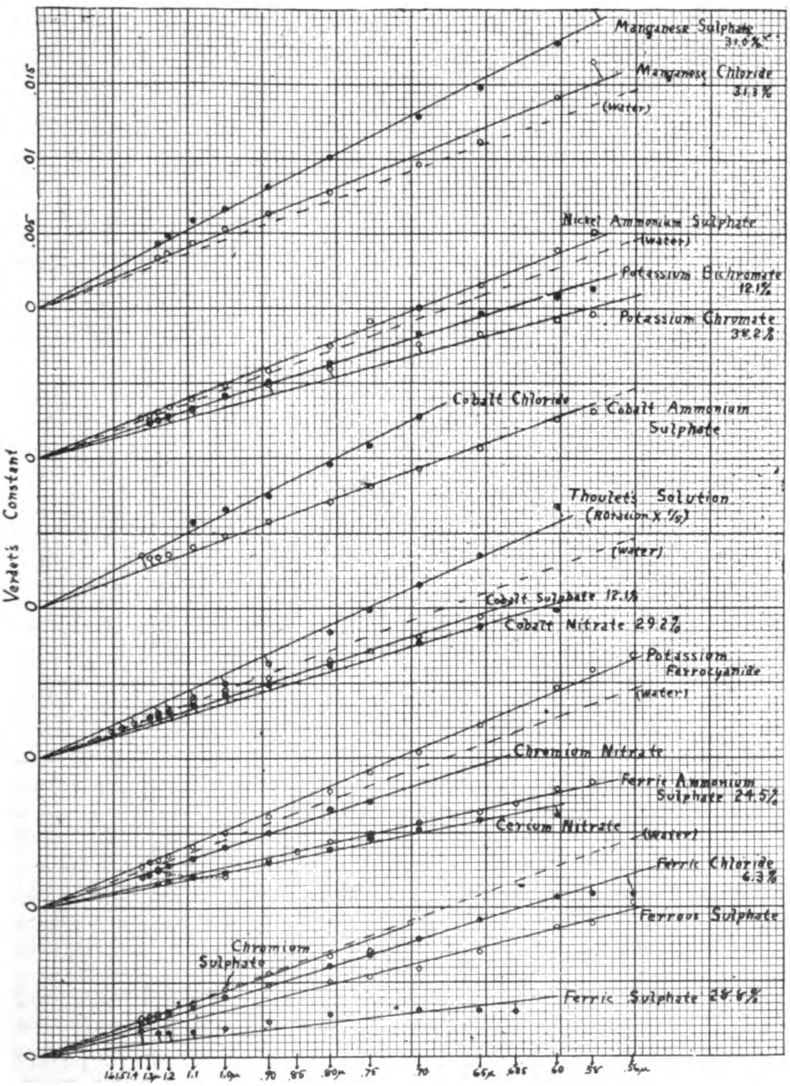

FIG. 5. Rotation curves for concentrated aqueous solutions. For explanation of plotting see Fig. 1

In a previous research the writer[13] found that films of iron showed a magnetic rotation which increased out to a wave-length of 1.5μ and then stayed nearly constant to 2μ. The fact that only the slightest trace of such behavior is exhibited by iron salt solutions is a bit of evidence for this particular phenomenon of magnetism being regarded as molecular rather than atomic in character.

A series of e/m calculations was also carried out for these solutions. In general the values fall off rapidly with increasing wave-length, as in the case of pure water.

Negatively Rotating Liquids. Our previous discussion has been entirely on positively rotating liquids. Fig. 6 shows the measurements on the three negatively rotating liquids tested, viz., strong solutions of ferric chloride and potassium ferricyanide, and the

TABLE 2.—*Summary of data for aqueous solutions at 23°C*

		Ferric Chloride (1)	Ferric Chloride (2)	Potassium Ferricyanide	Potassium Ferrocyanide	Ferric Sulphate	Ferrous Sulphate	Ferric Ammonium Sulphate	Cobalt Sulphate	Cobalt Chloride	Cobalt Nitrate
Density		1.523	1.049	1.187	......	1.446	......	1.250	1.322	1.296	1.308
%		47.8	6.3	31.4	......	......	......	24.5	......	......	29.2
$\lambda = .6\mu$	R	....	.0107	−.0233	.0147	......	.0087	.0078	......	......	.0099
	n	....	1.3487	1.3878	......	1.4234	......	1.3881	......	......	1.3892
$\lambda = .8\mu$	R	−.0399	.0061	−.0056	.0078	.0029	.0050	.0044	.0065	.0096	.0060
	n	1.4941	1.3442	1.3810	......	1.4160	......	1.3817	1.3816	1.3985	1.3835
$\lambda = 1.0\mu$	R	.0215	.0040	−.0017	.0051	.0020	......	.0021	.0046	.0066	.0043
	n	1.4860	1.3407	1.3768	......	1.4113	......	1.3779	1.3778	1.3945	1.3796
$= 1.25\mu$	R	−.0110	.0026	−.0001	.0032	.0016	......	.0026	......	......	.0029
	n	1.4803	1.3366	1.3731	......	1.4061	......	1.3730	......	......	......

[13] Phil. Mag. (6) *18*, p. 74; 1909.

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TABLE 2.—(Continued)

		Cobalt Ammonium Sulphate	Nickel Ammonium Sulphate	Cerium Nitrate	Manganese Chloride	Manganese Sulphate	Chromium Nitrate	Chromium Sulphate	Potassium Chromate	Potassium Bichromate	Thoulet's Solution
Density		1.106	1.054	1.202	1.326	1.369	1.087	1.140	1.372	1.085	2.445
%		......	......	......	31.3	31.0	......	......	38.2	12.1	......
$\lambda = .6\mu$	R	.0126	.0139	.0062	.0141	.0178	......	......	.0092	.0108	.0841
	n	1.3552	1.3438	1.3650	......	1.3948	......	......	1.4276	1.3540	......
$\lambda = .8\mu$	R	.0071	.0075	.0039	.0078	.0101	.0066	.0067	.0060	.0064	.0422
	n	1.3505	1.3397	1.3594	1.4069	1.3896	1.3468	1.3553	1.4180	1.3487	1.5685
$\lambda = 1.0\mu$	R	.0049	.0048	.0024	.0054	.0066	.0041	.0045	.0041	.0042	.0249
	n	1.3470	1.3362	1.3460	1.4030	1.3857	1.3431	1.3514	1.4127	1.3447	1.5590
$\lambda = 1.25\mu$	R	.0035	.0030	.0016	.0034	.0043	.0025	.0028	.0026	.0025	.0152
	n	1.3432	1.3324	1.3419	1.3987	1.3811	1.3391	1.3472	1.4079	1.3407	1.5525

fuming liquid titanium tetrachloride. This last is of interest in that it is the only known diamagnetic substance showing negative rotation.

As already mentioned several investigators, notably Siertsema[14] have concluded on the basis of visible spectrum measurements that the rotation dispersion of such substances is much greater than in positively rotating liquids, being more nearly proportional to the inverse fourth power of the wave-length. Now the curves of Fig. 6 show that over this wider spectral region both titanium tetrachloride and the strong ferric chloride solution obey fairly closely an *inverse cube law*. Potassium ferricyanide, on the other hand, shows a much greater dispersion than this—greater even than the inverse fourth power.[15]

[14] Amst. Proc. *18*, p. 925; 1916.
[15] Cf. Siertsema, Arch. Neer. (2) *5*, 447, 1900.

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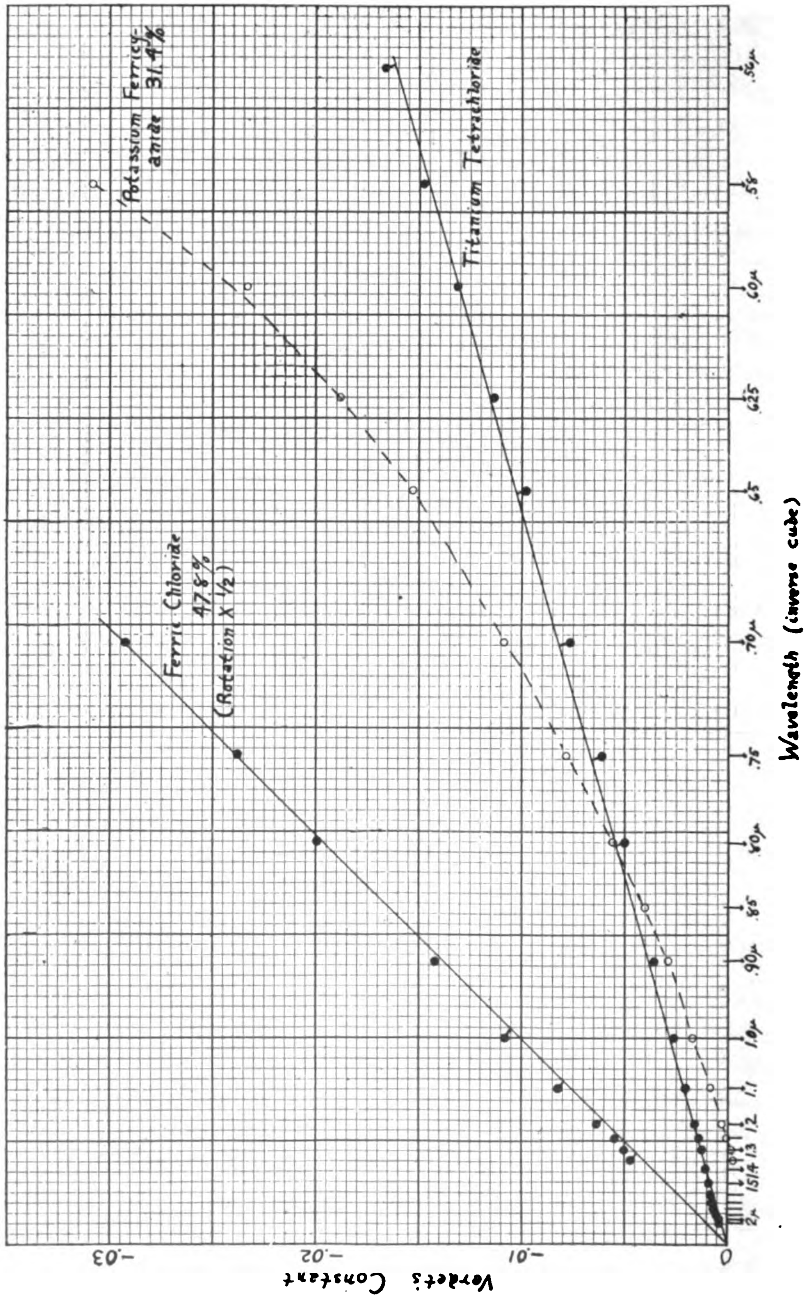

FIG. 6. Curves for negatively rotating liquids. For explanation of plotting see Fig. 1

Other details. A carbon bisulphide solution of iodine gave rotation and refractive index measurements almost the same as the pure solvent. The rotation curve for colloidal iron oxide in water could not be definitely distinguished from that for pure water. Similarly *n*.- and iso-butyl alcohols gave practically identical curves.

Two solutions of nickel salts, viz., the nitrate and chloride, were tested but were found to absorb too strongly for measurement save in relatively narrow transmission bands at $.8\mu$ and $.9\mu$, respectively. Incidentally it may be remarked that these salts in concentrated solution would constitute admirable filters if it was desired to utilize only this portion of the infra-red.

In conclusion I am glad to acknowledge my indebtedness to the Rumford and American Association funds for grants; to members of the Chemistry Department, as already mentioned, for their kindness in preparing pure liquids; and to Professor H. A. Lorentz, who, on the occasion of his recent visit to this laboratory, was good enough to give the writer the benefit of his criticism. I should like also to express my appreciation of the work of my assistant Mr. N. G. E. Sharp, who made all of the observations with painstaking care.

SUMMARY

1. The magnetic rotation and refractive index of twenty pure liquids and as many aqueous solutions of metallic salts have been measured for the spectrum range between $.56\mu$ and 2.3μ.

2. Positively rotating liquids show a rotation very nearly proportional, in most cases, to the inverse square of the wave-length, indicating that infra-red absorption bands have small influence on the rotation in this region.

3. The few negatively rotating liquids show a much greater rotation dispersion. This is nearly the inverse cube for titanium tetrachloride and concentrated ferric chloride solution, while for potassium ferricyanide solution it is almost as the inverse fifth power of the wave-length.

4. Values of e/m calculated from these measurements on positive liquids show in general a diminution for longer wave-length, as might be expected. Several types of curves may be distinguished in this connection.

PHYSICAL LABORATORY,
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JULY 12, 1922.

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INSTRUMENT SECTION

THE CLASSIFICATION OF OPTICAL INSTRUMENTS

By T. Smith

ABSTRACT

Exception is taken to the classification of optical instruments by the signs of their powers, and an alternative division comprising five classes is proposed, based upon the separation of the four Gaussian constants into two groups according to their signs. This classification cannot be modified by the addition to the system of inverting prisms and the like, and the properties usually associated with the sign of the lens in reality depend upon its class according to the new system. Each class may have systems of positive or of negative power.

The elementary theory of optical instruments which we owe to Gauss has the great merit that attention may be given wholly to the events which take place in the object space and the image space, the method by which the rays are altered in direction and position, and even the position of the instrument which causes these changes, being unimportant. This treatment is highly advantageous when the purpose is to consider the general correlation between incident and emergent rays, but in the practical applications of the theory to the construction and use of real instruments the position of the instrument is of the greatest importance. This would be rendered more apparent if it became customary at a later stage of the theory to consider the position of the instrument in relation to the cardinal points; such a development leads naturally to a classification of instruments in which there are five groups, important special cases such as telescopes being regarded as borderline cases or as members of two groups. The division proposed is more fundamental than the usual separation of lens systems into positive and negative combinations, a distinction which is not generally correct as regards many of the properties usually associated with them.

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The properties of a system, including the positions of the extreme surfaces relative to the cardinal points, are given by the four Gaussian constants A, B, C, D,[1] which satisfy the identity $BC-AD=1$. The ordinary division depends wholly on the sign of A, the power of the system. The essential theoretical distinction between positive and negative lenses is indicated in Figs. 1 and 2 by means of three incident parallel rays identified by the letters P, Q, R: the position of the instrument is immaterial and both object space and image space are supposed to extend to

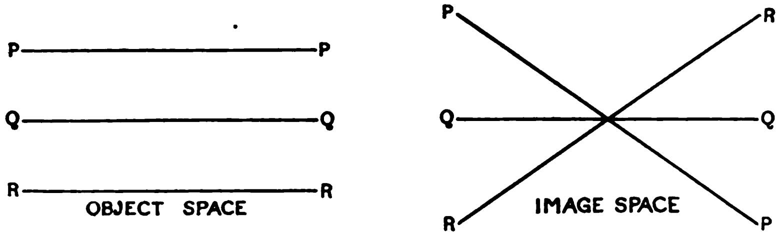

Fig. 1. Positive Lens.

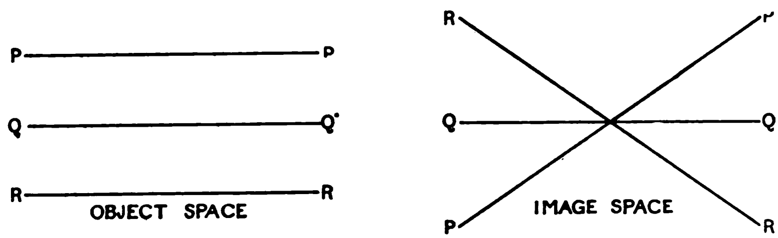

Fig. 2. Negative Lens.

infinity in every direction. In the classification now proposed the basis is the grouping of the constants according to agreements or disagreements of sign. Let the constants of one sign be included in one bracket, and those of another sign in another bracket. There are then six possible groups

[$ABCD$], [ABC] [D], [A] [BCD], [AB] [CD], [AC] [BD], [AD] [BC], it being of no consequence which bracket of a pair corresponds to

[1] These constants A, B, C, D are respectively $\kappa_{1,n}$, $\frac{\partial\kappa_{1,n}}{\partial\kappa_1}$, $\frac{\partial\kappa_{1,n}}{\partial\kappa_n}$, $\frac{\partial^2\kappa_{1,n}}{\partial\kappa_1\partial\kappa_n}$ in the notation used in calculations.

a positive and which to a negative sign. An algebraically exhaustive list would include the groups [ABD]]C] and [ACD] [B], but these must be excluded since they require $BC-AD$ to be negative. The actual number of classes is one less than the number of these groups, for if a system is considered in the reverse direction B and C are interchanged. It follows that the two groups [AB] [CD] and [AC] [BD] correspond to a single class of instrument.

The positions of pairs of conjugate points on the axis are given in terms of A, B, C, D, by the relation

$$Axx'-Bx'-Cx+D=0, \qquad (1)$$

where x is the distance from the first surface to the object point, and x' is the distance from the last surface to the image point, both considered positive when measured away from the system into the surrounding medium. It is at once evident from this equation that the positions of conjugate points depend upon the ratios of these constants, and that their relative signs rather than their absolute signs are important. The particular property affected by a change of all the signs without a change of magnitude of these four quantities is the inversion of the image of an object occupying a particular position, but in modern instruments the designer need pay little attention to this feature of a system as he has several devices which he may adopt to modify this result without altering the values of the four constants. The properties represented by the ratios of the constants on the other hand are fundamental and cannot be influenced by such devices as the introduction of reflecting surfaces.

From the equation connecting x and x' it is easy to see what properties are associated with each of the groups that have been described. Suppose that F (Figs. 3 to 8) is the point at which incident parallel rays are brought to a focus, that U is the image of the vertex of the first surface in the system, and that V is the vertex of the last surface. Then F, U, V are three points on the axis in the image space of which V is necessarily real while F and U are only real if they are encountered by a wave which has traversed the system after it has passed through V. There are six cases to consider. For U may be encountered by the wave

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before it reaches F or after it has passed through F, and V in either case may precede both, fall between or follow both U and F. If the points are placed in the order in which they are met by the wave, the following correspondences may be derived at once from equation (1):

$$VUF = [ABCD], \qquad VFU = [AC]\ [BD],$$
$$UVF = [ABC]\ [D], \qquad FVU = [A]\ [BCD],$$
$$UFV = [AB]\ [CD], \qquad FUV = [AD]\ [BC].$$

The general properties of members of any class may be inferred either from equation (1) or from the order of the letters F, U, V if it is remembered that the image of the real part of the object space axis extends from F in the positive direction (through infinity if necessary) to U. Either method shows that in every group with the exception of the last, $[AD]\ [BC]$, there is a range of the axis in which a real image of a real object may be formed. The portion of the image axis which corresponds to the real object space axis is indicated by a thicker line in the image.

Fig. 3. [ABCD]

Fig. 4. [ABC][D]

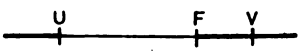

Fig. 5. [AB][CD]

Fig. 6. [AC][BD]

Fig. 7. [A][BCD]

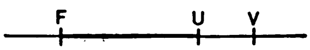

Fig. 8. [AD][BC]

Among the special cases it will be noted that thin lenses, which have $B = C = 1$ and $D = 0$, are members, if the power is positive, of groups $[ABCD]$ and $[ABC]\ [D]$; if the power is negative they belong to groups $[A]\ [BCD]$ and $[AD]\ [BC]$. When the thickness are small without being exactly zero the groups are those mentioned second in each case. It is because the negative lenses ordinarily used belong to the class $[AD]\ [BC]$, and not because the power is negative, that all the images they form are virtual.

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It is not difficult to construct positive combinations which have this property.

Other special classes comprise the telescopes. For instance, prismatic binoculars, gun-sighting telescopes and astronomical telescopes belong to either [*ABCD*] or [*A*] [*BCD*]. On the other hand, Galilean binoculars belong to [*ABC*] [*D*] or [*AD*] [*BC*], and their undesirable properties are wholly attributable to the class to which they belong and have no connection with the erectness of the images they yield.

The adoption of this classification will lead naturally from the Gaussian treatment of rays to the consideration of entrance and exit pupils, an important branch of geometrical optics which has not until lately, in this country at all events, received a proper amount of attention. It would be convenient to have short descriptive titles for each group, but the writer wished to have the opportunity of hearing the views of other workers before making any proposals of this kind.

EDITORIAL NOTE

The above paper on "The Classification of Optical Instruments" was read by the author before the Optical Society of London, June 8, 1922. It is published here by arrangement between the Optical Society of London and the Optical Society of America, in hopes that the author's proposal will have as wide discussion as possible.

For the benefit of such readers as may perhaps not be familiar with Mr. Smith's other optical writings, it may be well to state that "the four Gaussian constants A, B, C, D" which he employs have the following meanings (as he explains also in a footnote):

A denotes the refracting power of the system, usually denoted in this Journal by F or $F_{1,m}$, if the system is supposed to consist of m refracting surfaces; (however, Mr. Smith uses κ instead of F and n instead of m.)

$B=\dfrac{\partial A}{\partial F_1}$, where F_1 denotes the refracting power of the first surface;

$C=\dfrac{\partial A}{\partial F_m}$, where F_m denotes the refracting power of the mth surface;

$$D=\frac{\partial^2 A}{\partial F_1 \partial F_m}=\frac{BC-1}{A}.$$

Now if the vertices of the first and last surfaces are designated by A_1 and A_m, respectively, it may be shown (*cf. J.O.S.A.*, IV, pp. 246, 247; 1920) that the positions of the focal points, (F, F') are defined by the simple relations:

$$A_1F=-\frac{B}{A},\quad A_mF'=\frac{C}{A}.$$

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If M, M′ designate the positions of a pair of conjugate points on the optic axis, the symbols x, x' employed by Mr. Smith may be defined as follows: $x = \mathrm{MA}_1$, $x' = \mathrm{A}_m\mathrm{M}'$. Assuming that the optical system is surrounded by air on both sides, and employing the Newtonian abscissa-equation, we derive immediately the relation:

$$\left(\frac{B}{A}-x\right)\left(x'-\frac{C}{A}\right)=-\frac{1}{A^2},$$

which, by virtue of the relation $BC-AD=1$, reduces to the equation given by Mr. Smith, namely:

$$Axx'-Bx'-Cx+D=0.$$

For the three points which Mr. Smith calls F, U and V, it follows that

$$\mathrm{VF}=\frac{C}{A},\quad \mathrm{VU}-\frac{D}{B},\quad \mathrm{UF}=\frac{1}{A\cdot B}.$$

The five classifications above are evident from these relations.

J. P. C. S.

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SUMMARY OF THE LITERATURE RELATIVE TO THE FORMATION OF FILM ON POLISHED GLASS SURFACES

By George W. Morey

The stability of polished glass surfaces is of prime importance in the design and manufacture of optical instruments, and all factors affecting their stability are worthy of careful consideration. Some glasses are inherently inferior in their resistance to the corrosive action of water and weak acids, and such glasses are to be avoided whenever possible. Experience has shown the relative reliability of different glass types and glass compositions, and tests have been devised which enable the optician to avoid actually inferior types.[1] But glasses possessing a high degree of resistance to such "weathering" action often give trouble by becoming covered with a coating resembling that produced by the weathering of an inferior glass, but actually due to entirely different causes. This particular type of coating has been called "film," and the following report summarizes the information contained in the literature on the formation, appearance, cause, and prevention of "film" on polished glass surfaces.

The phenomenon in question is thus described by Ryland:[2] "In every type of enclosed optical instrument, instances, more or less numerous, of film will be found. By film is meant the coating of lens and prism surfaces with what is apparently a thin deposit of moisture. This has caused much trouble and annoyance, many instruments being rendered quite useless until taken apart and cleaned. The film appears to have several characteristics. It apparently consists of a series of globules of moisture, forming patterns on the surface similar to those made by cleaning with a linen rag which has seen much service. Often a series of larger globules will occur, each surrounded by a clear space.

[1] G. W. Morey, J. Soc., Glass Technology, *6*, p. 20–30; 1922.

[2] H. S. Ryland, Trans. Opt. Soc., *19*, p. 178–83; 1918–19.

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Parts of a surface under pressure from mounts will be clear for a small space surrounding the points of contact, the clear space being fringed by a denser deposit. Prisms will often show a distinct pattern of the seat, the part covered by metal having a fairly dense deposit, a fairly clear space occurring where the surface is free, becoming again more dense away from the metal. A cut, scratch or hole in the surface is generally surrounded by a clear space, beyond which the deposit is again more dense.

"If a drop of water from a pipette is allowed to run over a filmed surface, it will leave a track which is quite free from any trace of scum. Microscopic examination shows no injury to the surface after filming. Instruments such as telescopes, in which the air is occasionally changed, very seldom film. When they do it is generally the case that they have been out of service for some time."

The above description of filming agrees well with that of other writers. That, even in cases in which the film appears uniform to the unaided eye, it actually consists of tiny discrete drops, is stated by Ryland (loc. cit., p. 183) and by Jones.[3] The tendency of film to form on reticules ("graticules") and similar etched surfaces is often mentioned;[4] an excellent photomicrograph of such a deposit in a reticule from a prism binocular is given by Martin and Griffith.[5] That such deposits are usually found in instruments which after assembly are made air and water tight as opposed to those in which there is opportunity for circulation of air is brought out by Jones, by Ryland and by Beck. The latter also states that "The Admiralty informed him that they have never met with it (film) inside submarine periscopes which are hermetically sealed and filled with dry air. He was told that it had not been met with in the Aldis unit sight when properly sealed, except in some experimental cases when a coating of

[3] Remarks on the filming of glass. By H. S. Jones of the Inspection Dept., D. I. O. S. Report submitted by Lt. Col. A. C. Williams, Dept. of Scientific and Industrial Research, Standing Committee on Glass and Optical Instruments.

[4] Discussion on the filming of glass held by the British Optical Instrument and Manufacturers Association.

[5] L. C. Martins and C. H. Griffiths. Trans. Opt. Soc. London, *20*, p. 135–54; 1920.

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glycerine had been placed on the internal surfaces to pick up floating dust particles." Martin and Griffiths state that film is most often found in reticules, next in field lenses of eye pieces, next in prisms.

It follows from the above that in enclosed optical instruments a deposit, consisting of more or less discrete drops, tends to form on the glass surfaces, most especially on reticules. It also follows, e.g., from the data furnished by Beck, that such film is not met with even in enclosed instruments, when such instruments are assembled with great care, as is the case with periscopes. The important conclusion that the film is not a permanent attack of the glass surface, but on the contrary the original surface appears unchanged even when examined under the microscope after washing off the film with distilled water, is confirmed by Martin and Griffiths. The latter report: "As far, at least, as the globular deposit is concerned, the surface of the glass shows no trace of corrosion, when examined under the microscope, after the deposit of film has been washed off."

The cause of film has not yet been established. Two factors, however, seem to be necessary; the presence of water (moisture) and the presence of dirt or grease, especially the latter. French,[6] Chalmers,[7] and Wright[8] emphasize this point. Ryland writes, "A number of experiments have been tried in order to determine the cause of film. Certain types of black, notably the japan black and those containing pitch or bitumen, almost always cause film; aluminum machined and not cleaned afterwards; a small trace of animal matter or beeswax in the closed space will give trouble." Martin and Griffiths state that "moisture alone has no affect in producing the globular deposit" and that "the presence of lubricants does not produce globular deposit nor disintegration of surfaces unless moisture is present." The consensus of opinion is that, irrespective of the stability of the glass, elimination of dirt, grease and moisture are essential to the prevention of film.

[6] See discussion of the paper by Ryland, loc. cit.

[7] See discussion of the paper by Martin and Griffiths, loc. cit.

[8] "The Manufacture of Optical Glass and of Optical Systems," Lieut. Col. F. E. Wright. Ordnance Dept. Document No. 2037.

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Ryland, after outlining precautions to be taken in cleaning, writes, "The use of these precautions will apparently make even poor glass remain free from film." French, Beck, Chalmers, Jones and Wright all agree as to the imperative necessity of meticulous cleanliness in assembly. Jones states that, "Generally speaking, it may be said that those firms who had taken the greatest care with the cleaning to ensure absolute cleanliness had the least trouble with film, whilst those who had taken fewest precautions to ensure cleanliness had most trouble." He also cites this case: "One firm who have had very considerable trouble from film, noticed that not a single instrument in which lenses etc. had been cleaned by the foreman of the assembly department had been rejected for film, and that the ordinary cleaning methods were used, i.e., without special precautions."

The nature of the glass, however, is probably not without importance. While Ryland asserts that proper precautions will prevent film on even poor glass, he states that "Different glasses however appear to have different susceptibility to film; the most troublesome glass I have found to be one made of borosilicate crown." In the discussion on filming held by the British Optical Instrument Manufacturers Association, it was brought out that "generally speaking, glasses showing the greatest affinity for water give the most trouble," though in this connection it should be noted that borosilicate crown, designated by Ryland as the most troublesome, is not especially susceptible to weathering. In addition to laying stress on cleanliness in assembly it is stated that "Care should be taken in the choice of glass. Hygroscopic glasses are to be avoided." Martin and Griffiths in a series of experiments on the formation of film, were unable to detect any difference between ordinary crown and light flint. On the other hand they state that no film had ever been observed on reticules made by a certain German firm, who used a light barium flint, Jena O 463. The evidence collected by Jones on this point is mainly negative, with the exception of an instance cited in which, "One firm had a lot of trouble with the graticulated discs of prismatic binoculars owing to the formation of globular deposits. This has been entirely overcome by simply changing from plate

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glass to baryta light flint, without alteration of methods of cleaning." This latter case is probably one in which an actually poor glass had been used, as opposed to the majority of cases in which film was formed on fairly resistant glass, as a result of careless cleaning. It shows the necessity of choosing a good type of glass for the manufacture of such parts as reticules, though the light barium flint is probably not superior to many other glasses in common use.

Following are the precautions enumerated by Ryland as a complete preventative of filming:

1. No japan or bitumen black to be used on interior surfaces.

2. No pitch or beeswax to be used within the case.

3. All interior surfaces must be thoroughly cleaned and free from all animal matter such as finger marks. In the case of aluminum or other porus metal, heat treatment is necessary after machining to remove all trace of grease.

4. The lenses and prisms after ordinary cleaning to be dipped in hot running water with a pair of tweezers and wiped dry with clean linen (see 5 and 7) without being touched by the fingers again.

5. After washing the linen used for cleaning purposes, it must be well rinsed in hot water until all trace of soap is gone.

6. Dust must be removed with a clean camels hair brush and not by blowing.

7. The linen used for cleaning must be used in such a way that no part of it which has been touched by the fingers is brought into contact with the lens or prism surfaces.

8. All surfaces must be well polished and free from "orange peel" effect. If the pitch polisher leaves a trace of scum, a small quantity of glacial acetic acid added to the pitch when melted will prevent it."

While all writers do not agree that the above precautions will absolutely prevent film (Martin and Griffiths, Beck) there seems to be no doubt as to their being efficacious in most cases. In regard to the individual precautions, in some cases japan or bitumen do not appear to be at fault: "The black used by some American firms has not been found to be satisfactory in some instances,

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and improvement had resulted in the use of a black with a reliable composition" (Jones). That precaution No. 3 is essential appears to be without dispute. Precaution No. 4, dipping lenses, after ordinary cleaning, is recommended by Martin and Griffiths, whose experiments show clearly its efficiency.

From the facts enumerated, the following conclusions can be drawn with some degree of assurance.

1. Filming, i.e., the formation of more or less isolated drops on the polished surfaces of glass instruments, is a result of the deposition of water contaminated by greasy matter.

2. Both water and grease are essential to the formation of film, and hence the utmost care must be taken to thoroughly clean the glass surfaces themselves, to ensure that no moisture or grease, or material which may yield moisture or grease, is contained in the space within which the glass surfaces are confined.

3. The glass should be of good quality, at least as good as Class 3 as determined by the iodeosine test.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
JULY, 1922.

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A SUSPENSION TO ELIMINATE MECHANICAL DISTURBANCES

By Albert P. Carman

The writer recently was stopped for weeks in an investigation by troublesome vibrations due to various causes, but principally to the passing of heavy motor trucks. The laboratory is a brick building with very heavy masonry walls and with numerous masonry cross walls, and delicate instruments are not in general disturbed by mechanical disturbance even in the upper stories, but heavy motor trucks rolling over neighboring brick pavements with their concrete foundations shake the earth for considerable distances and with it the whole building. These earth vibrations seem to be particularly troublesome in the basement. Numbers of devices were tried to eliminate the vibrations, including a modified Julius suspension, but without success. Finally attention was called to a paper by Airy (Royal Astron. Society Monthly Notices, *17*, p. 160; 1856), which has suggested the solution which we have worked out for our difficulty. Airy devised a support for the mercury vessel of the reflex-zenith tube at Greenwich Observatory, using a series of platforms suspended by broad bands of vulcanized caoutchone, platform number one, carrying the mercury vessel, being suspended from supports carried on platform number two, and this being in turn suspended from supports carried on platform number three. Fig. 1 shows our scheme. A series of triangular wooden boxes, *A*, *B*, and *C*, were loaded with scrap lead. Box *A* was suspended from a rigid support resting on the concrete floor by three fine piano wires *DDD;* the box *B* was suspended from supports on *A* by pure gum tubing *EEE;* and the box on platform *C* was suspended from supports erected on *B* by another size and length of pure gum tubing *FFF*. The platforms were also connected at a number of points by small rubber bands hooked over tacks, so as to dampen quickly swaying motions. On each piano wire, there was a cylindrical weight of about 500 grams that could be clamped at any desired

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point on the wire, so as to eliminate some particular vibration which might be otherwise transmitted. The galvanometer rested on platform *C*. It is a high sensitivity Leeds and Northrup moving coil galvanometer of the ballistic type and is extremely sensitive to small mechanical vibrations. The device has proved to be effective in eliminating our vibration difficulties and it is reported here, with the thought that it may aid others who have

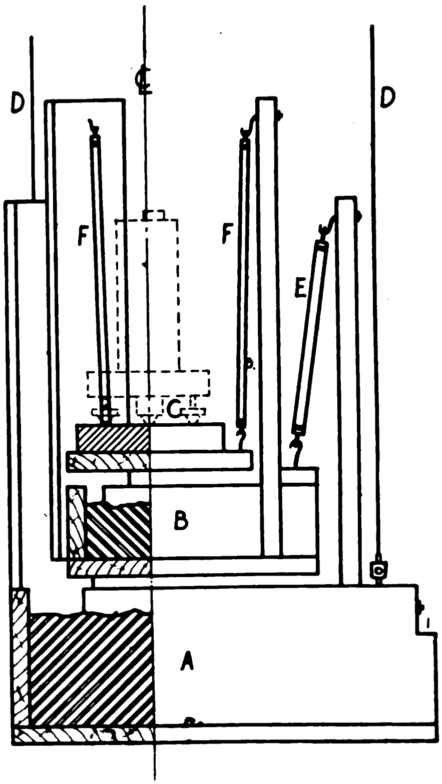

FIG. 1

similar disturbances. The device has the merit of being quickly and easily made, and of requiring no long tedious adjustment such as is demanded in setting up the usual Julius suspension. By changing lengths and sizes of suspension tubing and weights of platforms, and by adding one or more platforms, it ought to be possible to eliminate small vibrations in any case.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
MAY, 1922.

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A FORM OF IRON CLAD THOMSON ASTATIC GALVANOMETER

By B. J. Spence

ABSTRACT

A form of iron clad Thomson astatic galvanometer of high sensibility is described. It differs from the types described in the literature in that it is semi-portable, has a removable core containing coils and suspended magnetic system, has comparatively small weight and is easily adjusted to a high sensibility.

The early forms of the iron clad Thomson astatic galvanometer of high sensibility as used for radiometric study in connection with the bolometer or thermopile were bulky, heavy, and difficult to adjust for a maximum sensibility. The galvanometer has under gone a number of modifications diminishing somewhat the weight and bulk. More recently Coblentz[1] has designed a galvanometer of this type in which he has embedded the coils in soft iron. This galvanometer appears less bulky and is perhaps easier of adjustment than the earlier forms. In connection with the development of this galvanometer he has investigated the effect of coil construction, magnetic shielding, of different types of magnetic systems and of evacuating the region in which the system is suspended. For further details the reader is referred to the Coblentz article.

The writer has attempted the development of an iron clad galvanometer in which the coils and rotating magnetic system may be removed as a unit from the shielding and which may be adjusted to a high sensitivity with more facility than the earlier forms. The attempt has been also to embody in the design the results of the Coblentz investigation.

The four coils each 13 mm in diameter and 4 mm thick were wound on a mandrel whose surface was shaped approximately according to the expression of Maxwell.[2] Instead of winding the

[1] Bureau of Standards Scientific Papers, No. 282.

[2] Maxwell, Electricity and Magnetism, vol. 2, p. 360.

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coils in sections of double silk covered wire, they were wound with 12 ohms of No. 36 B. S. gauge enameled wire, boiled in half and half (50% resin and 50% bees wax) removed from the mandrel and the faces covered with gold leaf. The four coils were mounted in the two sections of a split hollow brass cylinder 15 cm long, 16 mm outer diameter and 10 mm bore (see Fig. 1). At 45 mm and 60 mm respectively from the bottom of each section were cut two circular holes 14 mm in diameter into which were cemented the four coils. When the sections of the cylinders were placed together the centres of the faces of the coils were opposite each other and 1.5 mm apart. The leads from each coil were carried to the top of each section along the flattened surface of each cylinder. At the top of one of the sections of the hollow cylinder was a removable head containing a screw which carried the suspended magnetic system. The screw could be raised or lowered and rotated for adjustment of the position of small magnets relative to the centres of the coils. When the magnetic system had been suspended by means of a fine quartz fibre the second section of the cylinder could be set in place without danger of breaking the fiber.

The magnetic system consisted of two groups of six each of tungsten steel magnets 1.2 mm long and fastened to a fine glass staff with a trace of shellac. At 30 mm from the lower group of magnets was fastened a mirror 1 mm × 1.5 mm. The system was magnetized and astaticized in the usual manner.

The shielding consisted of a cylinder of Swedish iron 18 cm long and 7.5 cm in diameter. Along the axis of the cylinder was bored a hole a trifle over 16 mm in diameter to a distance 4 cm from the bottom. A conical hole of small dimensions, 13 cm from the bottom was bored through the cylinder wall to the interior to observe the rotation of the mirror attached to the magnetic system. Over this hole was cemented a glass plate. After all machine work had been completed the cylinder was carefully annealed in an electric furnace and then keep away from contact with magnets. It was then mounted carefully on a suitable base with leveling screws. The coils and magnetic system were slipped into the shield and the whole properly adjusted until the magnetic

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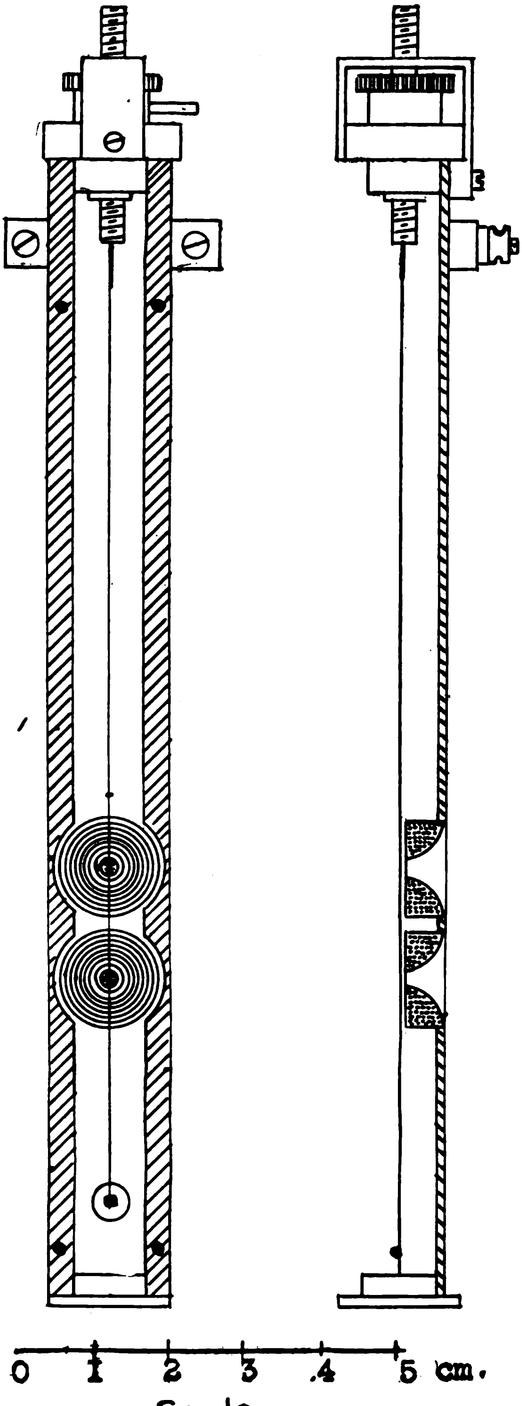

Fig. 1

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system swung freely. The ease of this adjustment to a large extent depends upon the care with which the machine work has been done. If the axis of the core contained in the cylinder is perpendicular to the plane of the base the adjustment can be carried out by placing a good level on the base.

In order to control the zero and adjust the instrument for high sensitivity a weak magnet of watch or clock spring was covered with paper and shellaced and slipped about over the cylinder.

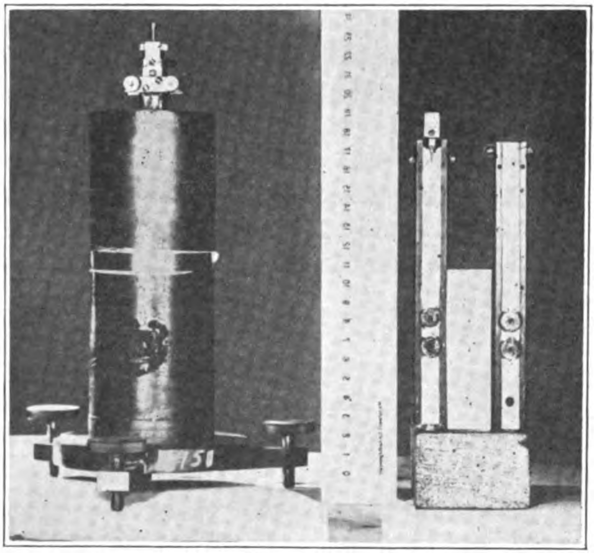

Fig. 2

By carefully manipulating this control magnet a high degree of sensitivity was attained in a comparatively short time. With this instrument of 3 ohms (coils in parallel) a sensitivity of 5×10^{-10} amperes with a scale at a meter distance could be attained readily. The period was about 1.5 seconds. The shielding appeared to be ample for this sensitivity. No zero drift manifested itself due to external magnetic disturbances. There was, however, considerable rapid zero fluctuation of about 2 mm due to building vibrations. These disturbances were amplified by the rotating system owing to the fact that the control magnet intro-

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duced a non-uniform field in the core. A slight displacement due to building vibrations shifted the system slightly into a field of different strength and produced and amplified vibration of the system. The sensitivity was increased to 8×10^{-11} amperes with a period of 3 seconds by increasing the shielding and operating the system in vacuo. The additional shielding was made of a roll of stove pipe iron 20 cm wide with the turns separated by paper and then placed on the base concentric with the Swedish iron. A cap was cemented on top of the Swedish iron shield and the pressure in the core lowered to 1 mm.

In conclusion it may be stated that a semi portable form of iron clad Thomson astatic galvanometer of high sensibility has been developed which is comparatively simple of construction, has small bulk and is comparatively easy to adjust. It had embodied in it the feature of removable core containing system and coils. I am indebted to Mr. F. Kung, our mechanician, who patiently worked out many details of the instrument.

NORTHWESTERN UNIVERSITY,
JUNE 7, 1922.

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A LOW VOLTAGE CATHODE RAY OSCILLOGRAPH

By J. B. Johnson

ABSTRACT

A sensitive cathode ray oscillograph tube is described which operates at a low voltage. The electron stream comes from a thermionic cathode, and is focused by the action of the ionized gas in the tube. Illustrations show examples of the use of the tube.

A cathode ray oscillograph tube operating at a comparatively low voltage was described by the writer some time ago before the American Physical Society.[1] Since then, the tube has been further improved and its operation studied so that now both the structure of the tube and the principles which have made the construction possible can be described in greater detail.

In the older types of Braun tubes the electron stream is produced by a high voltage discharge through the residual gas in the tube. This requires a source of steady potential of from 10 000 to 50 000 volts, an installation which is expensive, non-portable, and dangerous. In the new type of tube the low voltage operation has been obtained by the use of a Wehnelt cathode as the source of electrons, so that the lower limit of voltage is set by the effect of the electrons on the fluorescent screen and not by the voltage needed to obtain the electrons. At 300 volts the electrons produce quite bright fluorescence on the screen and the tubes are therefore designed to operate at 300 to 400 volts.

The external appearance of the tube is shown in Fig. 1. The electrodes are located at one end of the pear-shaped bulb, and the fluorescent material is deposited on the inside of the larger, flattened end. The tube is provided with a base which fits into a bayonet socket such as is used for vacuum tubes, and all the connections are made through the base. There are two orthogonal pairs of deflector plates inside the tube for electrostatic deflection, while magnetic deflection is produced by applying a field from the outside.

[1] Phys. Rev. (2), *17*, p. 420; 1920.

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The internal structure differs considerably from that of previous forms of Braun tube and it will therefore be described somewhat fully.

The Focusing

In some forms of Braun tube a sharp spot has been secured by using a very high voltage, and therefore high electron velocity,

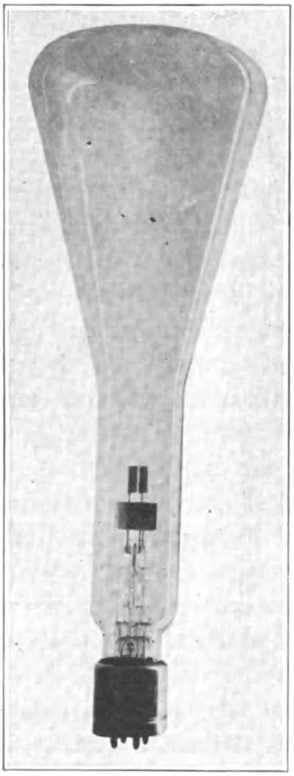

Fig. 1. The Cathode Ray Oscillograph Tube

so that after the electrons have passed through one or two fine apertures to make the beam parallel there is not time enough for the mutual repulsion to spread the beam again appreciably before the electrons strike the screen. With other tubes an external

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"striction" coil has been used which maintains a strong longitudinal magnetic field in the region between the anode and the cathode and which brings the electrons to a focus on the screen. In the low voltage tube the spreading of the electron stream is greater than in high voltage tubes because of the greater time during which the mutual repulsion of the electrons acts, so that some means of focusing must be used. The electrons can be brought to a focus by a longitudinal magnetic field so adjusted that each divergent electron makes very nearly one complete turn of a spiral and in travelling the length of the tube returns to the axis at the screen. In this way a very sharp spot can be produced, but the sensitivity of the beam to deflection is reduced very much by the directing magnetic field.

The method of focusing that is used in the present tubes grew out of the suggestion by Dr. H. J. van der Bijl, that a small amount of gas be introduced into the tube. This gas, at a pressure of a few thousandths of a millimeter of mercury, serves to reduce to 1 mm diameter a spot which would be 1 cm across in a high vacuum tube. The sharpness of the spot depends also upon the current in the electron stream so that the focus may be controlled by the cathode temperature. The mechanism of this focusing action will be explained later.

The presence of this slightly ionized gas also serves the purpose of preventing the accumulation of charges on the glass, and it provides for the discharging of the fluorescent screen so that the electrons can drift back to the metallic circuit.

THE ELECTRODE UNIT

With gas present in the tube, steps have to be taken to guard against arcing and the injurious effects of positive ion bombardment on the cathode. This is done by making the volume of gas surrounding the electrodes very small. For this purpose the cathode and anode, themselves small, are sealed into a short and narrow glass tube so that the volume exposed to both electrodes in common is less than 1 cm^3. All paths between the electrodes are then so short that at this low pressure there is not sufficient ionization to build up an arc.

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The structure of this unit, or "electron gun" is shown in Fig. 2. The cathode, *f*, is an oxide coated platinum ribbon of the same kind as the filament in our audion tubes. The anode, *a*, is a thin platinum tube 1 cm long and 1 mm in diameter, one end of which is about 1 mm from the top of the filament loop, the other end opening into the main tube towards the fluorescent screen.

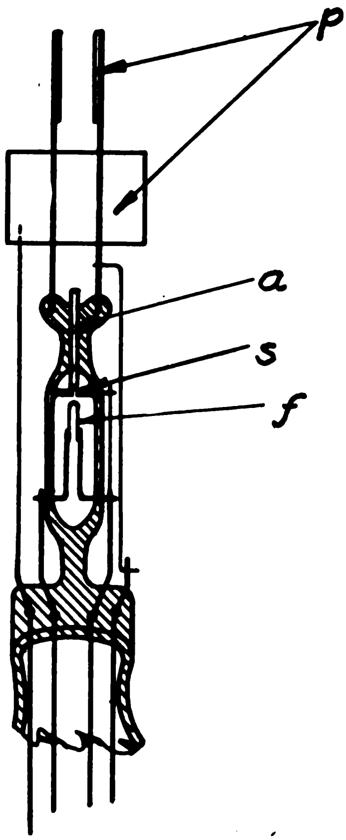

Fig. 2. The Electrode Unit

Between the cathode and the anode and connected to the cathode is a metal shield, *s*, with a small aperture through which the electrons must pass in going to the anode. Nearly all of the electrons must then go to the inside of the tubular anode, and a

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small fraction of them passes through the whole length of the anode and form the beam in the main part of the tube.

The deflector plates, p, are also mounted rigidly on this unit. In order to avoid large differences of potential in the tube, one plate from each pair is permanently connected to the anode, the variable potentials being applied to the other plates. The complete unit is mounted at the small end of the tube with the anode and deflector plates toward the fluorescent screen.

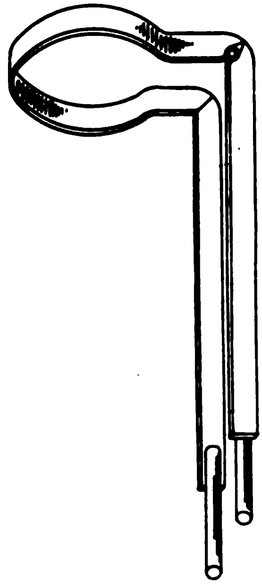

FIG. 3. The Thermionic Filament

THE FILAMENT

In some early forms the filament was bent into a simple hair pin loop which was placed close to the aperture in the shield. It was then found that the positive ions striking the filament from the direction of the anode soon destroyed the oxide coating and left the filament inactive. This trouble was largely overcome by placing the filament out of the direct path of the positive ions. The flat filament is now shaped into a ring as shown in Fig. 3, slightly larger in diameter than the aperture in the shield and is

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placed coaxial with the anode. The momentum of the positive ions then carries them past the active part of the filament and they strike where little damage can be done. The length of service of the tube is still limited by the filament life, but this has been increased by the above artifice so that the tube now gives around 200 hours of actual operation.

THE DEFLECTOR ELEMENTS

The deflector plates are made of German silver, which is non-magnetic and which has a high specific resistance that diminishes the effect of eddy currents when magnetic deflection is used. The plates are 13.7 mm long in the direction of the tube axis and the separation between them is 4.7 mm.

The sensitivity of the tube is such that the deflection of the spot is about one mm per volt applied between the deflector plates. When using magnetic deflection, a pair of coils 4 cm in diameter placed on the sides of the tube at the level of the deflector plates produce a deflection of approximately 1 mm per ampere-turn flowing in the coils.

The electrons striking the screen drift back to the anode structure, and most of them are collected by the deflector plates. There is also a small ionization current flowing to the plates. The tube is therefore not strictly an electrostatic device, and this must be kept in mind when using it. Fig. 4 shows the current flowing to the two free plates at various voltages with respect to the anode. With the large positive values of plate voltage the current to the plates is practically equal to the current in the electron stream and consists largely of the returning electrons. The small current in the other direction when the plate voltage is negative is a measure of the ionization in the tube.

THE FLUORESCENT SCREEN

The screen is spread on the inner surface of the large end of the tube, using pure water glass for binder. The active material consists of equal parts of calcium tungstate and zinc silicate, both specially prepared for fluorescence. This mixture produces a generally more useful screen than either constituent alone. The

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pure tungstate gives a deep blue light which is about 30 times as active on the photographic plate as the yellow-green light of the silicate, while the silicate gives a light which is many times brighter visually than that from the tungstate. By mixing the two materials in equal parts a screen is produced which is more than half as bright visually as pure zinc silicate and more than half as active photographically as pure calcium tungstate.

For mechanical strength the end of the bulb which carries the screen is rounded outwards so that the screen is not a plane sur-

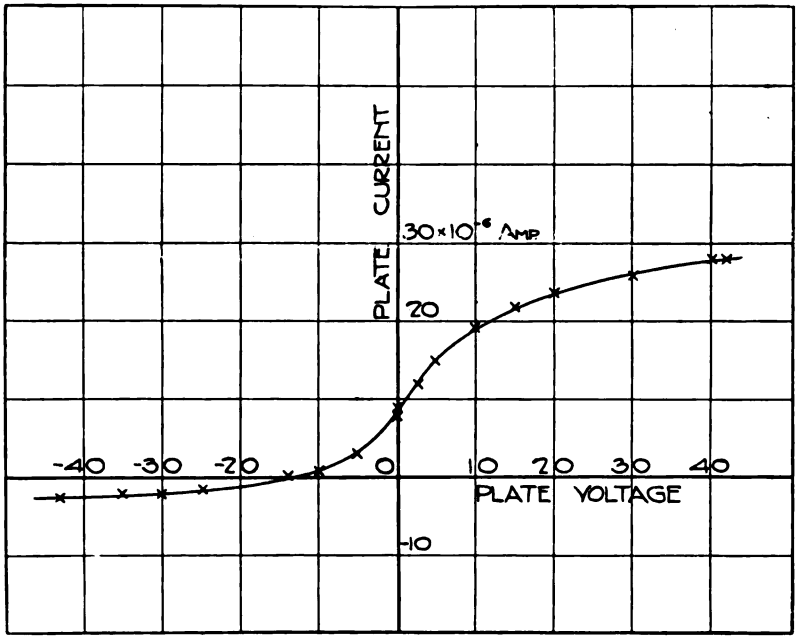

FIG. 4. Current to Deflector Plates

face. This introduces a distortion of the fluorescent pattern which in most instances is negligible. If the pattern is recorded by a camera whose lens is D cm. from the end of the tube, then the apparent reduction of the deflection produced by the curvature of the bulb is given in terms of the deflection y approximately by

$$\Delta y = \frac{20+D}{400D}\, y^3 \text{ cm.}$$

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THE FUNCTION OF THE GAS

The part which the gas plays in focusing the beam of electrons is an interesting phenomenon which depends upon the difference in the mobilities of electrons and positive ions. The electrons of the beam are pulled toward the common axis by a radial electric field produced by an excess of positive electricity in the electron stream and an excess of negative electricity in the space outside the beam. This distribution is produced as follows. Some of the electrons of the stream, in passing through the gas, collide with gas molecules and ionize them. Both the colliding electrons and the secondary electrons leave the beam but the heavy positive ions receive very little velocity from the impact and drift out of the beam with only their comparatively low thermal velocity. Positive ions therefore accumulate down the length of the stream and may exceed in number the negative charges passing along. At the same time, electrons are moving at random outside the stream, producing negative electrification. There is then a field surrounding the stream which tends to pull the electrons inward. If there were only the mutual repulsion between the electrons to compensate for, this would be done when the number of positive ions in the beam equals the number of electrons. There is in addition an original divergence of the beam which must be overcome. If this divergence is assumed to be one degree from the axis and the electron current 2×10^{-5} amp, then a simple calculation shows that the radial field required to pull the beam to a focus at the usual distance is about one volt per cm. This field strength is produced, with beams of the ordinary intensity, if there are four positive ions for each electron in the stream, a condition which seems not unreasonable.

The number of ions per electron in the stream is probably constant as the current in the stream is varied, since the conditions of collision and recombination are not altered. When the current is increased, therefore, the total positive ionization of the beam increases, the field around the beam becomes stronger, and the electrons are brought to a focus in a shorter distance.

These deductions have been confirmed experimentally. That the focusing of the stream depends upon the current flowing

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was one of the earliest observations made in developing the tube and this method had been used ever since to obtain a sharp spot. The point of convergence can be seen moving in the manner expected when the current is changed, and the effect has been further verified by using a tube with a movable fluorescent screen so that the length of the electron beam could be varied. The presence of the electric field around the beam was shown by the effect of two beams on each other, in a tube in which there were two electron streams crossing each other at right angles at their mid-points, each falling on a fluorescent screen. When one beam was moved away from the other by a field between the deflector plates, the second beam moved as if attracted by the first. The directed electrons in each beam were attracted toward the positive ionization in the other, and for one particular adjustment of the tube the displacement was such as would have been caused by a field of about 3 volts per cm, a result not far different from that previously calculated.

Since the beam must produce its own positive ionization some time must elapse before it can produce by collisions the required number of positive ions. Calculation shows this time to be of the order of 10^{-6} second. When the beam moves it has to build up the ionization as it goes along, and we should expect that when deflected very rapidly it might no longer be focused, due to lack of positive ions in its path. A test was made of this by applying a high frequency potential on the deflector plates so that the spot described an elliptic pattern. At a frequency of 10^5 cycles per second the line was still sharp, but at 10^6 cycles there was a noticeable widening of the line which is probably to be ascribed to imperfect focusing at this high speed.

In these experiments the evidence all points to the view that the focusing of the electrons is caused by an excess of positive charge in the beam itself, produced by the ionizing collisions of the electrons with the gas molecules. Further confirmation is found in the fact that a focus is much more readily obtained in the heavier gases having slow molecules, such as nitrogen, argon or mercury vapor, than in hydrogen and helium where the mean velocity of the molecules is greater. The tubes are therefore

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filled with argon, the heaviest available permanent gas which does not attack the electrodes. The best pressure for the length of tube adopted and for the current which can be obtained in the beam is 5 to 10 microns, and this leaves considerable latitude for the adjustment of the electron current to get a sharp focus.

Examples of the Use of the Tube

Because of the small amount of auxiliary apparatus required with this form of Braun tube it has proved to be a very convenient

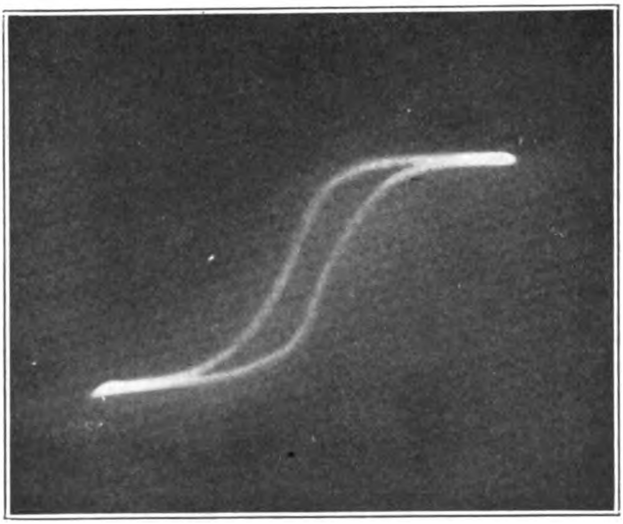

Fig. 5. Iron Hysteresis Curve

laboratory instrument. It has found application in studying the behavior of vacuum tubes and amplifier and oscillator circuits, of gas discharge tubes, of relays, and of numerous other kinds of apparatus, both at low and at high frequencies. Some reproductions of photographs of various types of curves are given below to illustrate the kind of results which are possible with this oscillograph.

Fig. 5 shows the hysteresis curve of a sample of iron wire. The wire was placed in a small solenoid with one end toward the side of the tube. The magnetizing current passed through a resistance,

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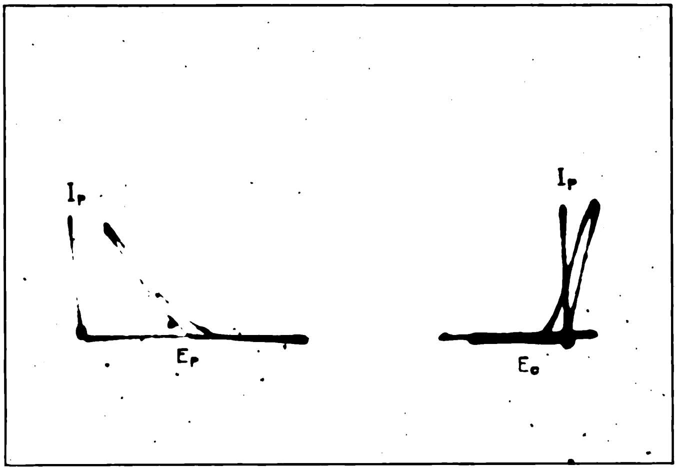
the voltage drop of which was applied to one pair of deflector plates so as to give a deflection proportional to the magnetizing field. The stray magnetic field from the iron itself produced the deflection proportional to the induction. Alternating current was used, and the exposure was 20 seconds with lens opening *f* 6.3 and speed roll film.



a *b*

FIG. 6. Characteristics of Oscillating Vacuum Tube

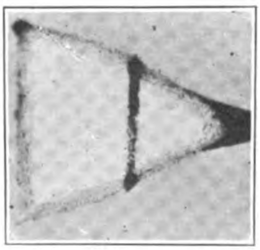


FIG. 7. Modulation of High Frequency Current

In Figs. 6a and 6b are shown the current-voltage relations of an oscillating vacuum tube. The axes were obtained by grounding one or the other deflector element.

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The measurement of modulation in a radio transmitting set has been reduced to a fairly simple process by means of the cathode ray tube. The low frequency modulating voltage, controlled by the voice, is applied to one pair of deflector plates, while the radio frequency output, with amplitude varying according to the low frequency voltage, is applied to the other pair of deflector plates. The resulting pattern on the screen is a quadrilateral of solid fluorescence, since the two frequencies are not commensurate. The two vertical sides indicate the greatest and the least amplitude of the high frequency, while the other two sides show the current-voltage characteristic of the transmitter. Fig. 7 shows such a pattern (retouched), the edges being much brighter than the center. The exposure was two minutes using a Seed 23 plate and f 6.8 lens opening.

Research Laboratories of the
Western Electric Co., Inc. and the
American Telephone and
Telegraph Company
New York City

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SPECIFICATIONS FOR RECORDING VAPOR-PRESSURE THERMOMETERS AND FOR PRESSURE GAGES

By Frederick J. Schlink

The specifications for recording thermometers and for reference standard pressure gages, the first of which are presented below, are believed to be the first specifications applying to these instruments ever published, although with respect to pressure gages of the more usual or service types, some codes of specifications more limited to constructional arrangements and details have been available heretofore. The present specifications for service pressure gages will be found to differ from existing codes in many important essentials, and to extend the scope and precision of specified requirements to a significant degree. Points of marked difference from common practice will be noted in the following: the zero stop pin eliminated, permitting secular changes of the Bourdon tube to be noted on the dial, as they occur; graduation circles required to fill substantially the whole dial diameter; the rated maximum pressure capacity established as the actual upper operating limit of the gage, rather than 100% higher, as is now a common practice with some makers; anomalous graduations in the zero region of the dial eliminated (for instance, four graduations in the first five pound interval have been almost the regular practice in American made gages, following some traditional error of an early gage maker; the present specifications prefer to permit omission of these first few graduations, in the region where the scale factor is not constant, and backlash is a special source of difficulty). Other points are the restriction of the radial length and thickness of graduation lines so as to increase the accuracy of readings; and the basing of the tolerances of error on the value of the minimum graduation. This latter, in a very real sense, the maker holds out as a measure of the precision of his instrument and to this therefore the tolerances should be clearly related

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when once the user has decided how small the minimum graduation need be to meet the requirements of his service. It may be said in passing that the tolerances herein set down are perfectly practical and attainable ones in respect to new gages of good manufacture with the types of graduation customary in the United States (40 to 80 graduations in 270° of arc). Considerable purchases have been made under the requirements of these specifications at little or no increase in price, but with a very significant increase in quality over the ordinary product, which experience and extensive investigation have shown to be surprisingly variable as between different makers and even between different samples of a given maker's product.

These specifications were used to control purchases of all instruments of the types named, employed in production departments of the Firestone Tire and Rubber Company at Akron, Ohio; and the marked success met with in their application in spite of the radical changes in construction and details which in some respects they require, has led to their publication in the hope that they may be of general interest and utility.

The degree of precision attainable with properly selected instruments of these two types under proper conditions of use, is quite surprising, and such as to make them adaptable to many of the uses of the physicist. For instance pressure gages, when used under proper cyclicization before reading, or when jarred energetically by a securely mounted buzzer, can give results accurate to $\pm 0.1\%$ under laboratory conditions when a calibration curve is used; while a recording thermometer of the vapor-pressure type, meeting these specifications in all essentials, will record temperatures over a limited range, say 25°, within $\pm 0.1°$ C, when a calibration curve is used, and the instrument is occasionally checked at some convenient reference temperature. In fact there are applications wherein the vapor-pressure recording thermometer, in view of its freedom from variable stem immersion errors, may afford a net accuracy definitely superior to that of a good mercurial thermometer.

June 25, 1922.

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SPECIFICATIONS FOR VAPOR-PRESSURE TYPE RECORDING THERMOMETERS AND CHARTS

June 11, 1920.

1. General Requirements

a. *Type.* These recording thermometers shall be of the vapor pressure type, so as to be substantially unaffected in their readings by the temperature of the tube connecting the bulb and the registering element.

b. *Use.* These recording thermometers are to be used to determine the temperature of (here define conditions of use fully and specifically to permit of proper design and choice of material of bulb, etc.)

2. Identification of Instruments

a. Each recording thermometer shall be equipped with an identification plate plainly visible without requiring the removal of any part of the mechanism, on which shall be clearly and permanently inscribed: the type and maker of the instrument; the month and year of manufacture; the range of registration; identification number of the chart used; and the intended relationship between the elevation of the bulb and that of the recording element. The information furnished shall be at least as complete as in the following specimen:

Recording thermometer, vapor-pressure type.
Maker: The Jones Company—Mar. 1919, (or 3-'19.)
200–300°F. Chart 721.
Bulb 10 ft. below Bourdon Tube.

3. Case Construction

a. *General.* The construction of the case, in respect to strength, rigidity and dust tightness, shall be the equal of that used on ———— recording thermometers, as described and illustrated (name catalog and page or other defining reference).

b. *Door and Hinge.* The door hinge shall be of strong and durable construction with an adequate amount of metal to withstand such bending and twisting strains as the door may commonly be subjected to in plant service. The door shall be equipped with plate glass of good quality so set into the door frame as to comply substantially with the requirements as to dust tightness provided for above. The hasp and staple of these recording thermometers shall be adapted to take, easily, a lock having a staple diameter of 7/32″, the radius of the center line of this lock staple being approximately 5/16″.

c. *Finish.* The surface finish of these instruments shall be neat and shall afford a protective coating of such thickness and durability as to protect the material of the case from the effects of moisture and abrasion in reasonable plant service.

d. *Holes for Mounting.* These recording thermometers shall be provided with at least three screw holes approximately 0.25 inch in diameter, by which the instrument may be secured to a supporting surface. If the case is of circular outline, the screw holes shall be spaced at equal angles, one of these holes to be located exactly at the top or bottom position.

4. Responsiveness

a. Time response.

The time-quickness of response of these recording thermometers to changes of temperature shall be the maximum possible without requiring too great reduction of

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the bursting strength of the bulb. The minimum wall thickness and maximum heat conductivity of the bulb shall be employed consistent with the requirements of strength, corrosion resistance, and other essentials.

b. Temperature response.

These recording thermometers shall respond to a change of temperature of 0.3°F. or less; that is, the passiveness, as defined on page 745 of Bureau of Standards Scientific Paper 328[1] shall not exceed 0.3° F. over the temperature region between ——— and ———° F. The present specification refers to the minimum change of temperature as the result of which the instrument will show a discernible change of indication, regardless of the direction or amount of the antecedent movement of the recording pen. This factor is to be clearly discriminated from that covered by the preceding specification, which refers to the promptness of response.

5. Arrangement of Bulb

a. The sensitive and active portion of the bulbs of these recording thermometers shall be so constructed and so isolated by the distance of their projection beyond the union or by necking down of the bulb wall back of the bulb or by these and other suitable means,—from the remainder of the instrument tubing and connections, that when the bulb connection is screwed within a standard pipe fitting or the walls of a vessel containing saturated steam in movement past said bulb, the temperature readings will be substantially unaffected by the temperature of the walls of the vessel at and adjacent to the point where the bulb fitting is applied. The maker shall specify the minimum radial distance to be provided around the bulb when the stem is screwed longitudinally into a standard pipe fitting in order that the temperature readings will not be affected to the extent provided for in the tolerance hereinafter to be specified. No separable sockets are to be furnished or used in calibration. A union, to avoid the necessity of twisting the capillary tubing in applying the instrument, shall be provided, of standard pipe thread size not in excess of ¾″.

6. Armoring of Capillary

a. The protecting armor, if any is employed, around the capillary tube, shall be similar and equal to that used in the ———— Company's flexible, bronze-armored capillary tubing. Protecting armor will not be essential provided that a sufficiently rugged solid wall copper or bronze tube of reasonable flexibility is furnished, subject to prior approval of sample by the purchaser. The capillary tube and its armor, at the point where these enter the case of the instrument shall be reinforced and rigidly and permanently secured to the case in such a manner as to protect the capillary tube from strains of a character likely to deteriorate or otherwise damage it.

7. Clocks

a. The clocks to be furnished in these recording thermometers shall be of such construction that they will run at least 50 hours on one winding and during the first 24 hours of running their deviation from correct time shall not exceed 5 minutes at a temperature of 80° F., while during the second 24 hours the deviation shall not exceed 10 minutes. The clock movements shall be enclosed in dust-tight metallic boxes within the case of the recording thermometer.

[1] Variance of Measuring Instruments and its Relation to Accuracy and Sensitivity, by the present author.

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8. Recording Pens

a. The pens of these recording thermometers shall be of the V-type and shall be made of material that will not corrode under the action of an ink composed of an aqueous and glyceric solution of dye. These pens shall be accurately and carefully finished and rigidly and neatly secured to the pen arm. The means of attachment of the pen to the pen arm is preferably to be one that will permit of removal without the necessity of unsoldering a joint.

9. Quality of Record

a. The pen line drawn by these recording thermometers shall be of uniform width not exceeding .005 inch and the pen shall draw a full, solid line at any speed of travel relative to the chart not exceeding 2 inches per minute.

10. Pen Arm Lifter

a. These recording thermometers shall be equipped with a pen lifting device so constructed and positioned that the chart may be readily removed without danger of straining the pen movement or pen arm, and operating so as to return the pen to the recording position automatically on closing the door of the case.

11. Pen Arm Adjustment

a. The pen arm adjusting device used on these recording thermometers shall be of such construction that it will require no lock nut or equivalent device. Its movement shall be sufficiently stiff, or so restrained by the action of a split nut or screw or other permanently effective means, that it can be accurately controlled in its movement, and will maintain its setting permanently.

12. Attachment of Chart Clamping Screw

a. The chart clamping screw or other device used to secure the chart to its driving arbor, shall be loosely connected to a fixed part of the case of the recording thermometer, by a convenient length of chain, so as to prevent inadvertent loss of this screw during the chart-changing operation. This chain must be so arranged as not to interfere in any way with the movement of the recording pen, or the clock arbor, or with the closing of the case door.

13. Accessories

a. Each recording thermometer shall be accompanied by 100 charts of the proper type, one bottle of non-corrosive, slow-drying ink, and a clock winding key. No padlocks or other locks for the cases are to be supplied unless specifically stated in the order.

14. Charts

a. *Paper and Printing.* The charts shall be made of a quality of paper such that excessive alteration of dimensions with changes in atmospheric humidity, and lateral spreading of the record line, will not occur. The arrangement and numbering of lines and graduations shall be such as to afford a maximum of ease and accuracy in reading. Preference will be given to charts printed in olive green or gray or other neutral tint.

b. *Centering.* There shall be no measurable difference between the diameter of the centering hole of the chart and that of the arbor or spindle upon which it is mounted when in use on the instrument. The burring of centering holes or of peripheries in any charts or eccentricity of punching in excess of .003 inch, will constitute cause for rejection of the whole lot of which they form a part.

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15. TOLERANCE

a. These recording thermometers shall exhibit no error at any point of their graduated scale, due to any cause, greater than ± 1°F.; nor greater than ±0.5°F.[2] at any single point of their range of graduation specified in the order; provided that in respect to the former value no tolerance smaller than that equivalent to .03 inch of pen movement and in respect to the second value no tolerance smaller than that equivalent to .015 inch of pen movement, shall be applied.

16. PERMANENCE OF ADJUSTMENT

a. The quality and aging of the Bourdon tubes shall be such that no secular change of reading at a given temperature subsequently applied, greater than that equivalent to .015 inch of pen movement, will occur during an interval of one month, during which the instrument is out of use or in stock. Moreover when subjected to an oscillatory temperature test having an amplitude between the innermost graduation of the chart and an upper limit lying 10° F. or more below the outermost graduation circle carried out at a rate not to exceed one cycle every five minutes, over a period of twenty-four hours, these instruments shall develop no errors in excess of the tolerances provided in the foregoing specification.

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29 W. 39TH ST., NEW YORK CITY.

[2] These limits applicable, like that in Specification 4-b, to instruments operated through a temperature interval not exceeding 250° F. For larger intervals of indication, the tolerance will need be proportionately increased.

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SIEVE TESTING APPARATUS

By L. V. Judson and R. E. Gould

There has recently been developed at the Bureau of Standards a new projection apparatus intended primarily for the testing of sieves, but readily adaptable to various other purposes.[1]

The Bureau has found by experience that in testing sieves for conformity to the "Standard Specifications for Sieves," the most reliable results are obtained by measuring the wire diameters and determining the number of wires per centimeter, and then computing the opening by the formula

$$O = \frac{10}{N} - W$$

where O = average opening in millimeters
N = number of wires per centimeter
W = average diameter of the wires in millimeters.

Until recently the wires have been measured directly by means of a micrometer microscope. As this process is both tedious and fatiguing to the eyes, a better method was sought. The projection method developed is much quicker and much less wearing on the observer than was the method formerly used. Measurements can be taken on any number of the warp and of the shoot wires of the cloth, and the cloth also examined for maximum openings in a small fraction of the time formerly required.

The final development of this apparatus consisted of several steps: The construction of a suitable light-tight box of proper dimensions; the selection of a microscope combination to give the best general results, of the light source and its location, and of the screen on which the image is cast; the development of a method of measuring this image so as to avoid parallax, of a means of reducing to a minimum the color bands on the edge of the image, and of

[1] The final form of this apparatus was developed after seeing photographs of a projection apparatus developed by Mr. Schoof of the Greenfield Tap & Die Corporation.

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a device for focusing and for moving the sieve at right angles to the beam of light. The apparatus at present consists of a light-tight box about 40 cm square and a meter long with a microscope mounted on one end and a ground glass plate 2 mm thick in the other end. The source of illumination is a microscope illuminator containing a concentrated filament lamp, 6 volts, 108 Watts, connected through a transformer to a 110-volt alternating current supply circuit. The light passes through a lens in the end of the illuminator and is focused on the objective of the microscope.

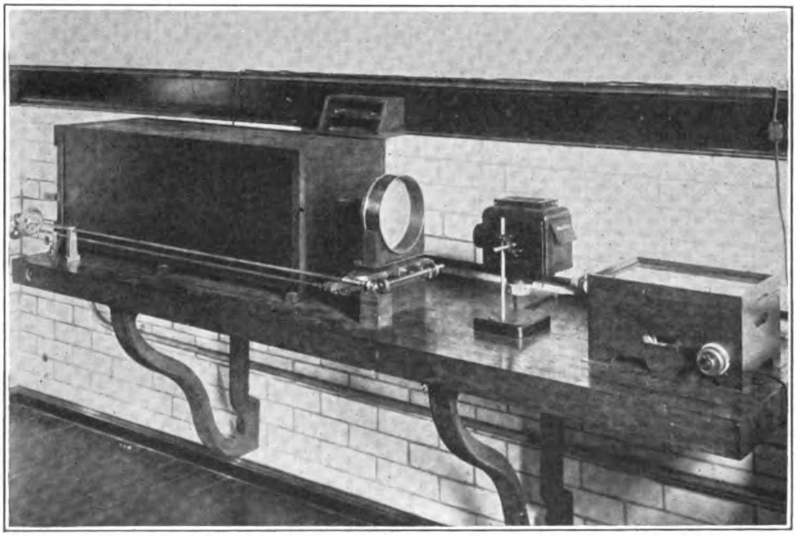

FIG. 1. Sieve Testing Apparatus

After passing through the microscope it diverges to the ground glass plate which is mounted with the ground side in. A 50-cm steel scale is mounted against the inner face of the ground glass screen in such a way that the graduations of the scale may be seen through the glass. The position of the scale allows a direct reading on the edges of the image cast by the wire of the sieve and avoids parallax due to the thickness of the glass. It was found that by oiling the ground surface slightly, the visibility was greatly increased without diminishing the distinctness of the image.

A frame for holding the sieve is placed on a platform so ar-

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ranged as to permit a lateral motion of about 8 inches, and also motion at right angles for focusing. Long rods, extending to the end of the apparatus at which the observer is seated, enable the observer to move the sieve without leaving his place, the lateral motion being accomplished by means of a rack and pinion and the focusing by the use of beveled gears. A green filter is held before the objective by means of a clamp fastened to the tube of the microscope. The filter relieves eye strain very considerably and practically eliminates the color bands otherwise appearing on the edges of the image.

In use, the sieve is mounted in its holder on the focusing platform, between the illuminator and the objective of the microscope, and is focused by the observer until a sharp image is seen on the ground glass. Measurements are then taken in millimeters by reading the positions on the steel scale where the two edges of the image of the wire cross it, a reading glass being sometimes used. The sieve is then moved across the field, readings being taken at several places on the cloth, until the whole diameter of the sieve has been covered, care being taken at the same time to watch for the uniformity of spacing and to measure any excessively large openings. The sieve is then rotated through 90° and the process repeated.

The magnification of the apparatus may be determined by placing a standard wire of known diameter in place of the sieve and making several readings on its image. This should be done at least twice a day to guard against any possible change in magnification while the apparatus is being used.

By using a microscope having a tube about 15 cm long and an eyepiece with a magnifying power of approximately three diameters, together with a 16 mm objective, a magnification of about 260 diameters is obtained. This is found to be very satisfactory for the fine-mesh sieves. Measurements good to 0.2 mm can be made of the image as seen on the ground glass plate, individual readings repeating to 0.5 mm or better. This gives an accuracy of better than 0.001 mm for the average wire diameters and width of opening.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
JULY 24, 1922.

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THE FILM METHOD OF MEASURING SURFACE AND INTERFACIAL TENSION

By A. W. Fahrenwald

Many methods of measuring surface tension are available. The most important among them are (1) the drop-weight method (2) the capillary rise method (3) the Jaeger or capillary bubble method (4) vibrating jet methods (5) methods measuring the tension required to detach a ring, sphere or disc from the surface of the liquid, and (6) the film method.

More attention has been given to the drop-weight method than to any one of the other methods mentioned. The work of Morgan, Richards, Ferguson and of Harkins and his students has resulted in a high refinement and reliability of this method. However, it should be pointed out that there are factors that enter into its accuracy that cannot be overlooked if trustworthy results are to be obtained. They are (1) the rate of dropping (2) the diameter, shape, and material of dropping tip, and lastly the proper corrective factors must be applied if absolute figures are desired. Further the drop-weight method measures the tension in relatively fresh surfaces, and it is, therefore, not satisfactory when the tension of relatively older surfaces is desired. A method that measures the tension of fresh surfaces is not nearly as sensitive to slight changes in interior concentration of a liquid as one that measures the tension in older ones. Another serious objection to the drop-weight method when working with many liquids, each having different rates of adsorption of certain surface tension lowering constituents in its surface, is the strict requirement of rate of dropping. Even though the rate of dropping be absolutely constant for all measurements there is still the uncertainty of reliability of the relative value of the results due to the different rates at which the different substances go into the water surface. For the above reasons it proved to be unsatisfactory in working with oil and water emulsions and solutions in the determination of oil adsorption by minerals.

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The second method is accurate but considerable skill and care are required in making measurements by its use. Clean and uniform capillary tubes and zero contact angles are pre-requisites. It is rather slow and is not a good method for every day laboratory work.

The Jaeger or capillary bubble method[1] depends on the measurement of the maximum pressure required to force out a bubble of gas from a capillary tube whose circular knife edge opening dips just below the surface of the liquid. As Ferguson[2] has pointed out, this method is more accurate than the capillary rise method.

Ferguson's work was on pure liquids and it is doubtful if this statement would apply in the case of liquids containing contaminants.

As with the drop-weight and vibrating jet methods, it measures the tension of fresh surfaces only. The tension in an old surface in which adsorption of the surface tension lowering constituent has been partially or entirely complete, cannot be determined by this method. It is therefore unsuitable for determining the adsorption of oils by minerals where the surface tension method is used.

To illustrate how the surface tension of an oil-water emulsion depends upon the age of the surface the following figures are given for the surface tension of an emulsion of a steam-distilled pine-oil in water (23 mg in 100 cc water) by three different methods:

Jaeger Method	65	dynes at 20° C
Drop-weight Method	63	" " " "
Film Method on circulation[3]	57.5	" " " "

[1] "Investigation on Temperature Coefficients of the Free Surface Energy of Liquids." Part I Methods and Apparatus F. M. Jaeger, Verstag Akad. Weten Schappen, *23*, 330–65; 1914.

[2] "Surface Tensions of Liquids in Contact with Different Gases." Phil. Mag., *28*, Series 6, p. 403; 1914.

[3] The surface tension of an oil-water emulsion is brought to equilibrium by circulating the emulsion with a pipette. The emulsion is drawn up a number of times, into the pipette and allowed to discharge under the surface of the emulsion. This gives efficient circulation to the body of the emulsion without disturbing the surface film. This treatment is necessary with all oil-water emulsions, in order to get readings that can be duplicated. It takes from 1 to 15 circulations to bring the surface to a point where the surface tension remains constant.

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To further illustrate the advantage of the film method over the other two methods for the determination of adsorption of the oil from the above emulsion by minerals the following experiment is given:

Oil in Water mg	Surface Tension of Emulsion		Difference	Method
	Before Treatment Dynes	After Treatment Dynes		
23	65.0	65.2	.2	Jaeger
23	63.0	65.3	.3	Drop-Weight
23	57.5	60.5	2.0	Film

In many cases the Jaeger method will not show more than from .1 to .3 dynes increase in the surface tension of an emulsion before and after adsorption of some of the oil by mineral particles.

Vibrating Jet methods are entirely unsuitable for general work.

Methods under (5) give values in all cases higher than the actual surface tension. To obtain theoretical values a complicated formula has to be used. The reason is, that a column of the liquid is raised, by the ring or bar, above the surface of the liquid and in the case of a ring or bar values higher by from 1 to 10 per cent of the tension in the surface, depending upon the thickness of the edge pulled from the liquid, will be obtained.

Fig. 1 shows what happens when a straight edge is pulled from the surface of a liquid that wets it. The liquid in the portions *a a*, raised above the plane surface of the liquid is that due to capillary attraction of the liquid for the metal and is a measure of the tension in the liquid film. The liquid in the rectangular portion *b* is due to the attraction of the metal plate for the liquid molecules and the width and weight of this hydraulic column depends upon the thickness of the edge pulled from the surface, and surface tension measurements are too high by amounts which increase with the thickness of the edge used. For an edge ground to the thinness of a safety razor blade, the surface tension for water as measured by this method will be approximately one dyne too

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high. The liquid in the rectangular space *b* disappears to a negligable degree when the film forms.

The film method of measuring surface tension was first used by Mickaelson and Hall.[4] Their method or device for forming the films was essentially the same as that used by the present writer; however, they did not extend it to the use of measuring interfacial tensions. Physicists have recognized the sensitiveness of the method of measuring the tension in films. Allan Ferguson[5] includes it among the most sensitive of methods but states that owing to the impossibility of forming films of sufficiently long life for an accurate measure of the tension, it has never found practical application. Hall used an analytical balance for weighing

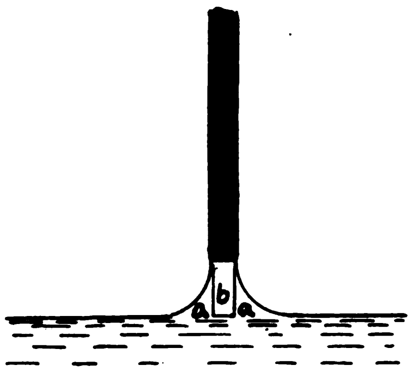

FIG. 1. Showing condition of the Liquid just before the Film is formed

the tensions in the films he worked with. This method is slow and cumbersome and the weighing process is too slow in the case of most liquids, to weigh accurately the tension in the formed film which in many cases endures for not more than a second.

This method of measuring surface tension, has, however, been taken advantage of and made practical by the use of an automatic balance, the sensitiveness of which is not quite as great as that of an analytical balance, but measurements reproducible to 1/10

[4] "New Methods of Measuring the Surface Tension of Liquids." T. Proctor Hall, Phil. Mag. Fifth Series, *36*, p. 385; 1893.

[5] "Critical Review of Some Twenty Methods of Measuring Surface Tension," Science Progress. Jan. 1915, p. 428.

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dyne per cm are easily possible. This is sufficiently close for a rather wide variety of technical work.

In text books on physics, values for the surface tension of water ranging from 73 to 81 dynes per centimeter at 20°C are found. The values in many cases the writer has found have been determined by one of the methods of measuring the force required to detach a ring or bar from the surface of the water. With this method zero contact angles between the object and the liquid are necessary, otherwise, low results will be obtained. For relative work these methods are satisfactory.

SURFACE TENSION BY THE FILM METHOD

The instrument is shown in Fig. 2. The knife edge to engage the liquid, can be made out of aluminum, copper, silver, platinum or other metal foil. It has been found that all of these metals, when thoroughly cleaned, have zero contact angles with water. The plate should be thin, not over .015 cm thick, and geometrically as nearly perfect with respect to the point of suspension as possible. The projections or horns *A* are to prevent the pulling of the liquid away from the ends of the edge and a correction has to be made for these as will be later explained.

The balance proper consists of a main support which is composed of tripod stand *B* and shaft *D*. To the latter is fastened the piece *C* that supports the swinging member E. The swinging member is a cork wheel about 2½ to 3 inches in diameter. Cork is used on account of its extreme lightness which adds to the sensitiveness of the instrument and because the axis on which it turns can be readily put through it. A steel bar 1½ mm in diameter passes axially through the cork, the ends of which are carefully ground to knife edges, and rest on steel or glass bearings. Around the periphery of the cork wheel is machined a small V shaped groove. Over the wheel and in this groove is hung a No. 50 silk thread (thread is used as its weight is negligible). To one end of the thread is hung the knife-edge which forms the liquid film. It must come down just below the graduated scale to be described later. To the other end of the thread is suspended a light aluminum pan into which enough fine sand is placed to just

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balance the knife-edge and other weight at that end of the string. Into the periphery of the cork wheel is stuck a light aluminum pointer about .85 mm in diameter and of sufficient length to provide a weight a little more than sufficient to just balance the maximum tension that will come on the knife-edge. A pointer of this diameter and about eight inches in length will be deflected through about an 85 degree arc if a gram of weight is placed in

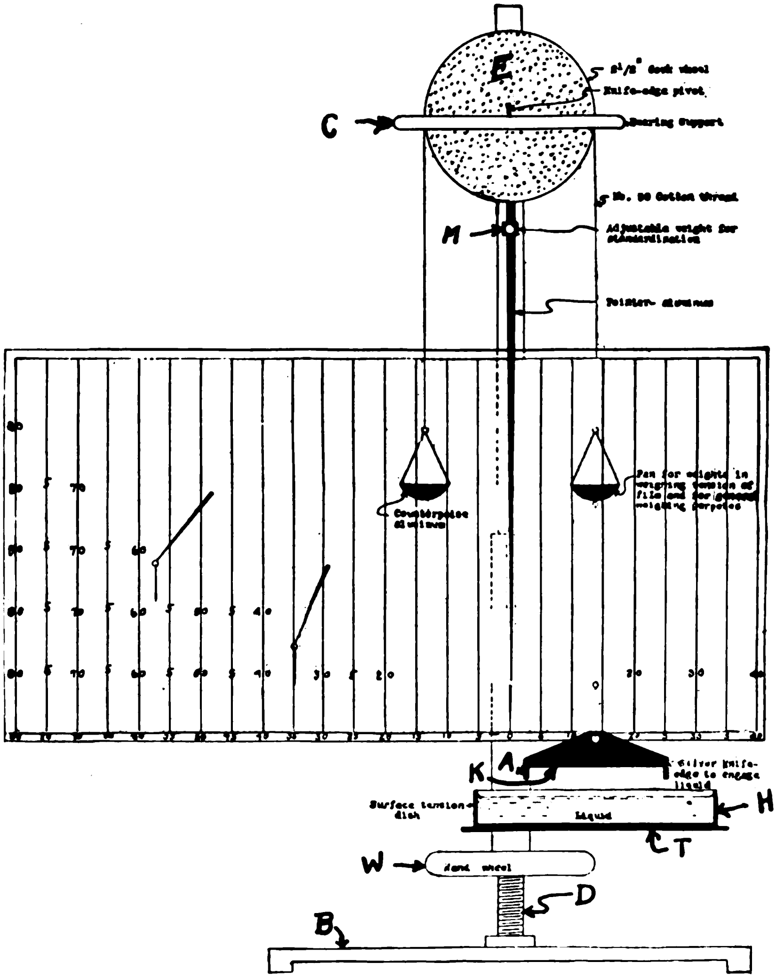

FIG. 2. Instrument for Measuring Surface Tension by the Film Method

the little pan on the knife-edge side of the spring. At the end of this aluminum pointer is hung a second very light and short aluminum pointer (about $1\frac{1}{2}$ cm long and about .3 mm diameter) which hangs freely in a loop as shown. This secondary pointer maintains a vertical position through all angles of the main pointer.

The use of the secondary pointer obviates the necessity of graduating the arc of a circle, when the straight pointer is used, the divisions of which have to increase from the zero position of the pointer to a horizontal one, and makes the use of plain cross section paper as a scale possible. On this scale equal horizontal distances traveled by the secondary pointer represent equal tensions on the knife-edge. The secondary pointer which remains in a vertical position through all angles of the main pointer is easily read against the vertical graduation lines from the scale, low-powered reading glass is of assistance in reading the position of the pointer.

A part of the instrument not yet described is the mechanism for raising and lowering the dish[6] that contains the liquid to be tested. It consists of a table *T* and a sleeve which slides up and down on the shaft *D* when a thumb-wheel *W* is operated.

PROCEDURE IN MAKING A SURFACE TENSION MEASUREMENT

A single measurement which takes less than two minutes time is as follows:—

The liquid to be tested is placed in the dish *H* which has vertical walls. By the use of the thumb-wheel *T* the dish is raised until the knife-edge is pulled into the liquid. The dish is then lowered rapidly by turning the thumb-wheel until it is seen that the edge is just about to become detached from the liquid, from this point on (now watching the pointer) the dish is lowered very slowly until the pointer is seen to slip back a few divisions. The slip indicates that the column of water raised above the surface is letting go and that the film is being formed. This is the hydraulic column of water mentioned under the fifth method of measuring

[6] In the case of volatile liquids and with other liquids when the best results are desired, covered vessels are used. For general work this precaution is hardly justified.

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surface tension which has to be corrected for in all methods using a ring or bar. For water and pure liquids the film lasts several seconds holding the pointer motionless at a given point on the scale. The tension is thus automatically measured. A film ½ inch long can sometimes be pulled out.

In making the correction[7] for the capillary rise of the liquid against the horns of the knife-edge which are actually dipping into the liquid at the time the tension in the film is being read, after the film has broken, the dish is again raised until the horns dip into it and until the surface of the liquid comes very close to the straight edge. The dish is then lowered a few turns. This is to insure a zero contact angle or to insure the same contact angle as existed at the time the film broke. The deflection resulting from the capillary action of the liquid on the metal horns which amounts to from two to four dynes depending upon the degree of wetting and the width and thickness of the metal horns should be subtracted from the first reading. The difference in the two readings, if the instrument is accurately standardized, is the surface tension of the liquid.

With volatile liquids measurements are made in closed vessels with a small opening through which the string, to which the silver knife edge is fastened, passes. No difficulty is experienced in forming films with volatile liquids such as benzene and alcohol.

If difficulty is experienced in forming a film for certain liquids practically the same result can be obtained by using a slightly thicker knife edge and by the same procedure as in the formation of the film with the exception that the plate is not pulled or detached from the liquid, but is pulled, by lowering the dish, until the pointer appears to remain constant through a turn or so of the thumb-wheel. At this point the lower edge of the plate is in the plane of the surface of the liquid and the question of the liquid in the portion *b* in Fig. 1 has been eliminated and we are

[7] The correction by dipping the horns into the liquid a second time may be avoided, by simply taking the reading for the film, and then allowing the pointer to come to rest after the film has broken, reading again and taking the difference. This procedure is less sure than the one described above.

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weighing the tension in the films on either side of the plate which is equal to the weight of the liquid in the portions *a a*.

This is the procedure that was used in showing that the contact angle between the various metals mentioned earlier and water is zero degrees. This method calls for zero contact angles and we have never had to resort to it as films are usually easily obtained.

By this method the surface tension of water has been determined to be 72.8 dynes per centimeter at 20° C. This is in very close agreement with the most reliable figures so far obtained. This method checks well with the drop-weight and Jaeger methods, for all pure liquids tried.

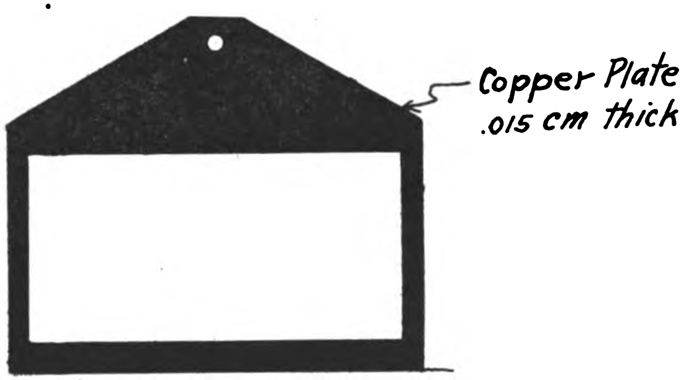

FIG. 3. A form of Frame used in Interfacial Tension Measurements

INTERFACIAL TENSIONS BY THE FILM METHOD

By the use of this instrument, the tension existing at a liquid-liquid interface can be almost as readily determined as for a liquid-air surface. A slightly differently constructed knife edge for forming the film is used. The design used is shown in Fig. 3.

The usual knife edge is removed, and the one shown here suspended in its place. It should be pointed out, however, that the knife edge or frame as used for interfacial tension measurements may be used also for surface tension work. A measurement is made as follows: If the interfacial tension oil-water, or benzene-water is to be determined, about 100 cc of water are put into a

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beaker and on top of this is placed a layer of benzene or oil about 1/16 inch thick. The beaker with its contents as explained is put on the adjustable stand T which is raised by means of the thumb-wheel W until the knife edge, Fig. 3, has passed below the level of the interface, forming and pulling the oil film with it. The film formation is shown in Fig. 4.

The deflection of the pointer, which will be to the right on the scale is read before and after the breaking of the film. The difference gives the interfacial tension in dynes per centimeter. The same results are obtained whether the film is pulled from the water to the oil (water film) or whether it is pulled from the oil to the water. When a metal knife edge is used, it is better to go from the oil to the water as the oil more easily wets the metal plate. If it is desired to go from the water to the oil, the plate should be put in the water before the oil layer is added.

To measure the interfacial tension between water and a liquid heavier than water the same procedure is used. The oil should be first put in the beaker, and a shallow layer of water poured on top of it. The knife edge is allowed to pass through the water pulling a film of it into the oil. For measuring the tension at an interface between two liquids of equal specific gravity, the oil is placed in a smaller rectangular vessel which is set into the beaker and the water is poured around and over it.

METHODS OF STANDARDIZATION

In calculating the theoretical surface tension of a liquid the simple formula $Y = \frac{980 \times W}{2L}$ where W is the weight required to swing the pointer through the same number of divisions of the scale as 2 L cm of the film, where L is the length of the straight edge between the horns and Y the surface tension of the liquid.

The instrument is standardized to read direct in dynes per centimeter by raising or lowering the adjustable weight M on the pointer until a film of pure water swings the pointer just through 72.8 divisions of the scale. This requires several trial measurements. However, the instrument need not be actually standard-

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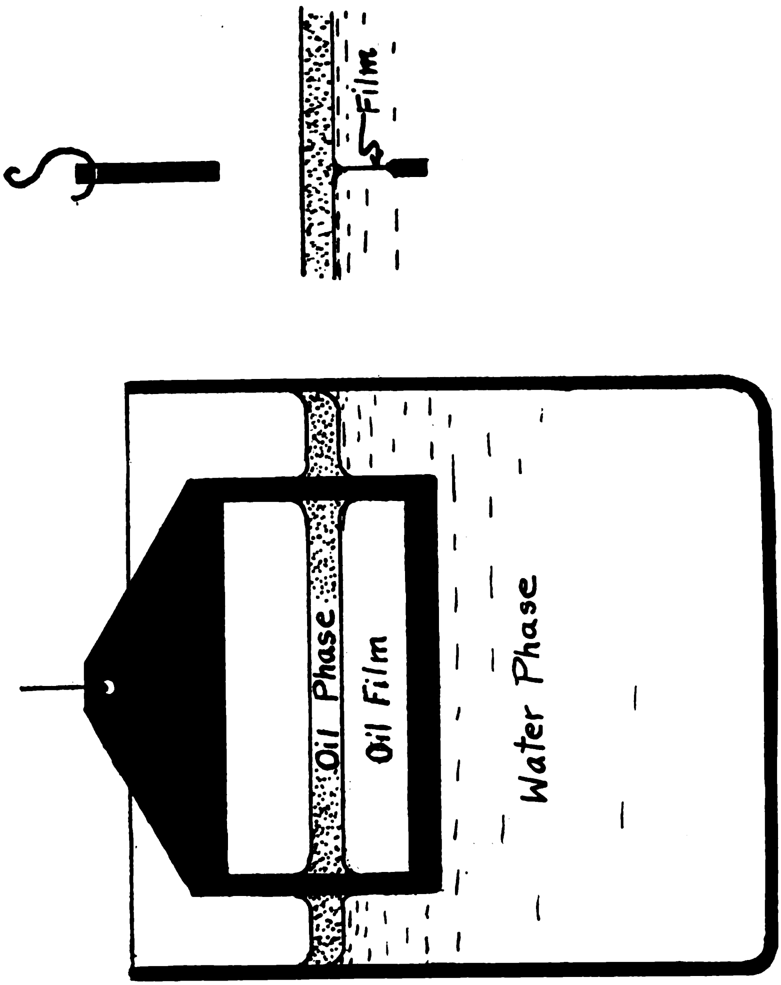

FIG. 4. Showing Film Formation Stage in an Interfacial Tension Measurement

ized; the surface tension of water can be taken as 72.8 or for practical purposes 73.0 and a factor used.

Several thousand surface tension and interfacial tension determinations have been made with this instrument in studying the theory of flotation. It has been found that with oil-water emulsions it is the only instrument that will give true static surface tension, that is, surface tension at the point where equilibrium[8] between surface concentration and interior concentration exists. Most other methods measure the surface tension of relatively fresh surfaces and for emulsions give values from 1 to 15 dynes higher than that at equilibrium.

The instrument should find a wide application for general laboratory use and in institutions in the teaching of molecular physics for which it is eminently suited. It is simple of construction and the average operators can get consistent and reliable results with it. It is independent of contact angle and maintains perfect adjustment from day to day. It easily gives values reproducible to 1/10 dyne/cm. It works well on volatile liquids and measurements are conveniently made in atmospheres other than air. It is also convenient for measuring interfacial tension between liquids.

A brief description of this instrument and its application to one problem involving the study of surface tension was given in the August 13, 1921, issue of the Mining & Scientific Press, San Francisco, California.

The writer is greatly and pleasantly indebted to Mr. R. B. Elder of the School of Mines, University of Idaho, for his helpful suggestions, and assistance in checking the instrument against other methods; to Professor A. A. Knowlton of Reed College, Portland, Oregon, for his critical examination of the instrument, and to Mr. S. N. Shanfeld for his assistance in carrying out many of the experiments which have established the reliability of this method of measuring surface tension.

BUREAU OF MINES,
WASHINGTON, D. C.

[8] To maintain this equilibrium the emulsion must be circulated frequently. When the point of constant surface tension is reached, the emulsion will, on standing and without circulation, show either an increase or decrease in surface tension. The surface tension usually rises.

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THE PASSAGE OF HYDROGEN THROUGH QUARTZ GLASS

By J. B. Johnson and R. C. Burt

SYNOPSIS

The *Rate of Flow of Hydrogen Through Quartz Glass* has been measured over the range of 300°C. to 900°C. Some measurements were also made with *Nitrogen and Argon*. A perceptible diffusion starts with hydrogen at about 300° C. and with the nitrogen at 600° C., and then in each case increases rapidly with the temperature.

A brief discussion is given on the *Possible Nature of the Flow* of gases through fused silica.

The heat resistive property of fused quartz has made this material valuable for the construction of many kinds of scientific apparatus. In some cases, however, its usefulness has been limited by the porosity to various gases at higher temperatures. O. W. Richardson[1] found that hydrogen and helium, and to a less extent neon, diffused through a quartz glass tube at 800°–1200° C. The rate of the diffusion has been measured when the pressure on both sides of the material was fairly high,[2] but the results thus obtained cannot safely be applied to high vacuum apparatus. The increased importance of electronic discharge tubes of larger power opens a new field for the use of fused silica, provided the material is used with proper regard for its limitation. The experiments to be described were therefore done to get more definite knowledge of the behavior of this material under conditions of high vacuum and high temperature. While the flow of hydrogen was studied more fully, some measurements were also made with nitrogen and argon.

The method used in making the measurements was to observe the pressure rise in an evacuated silica glass tube around which the gas flowed. A diagram of the apparatus is shown in Fig. 1. The furnace was an iron pipe around which was wound a heater of resistance wire and a covering of asbestos. A thermocouple

[1] Phil. Mag., *22*, p. 704; 1911.

[2] E. C. Mayer, Phys. Rev., *6*, p. 283, 1915; H. Wüstner, Ann. d. Phys., *46*, p. 1095, 1915.

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placed near the center of the furnace was used to measure the temperature. The gas from a commercial tank was passed through the furnace at a slow steady rate at atmospheric pressure. The quartz glass tube *Q* was placed axially in the furnace and was connected to a vacuum pump, a McLeod gauge and a volume bulb *V*, the total volume of the system being about 1500 cc. A side tube *T* with a volume of about 3 cc could be immersed in liquid air for freezing vapors out of the system. The difference in level of the capillary mercury columns of the gage at the time of taking a pressure reading was kept less than the vapor pressure of water. No condensed water, therefore, existed in the closed capillary, so that when the side tube was not cooled the gauge indicated

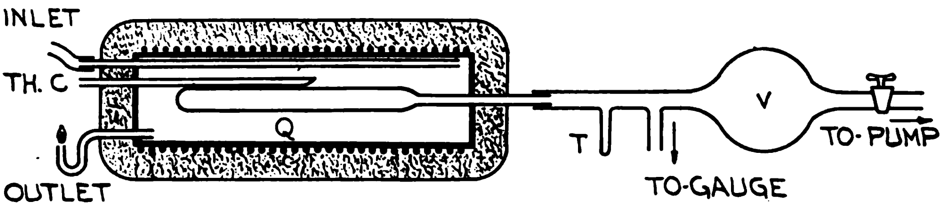

FIG. 1. Apparatus

approximately the total pressure, including that of water vapor. After the tube was pumped out the connection to the pump was closed during a run and the pressure read at short intervals while the temperature was kept nearly constant.

The materials tested were furnished by the Thermal Syndicate, Ltd., and were in the form of tubes 35 cm long, 1.5 cm in outside diameter and 1.5 mm wall thickness. Rough tests on several opaque varieties of fused silica, made from quartz sand, showed that these are unsuited for high vacuum apparatus even in the air at room temperature. The lowest rate of pressure increase observed was, for one of the tubes, about .001 mm of air per hour at room temperature and other tubes gave as high as ten times this value under the same conditions. The final work was therefore confined to the clear fused silica made from quartz crystals.

After the clear silica tubes had been heated up to drive off the vapors on the inner surfaces, no leakage of air or hydrogen at room temperatures was detected in as long a time as two weeks at a

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pressure of less than .001 mm. Upon again heating the tube a measurable leak of hydrogen started first at about 300° C, and then increased rapidly with the temperature. Fig. 2 shows a summary of the results obtained with three tubes. *R* is the rate in cc per hour at which gas leaked into the tube per unit area, reduced to one mm pressure and room temperature, corrections being made for the temperature of the quartz and of the tube *T*.

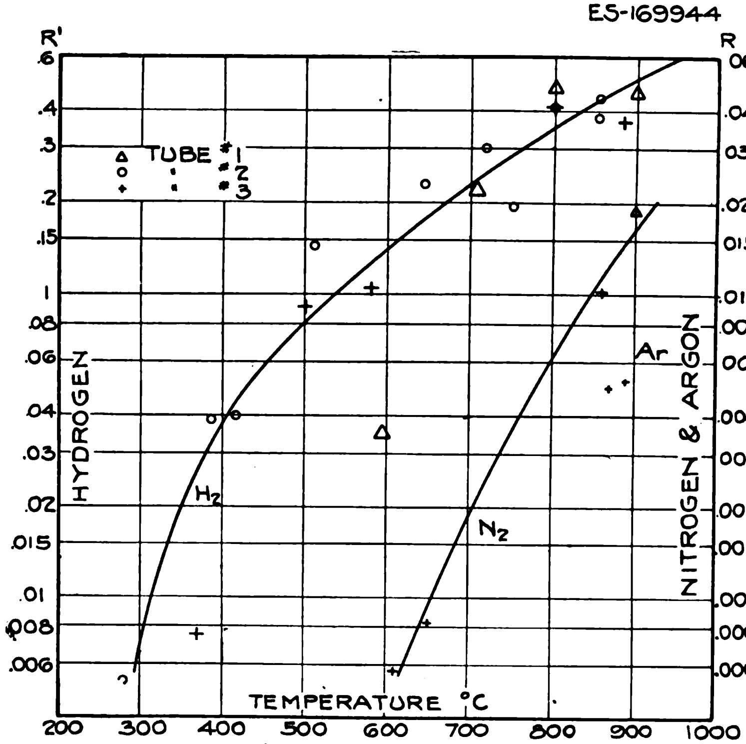

FIG. 2. **Rate of Flow of Hydrogen, Nitrogen and Argon**

The tubes were not quite of the same average thickness and the results have been reduced to correspond to a wall thickness of 1.5 mm, on the assumption that the leak is inversely proportional to the thickness. The runs with nitrogen and with argon are also shown in this figure. At 400° and 500° no leak of nitrogen as large as 5×10^{-4} mm was detected in 24 hours.

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The result of changing from one gas to another is shown in Fig. 3, which makes clearer the difference between leakage of hydrogen and the heavier gases. The curve at the top of the figure gives the temperature of the oven during the course of the experiment. The upper pressure curve gives the total pressure in the system, while the lower curve gives the pressure of permanent gas measured when the small side tube was immersed in liquid air. The difference between the two curves represents the pressure of condensible vapors, about 90% of which was water vapor as was shown by exposing the gas to a small amount of

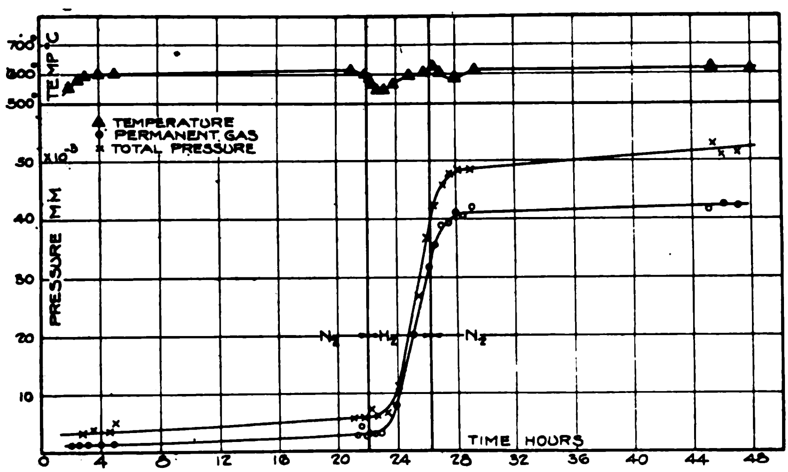

FIG. 3. Change of Gas Surrounding the Tube

phosphorus pentoxide. The presence of this condensible gas suggests the possibility that the rise in pressure was caused by gases continually given off from the walls of the tube rather than by a leakage through the walls, but the changes in slope show conclusively that there is a leakage of hydrogen at least. The constancy with time indicates that the lower rate of increase of pressure is caused by leakage of nitrogen as has been assumed and only to a small extent by release of gases from the inner surface of the tube.

The results of these experiments, though incomplete, justify a brief discussion of the nature of the flow of gas through fused silica. It seems improbable that a clear, dense substance such as

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quartz glass is porous to gases in the sense that chalk is porous. It is equally improbable that in this inert material there is a chemical diffusion as in rubber. An assumption which might readily be made is that there is a flow of gas through very fine holes or tubes in the material. This view gains apparent support from the fact that even clear fused silica is not quite free from fine striations, caused by drawn out air bubbles, a condition which is so obviously present in the satin finish material. According to the kinetic theory, however, the volume of gas passing through a tube when the diameter of the tube is small compared with the mean free path of the gas molecules is inversely proportional to the three-halves power of the molecular weight and directly proportional to the square root of the absolute temperature. When the experimental rates for the three gases are compared, their ratios fall within the correct range for the molecular weight relation, but the increase of rate with temperature is very much greater than that shown by flow through tubes and apertures, and indeed varies as the third or higher power of the temperature, so that we must look for some other explanation than simple flow along capillary tubes. The transfusion begins at the temperature at which structural changes are known to occur in crystalline silica, and this fact suggests that the passage of the gas may accompany a modification in the structure of the non-crystalline material.

The data enable us to judge what to expect with tubes and bulbs of ordinary dimensions. The curves show that at the same outside pressure hydrogen passes through the walls about 100 times faster than nitrogen or argon. Under normal conditions, however, the hydrogen content of the atmosphere is small, about four parts in 100,000 by volume, so that in air the rate of admission of nitrogen should be of the order of 250 times larger than that of hydrogen. To take a concrete example which may be met with in practice, we can probably say that a well evacuated bulb of one liter capacity can be kept in the air at 400° C for one hundred hours before the pressure reaches 10^{-4} mm, and the transfused gas will be largely nitrogen.

RESEARCH LABORATORIES OF THE
WESTERN ELECTRIC COMPANY, INC.,
AND THE AMERICAN TELEPHONE AND
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A SIMPLE APPARATUS FOR COMPARING THE THERMAL CONDUCTIVITY OF METALS AND VERY THIN SPECIMENS OF POOR CONDUCTORS

By M. S. Van Dusen

The apparatus to be described here has been recently used by the author to measure the thermal resistance of various contacts between metals, either with or without the addition of some cementing material, and to compare the thermal conductivity of metals and thin specimens of poor condùctors. The method consists essentially in comparing the temperature gradients in two materials placed in series, the rate of heat flow in each being the same. Heat flow in any direction other than that in which the

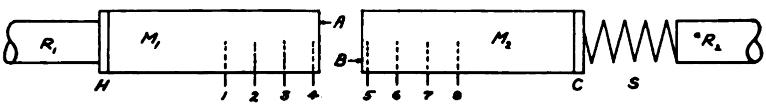

Fig. 1. Sketch of Apparatus

temperature gradients are measured as well as contact thermal resistances between the materials compared are made relatively small.

Fig. 1 shows a sketch of the apparatus as it has been used by the author, the particular object being to determine the thermal resistance of contact between metal surfaces under certain conditions. M_1 and M_1 are two solid brass cylinders, about 3 cm in diameter and 10 cm long, mounted coaxially. An electric heating unit H, consisting of a spiral of small copper tubing enclosing an insulated resistance wire, is soldered to one end of the cylinder M_1, and a water cooled cell C is placed on the opposite end of the cylinder M_2. Means are thus provided for producing a heat flow from H to C along the bars and through any material placed placed with the surfaces A and B. The rods R_1 and R_2 together with the spring S are used to apply a known force between the

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surfaces A and B. Measurements of axial temperature gradient in the brass cylinders are made by means of the copper-constantan thermocouples (No. 36 wire) numbered 1, 2, 3, 8, inserted in 0.7 mm radial holes drilled 1.5 cm deep at definite intervals along the bars. Thermal contact between the thermocouple wires and the brass was improved by filling the holes with oil. It was found that under these conditions the depth of immersion was sufficient to eliminate the effects of lead conduction, no appreciable change in the temperature indication occurring when the junction was moved 1 cm away from the axis. The couples 4 and 5 are placed near the surfaces A and B so that the temperature of these surfaces can be found by extrapolation of the temperature versus distance curves along the axis of the system. A small amount of insulation on the two cylinders will serve to reduce heat loss from the convex surface to a very small value relative to the heat flow parallel to the axis. If now the surfaces A and B are put in contact, a constant current supplied to H, and a constant flow of water to C, temperature measurements after the steady state is attained will serve to compare the thermal resistance of the contact with that of a unit length of the brass itself. Similar experiments can be made with the surfaces cemented together by any means whatever. Furthermore the effects of pressure can be found by the means provided for varying the force holding the cylinders together. The influence of the character of the metal surface can be determined by roughening or grinding in various ways. It was found that if the surfaces were ground fairly flat, (convex with radius of curvature about 40 meters) the thermal resistance of a dry contact with a pressure of about 1 to 5 kg/cm^2 was equivalent to the resistance of from 1 to 1.5 cm of brass. When the surfaces were wrung together with water or a glycerine solution the effect of the contact could not be detected, being equivalent to less than 0.5 mm of brass.

This fact suggested that the apparatus could be used to compare the conductivity of various metals or alloys with some standard metal the conductivity of which is known. The heat flow would not be seriously distorted by the contacts. There is no difficulty in grinding the surfaces on a flat lap to a sufficient degree of flat-

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ness. Cylinders of pure tin, lead, zinc, and 99.7% aluminum were prepared, each about 3 cm long and having the same diameter as the brass cylinders. The conductivities of these metals were compared at a mean temperature of about 40°C, the contact resistances being eliminated by the use of a dilute glycerine solution since pure water partially dries out on the hotter side in two or three hours. A specimen of brass cut from the same bar from which the apparatus was made was also compared, and no effect of the two contacts could be detected. Table 1 shows the results obtained, together with other values given in Landolt Bornstein tables. Zinc has been used as the standard and its conductivity has been assumed to be 0.265 cal sec^{-1} cm^{-1} deg^{-1}.

TABLE 1—*Comparison of Thermal Conductivity of Several Metals*

Metal	Thermal conductivity cal sec^{-1} cm^{-1} deg^{-1}	Probable best values at about room temp. taken from Landolt-Bornstein
Zinc (Standard).....	0.265	0.265
Aluminum (99.7%)	0.52	0.48
Tin................	0.160	0.155
Lead...............	0.085	0.083
Navy Brass.........	0.28	.....

No great accuracy was anticipated when the apparatus was designed, but the results seem to indicate a precision within 5%. The variation of the thermal conductivity with temperature was disregarded, since it is within the experimental error in the small temperature range investigated.

The apparatus can be also used to determine the thermal resistance of very thin sections of poor conductors, such as mica, paper, etc., as well as thin films of liquids. In the latter case the brass cylinders are held a fixed distance apart by three very small mica separators, and the liquid remains between the ends of the bars by capillarity. The typical curves in Fig. 2 illustrate the general nature of the experiments, and in particular some results on wet and dry contact. The fact that the curves are sensibly straight and have the same slope on each side of the

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break shows that the heat loss from the convex surface is negligible. The magnitude of the discontinuity in terms of the abscissa (i.e. distance along the axis) gives directly the thermal resistance of the contact or specimen in terms of centimeters of brass. Table 2 gives some results obtained with various contacts, as well as with layers of mica and paper. It will be noted that mica split into thin layers and subsequently squeezed together has a considerably greater thermal resistance than unsplit material.

TABLE 2—*Thermal Resistance of Miscellaneous Contacts or Thin Layers*

Nature of contact	Thermal Resistance in cm of brass	Pressure	Thickness of film	Remarks
		$\frac{kg}{cm^2}$	*mm*	
Dry	5.0	2.6	....	Rough ground surfaces
Wet with water	0.2	2.6	....	" " "
Dry	1.30	0.9	....	Finely ground flat surfaces
Dry	1.15	2.6	....	" " " "
Dry	0.85	4.4	....	" " " "
Wet with water	0.10	0.9	....	" " " "
" " "	0.00	2.6	....	" " " "
Light mineral oil	0.20	0.9	....	" " " "
" " "	0.10	2.6	....	" " " "
" " "	2.05	....	0.028	Spaced by 3 small mica chips
" " "	5.70	....	0.066	" " " " "
1 layer mica	3.30	2.6	0.024	
2 " "	4.85	2.6	0.048	
3 " "	6.05	2.6	0.075	
4 " "	7.20	2.6	0.100	
5 " "	8.70	2.6	0.125	
1 " "	5.80	2.6	0.234	
1 " "	0.80	2.6	0.025	Wet with water
1 layer paper	13.5	2.6	0.103	
2 " "	27.0	2.6	0.205	
3 " "	38.0	2.6	0.310	
4 " "	48.5	2.6	0.412	

It is believed that this form of apparatus can be employed in schools, either in class room demonstrations or laboratory work.

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The undergraduate student does not ordinarily obtain very clear conceptions in the subject of thermal conductivity, usually because he is not shown by actual experiment the analogy between electrical and thermal conduction. Experiments similar to the kind described in this paper follow closely certain experiments

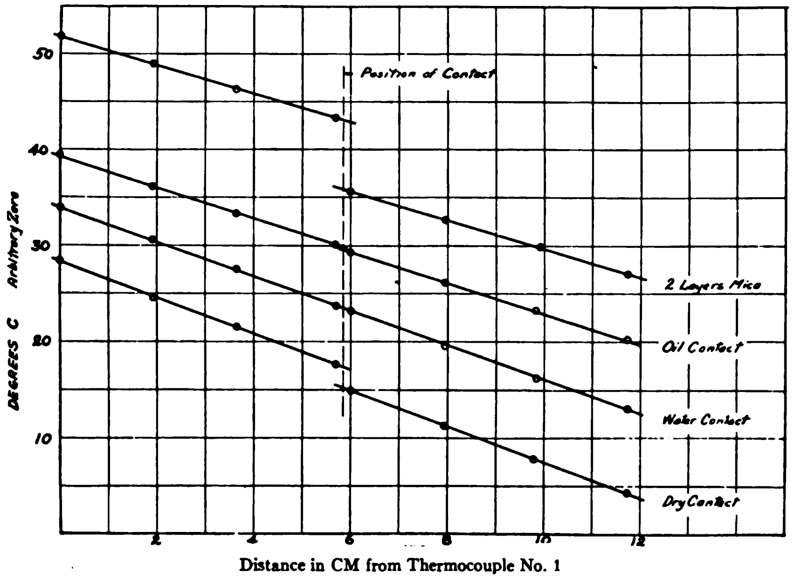

FIG. 2. Typical Temperature—Distance Curve

on Ohm's law which the student always performs in his first course in physics. Furthermore the student will obtain in a rather vivid manner correct ideas of the relative magnitude of various thermal resistances.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
JULY 22, 1922.

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AERONAUTIC INSTRUMENTS

By Franklin L. Hunt

CONTENTS

The instrument equipment of modern aircraft ordinarily consists of a group of ten or more instruments which are located on an instrument board in front of the pilot. They serve to assist him in the control of the altitude, speed and orientation of his aircraft and the behavior of the engine. In addition to the regu-

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lar equipment special instruments are frequently installed, such as navigating instruments where long distance flights are to be made, experimental instruments for airplane performance tests, or instruments for military use. The purpose of this paper is to describe briefly the various types of aircraft instruments which have reached a state of practical development such that they have found extensive use in service.[1] These may be conveniently considered in the order mentioned above. A group of typical airplane instruments is shown in Fig. 1.

ALTITUDE INSTRUMENTS

Altimeters. Altimeters are used to indicate the altitude of aircraft. They are the same in principle as aneroid barometers and have as the essential working element a corrugated metal capsule from which the air is exhausted and which is maintained distended by an external or an internal spring. With decreasing atmospheric pressure, such as is experienced when an aircraft climbs, the evacuated capsule expands under the action of the spring. This motion which is very small, amounting to a few thousandths of an inch only, is multiplied by a suitable transfer mechanism and used to operate a pointer moving over a circular dial. The dial is graduated either in feet or in meters in accordance with some impirical mathematical relation between the atmospheric pressure and the altitude. The dial is rotatable so that the zero of the instrument can be adjusted for fluctuations in ground level barometric pressure. The pressure-altitude relation used in calibrating American altimeters is based on the assumption of a uniform air column temperature of 10°C and the corresponding mean humidity. It is calculated from the constants used in Smithsonian Meteorological Tables 51 and 54, 4th revised edition, and neglects the small effect due to the variation of gravity. It may be expressed by the relation

$$H = 62900 \log_{10} \frac{759.6}{P}$$

where H is the altitude in feet and P the barometric pressure in

[1] For a more detailed discussion see Reports No. 125–132, inclusive, of the National Advisory Committee for Aeronautics, 1922.

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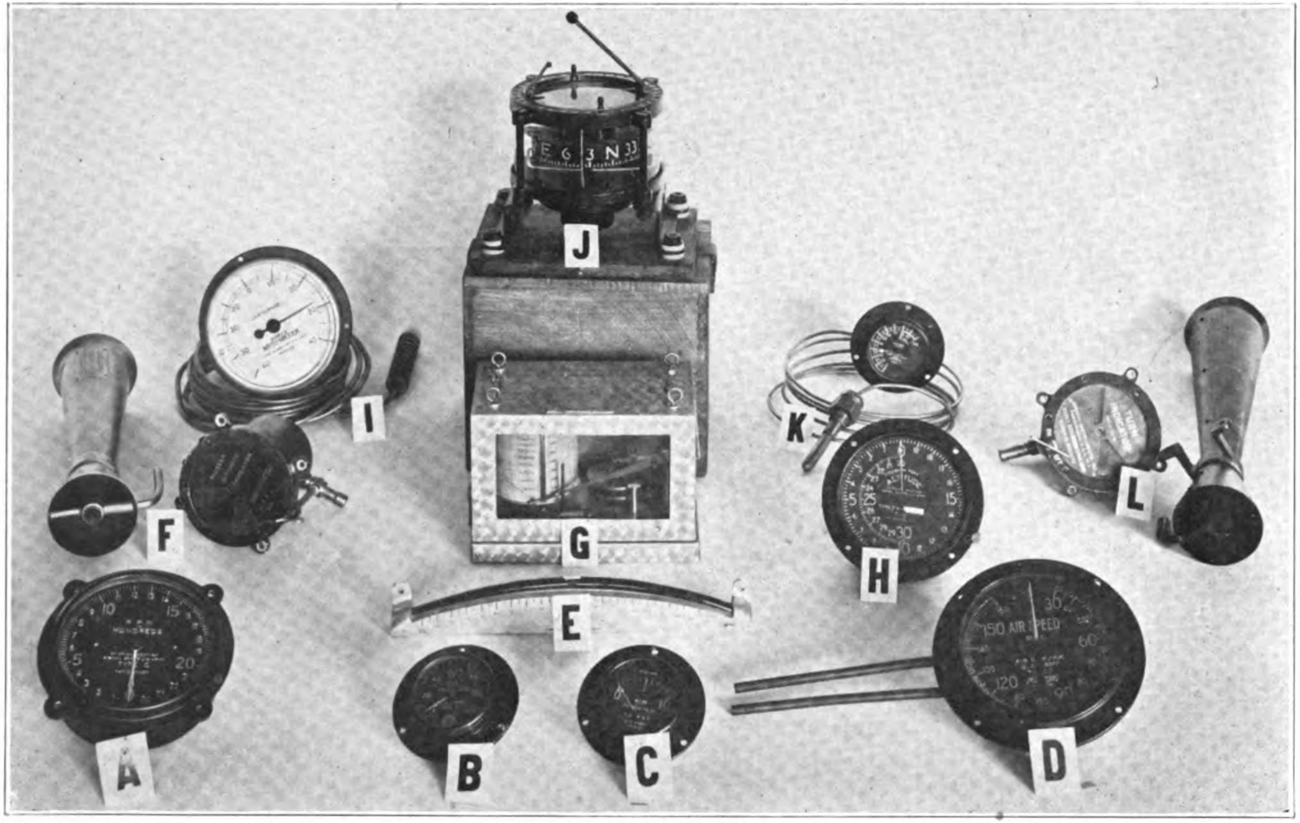

FIG. 1. AERONAUTIC INSTRUMENTS

A Tachometer, *B* Oil Pressure Gage, *C* Air Pressure Gage, *D* Airspeed Indicator, *E* Bubble Inclinometer, *F* Gyroscopic Pitching Indicator, *G* Barograph, *H* Altimeter, *I* Strut Thermometer, *J* Compass, *K* Radiator Thermometer, *L* Gyroscopic Turn Indicator.

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millimeters of mercury. Practically the same formula is used in Great Britain. A typical altimeter is shown in Fig. 2.

Barographs. Barographs are the same in principle as altimeters but are provided with a recording mechanism which gives a continuous and permanent record of the altitude thruout flight. A battery of corrugated evacuated capsules is ordinarily used instead of a single capsule as in the altimeter, and interior springs are more frequently used than exterior ones. The expansion of the battery of capsules with decrease of external pressure operates

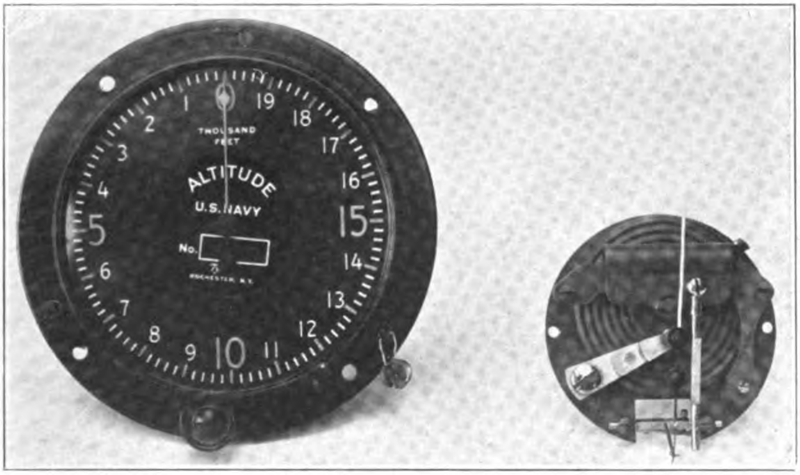

FIG. 2. Tycos Altimeter

a pointer which carries a pen and makes a record on a chart on a drum rotated by clockwork. The chart is graduated in feet or meters in accordance with some mathematical pressure-altitude relation, usually the same as that used on altimeters. Fig. 3 shows a typical aviation barograph.

Statoscopes. Statoscopes are used more frequently in lighter-than-air craft than on airplanes. They provide a sensitive means of indicating qualitatively whether an aircraft is rising or falling and help the aviator to maintain horizontal flight. The ordinary type consists of a closed air-chamber which is connected to the exterior air through a glass U-tube containing a

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small quantity of colored liquid, thus forming a trap which seals the air in the container. Heat insulation is used to prevent the expansion and contraction of the confined air with changes of external temperature. When the aircraft rises or falls the pressure of the air inside the container becomes greater or less than that of the external air, according as the aircraft is ascending or descending, and the liquid in the trap which is visible to the avia-

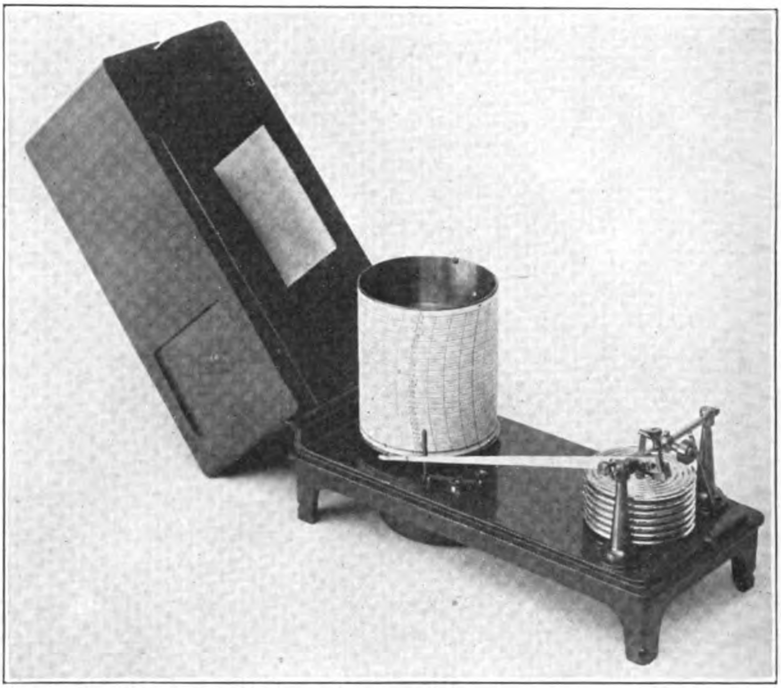

FIG. 3. Friez Barograph

tor is forced in one direction or the other, indicating a change of level. When the difference in pressure becomes sufficiently great equilibrium is re-established by air being forced past the liquid in the trap, after which the liquid again collects in the trap as previously. The frequency with which the air is forced past the liquid or as it is ordinarily expressed the rate at which the bubble

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breaks is a rough measure of the rate of ascent or descent. Statoscopes can be made to detect changes in level of from five to ten feet. A typical instrument is shown in Fig. 4.

SPEED INSTRUMENTS

Air Speed Indicators. Air speed indicators show the speed of aircraft relative to the air. They give the speed with reference to the ground only in the absence of wind. The most commonly used types depend for their action on the pressure developed by the impact of the air stream caused by the motion of the airplane, or on the speed of rotation of small cup anemometers or air propellers. The indications of the pressure type are proportional to the density of the air so that the readings depend on the altitude. The anemometer type on the other hand shows practically no altitude effect. In the most usual form of the pressure type the pressures developed by a Pitot or Venturi nozzle located on one of the struts of the airplane is indicated by a sensitive gage on the instrument board. Usually it is also necessary to determine the static pressure at the point where the Pitot or Venturi nozzle is located. This is effected by using what is known as a static head which consists of straight tube closed at the end with a concentric ring of small holes or narrow slots at the sides. This tube is pointed in the direction of motion so that pressure within is maintained equal to that of undisturbed air without, the rush of air past the opening at the side of the tube being at right angles to these openings. Typical Pitot and Venturi nozzles are shown in Fig. 5A-B.

The indicator which is in effect a sensitive pressure gage ordinarily consists of one or more corrugated metal capsules enclosed in an air tight case or of an air tight case separated by a membrane of rubber or doped fabric into two air tight chambers. In early instruments a liquid manometer was sometimes used. The dynamic head of the pressure nozzle is connected to the diaphragm capsule and the static head to the air tight case, or in the doped fabric diaphragm type one head is connected to each of the air tight chambers. In some cases a combination of Pitot and Venturi nozzles is used to take advantage both of the pressure develop-

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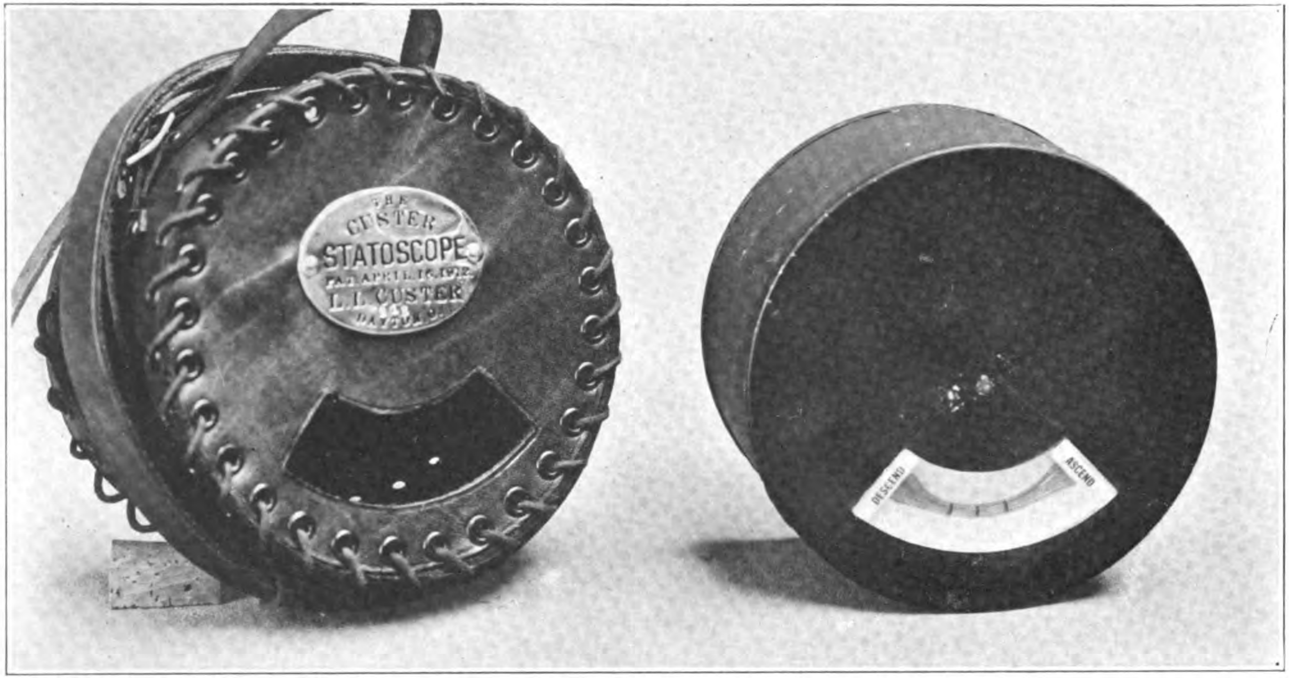

FIG. 4. Custer Statoscope

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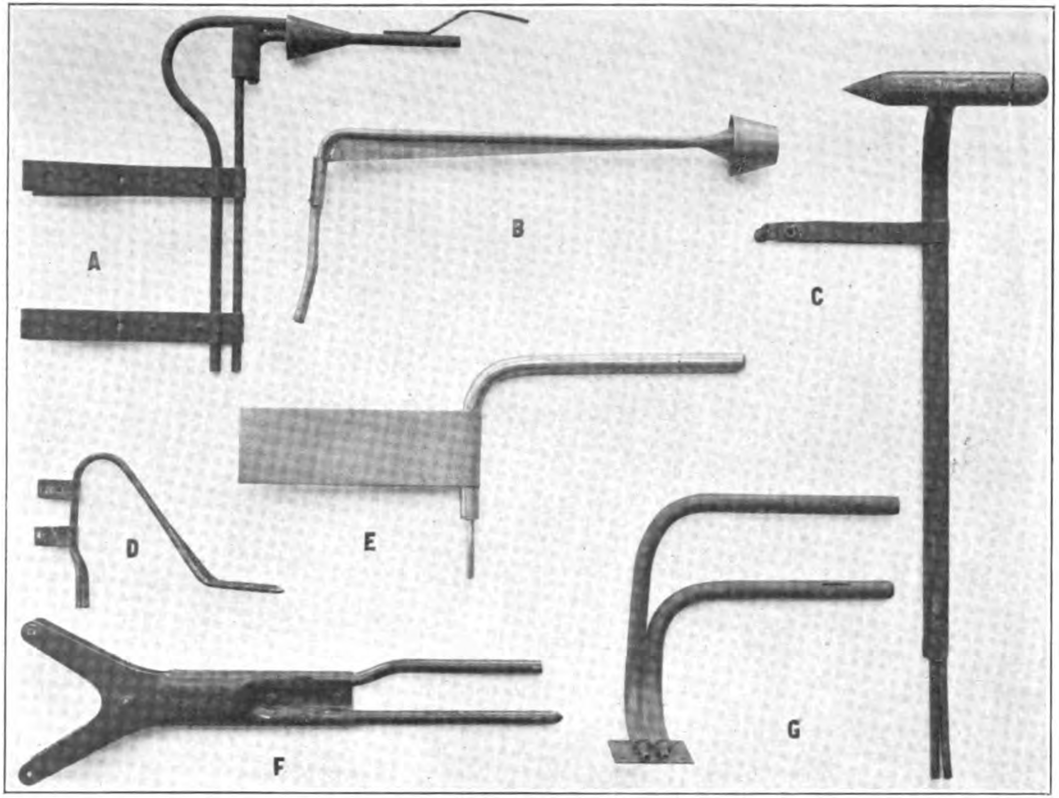

FIG. 5A. PITOT NOZZLES

A and *B* Nozzles with Static hoods, *C* and *E* Nozzles with concentric static heads, *D*, *F* and *G* Nozzles with separate static heads.

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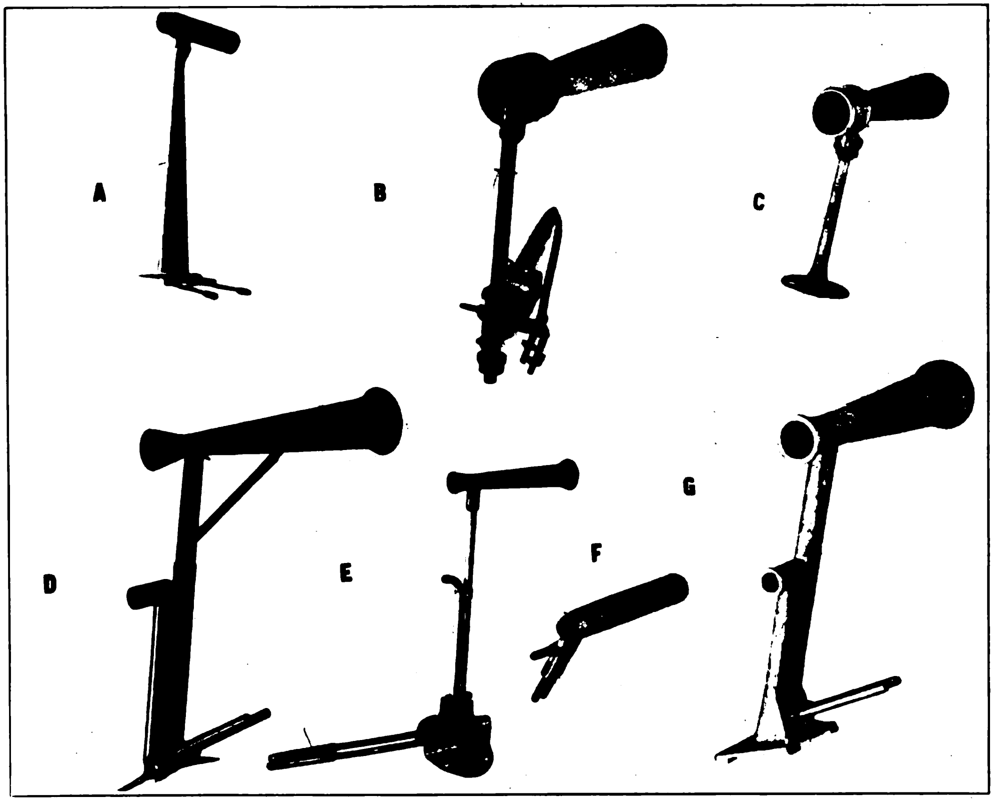

FIG. 5B. VENTURI NOZZLES

A, Single Venturi nozzle with concentric static head, *B* and *C* double Venturi nozzles, *D*, *E*, *F* and *G*, Pitot-Venturi nozzles

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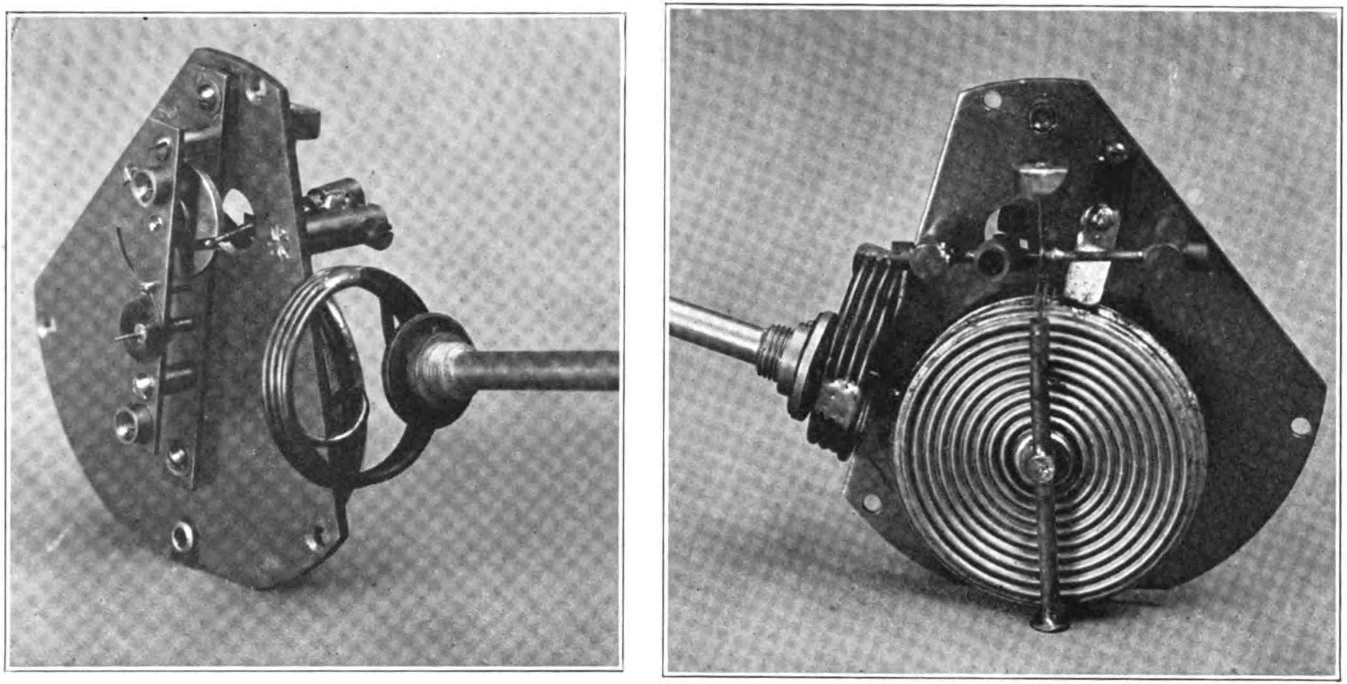

FIG. 6A. Bristol Double Diaphragm Air Speed Indicator

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ment by the former and the suction by the latter. Under these circumstances no static head is used, the Pitot nozzle being connected to one of the air tight chambers and the Venturi to the

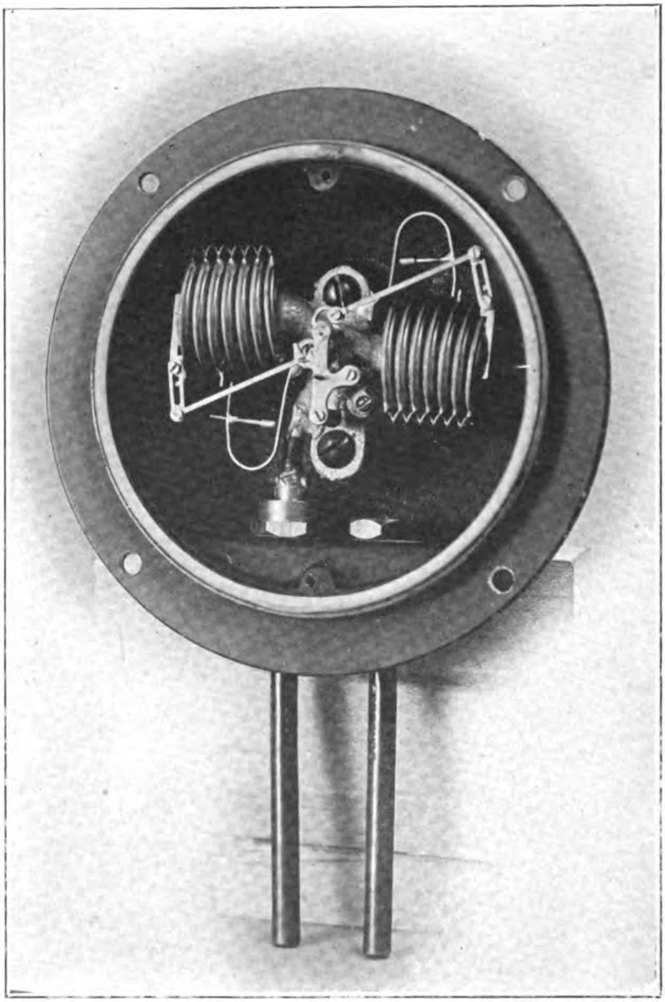

FIG. 6B. Foxboro Bellows Diaphragm Air Speed Indicator

other. The differential pressure developed by the nozzle causes the diaphragm to expand or contract according to the magnitude and direction of the excess pressure. This motion is carried by a

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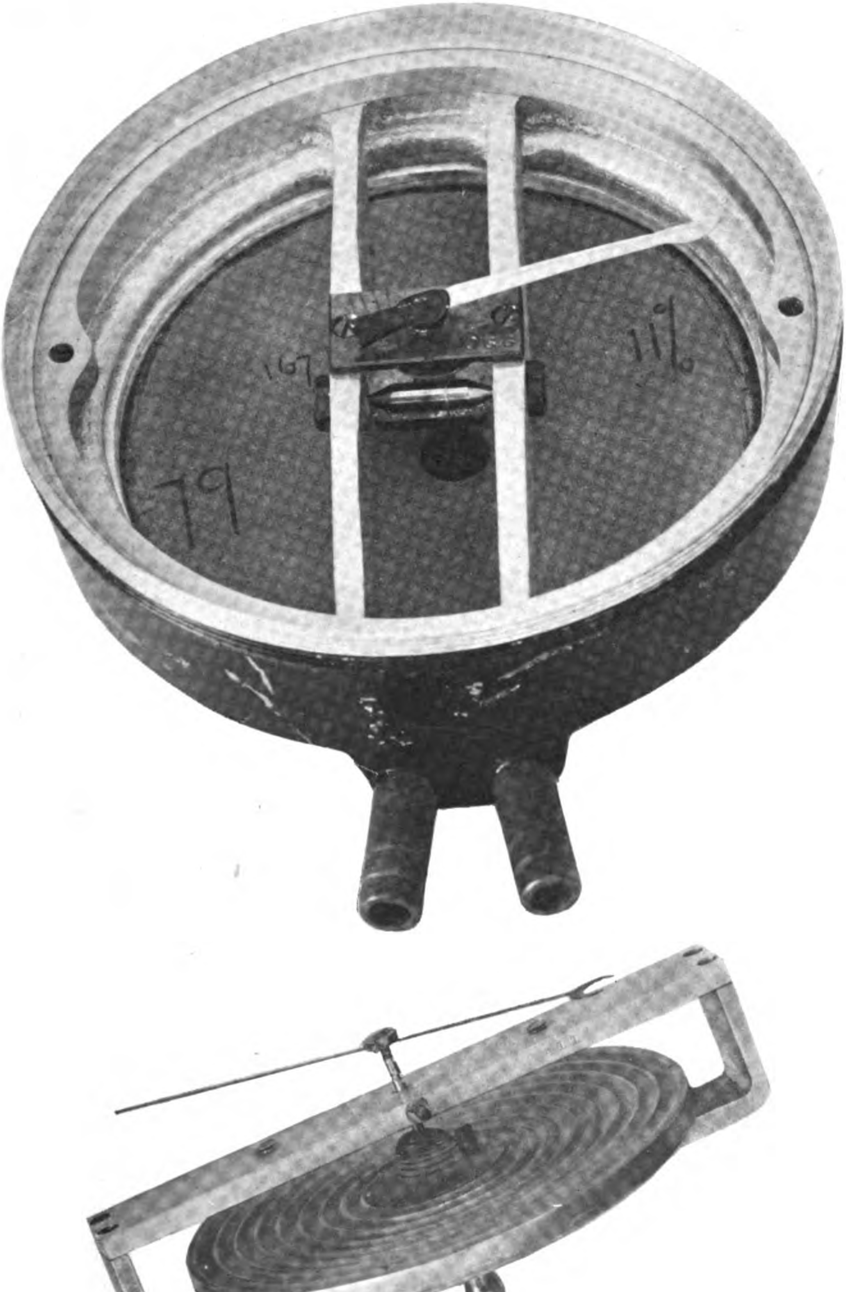

FIG. 6C. Ogilvie rubber diaphragm (above) Sperry single diaphragm (lower) Air Speed Indicators

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suitable transfer mechanism to the pointer, indicating corresponding speeds on a dial which is graduated in miles per hour or kilometers per hour in accordance with the pressure-speed relation of the nozzle used. Most nozzles of the Pitot and Venturi type obey the so-called ρv^2 law, i.e., the pressure developed is proportional to the density of the air and the square of the velocity of motion.

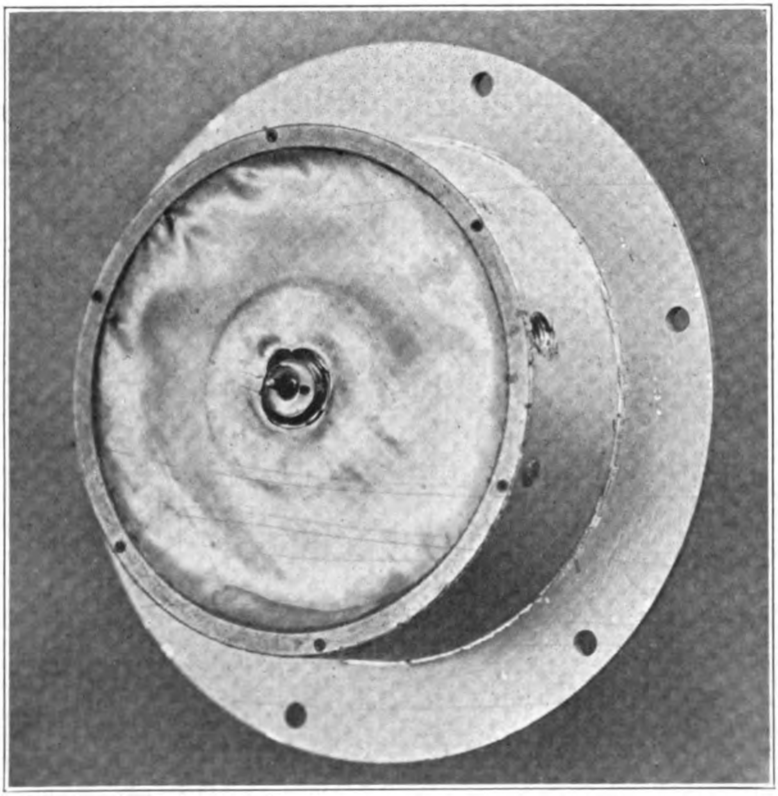

FIG. 6D. Clift Doped Fabric Diaphragm Air Speed Indicator

Typical air speed indicators of the Pitot and Venturi type are shown in Fig. 6A-E.

Instruments in which a flat plate perpendicular to the direction of motion is used to measure the air pressure have also been constructed. In these the plate is attached to a lever whose motion

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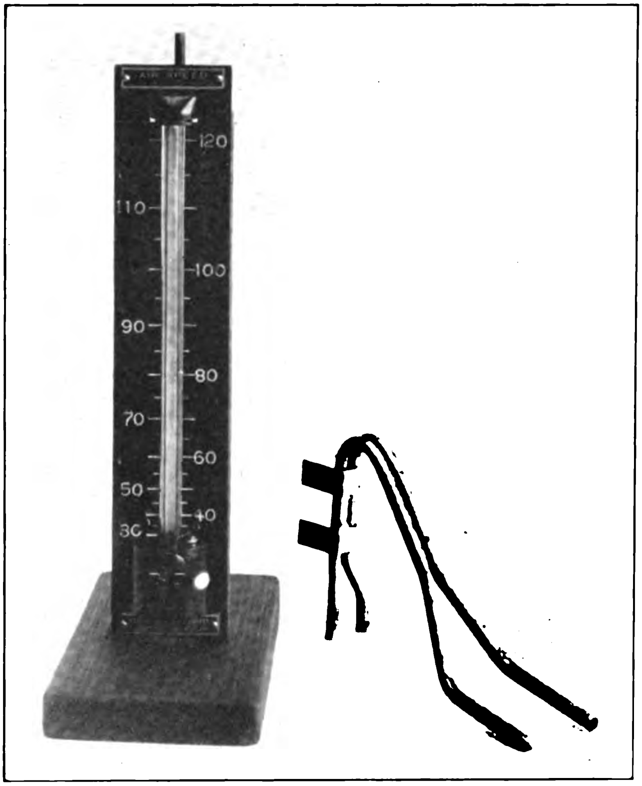

FIG. 6E. Pioneer Liquid Manometer Type Air Speed Indicator

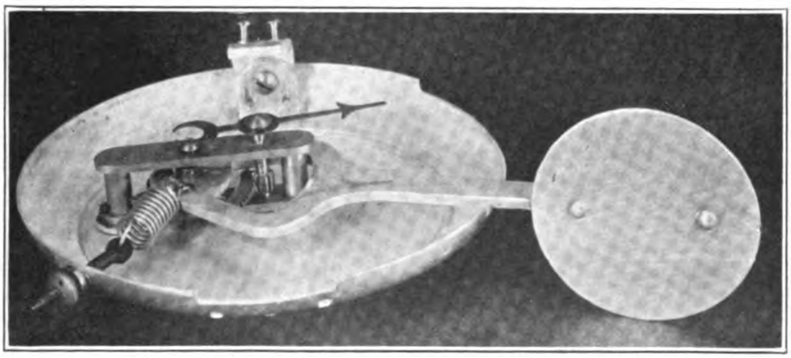

FIG. 7. Pensuti Pressure Plate Air Speed Indicator

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is resisted by a spring. The displacement of the lever which depends upon the speed is used to indicate the air speed directly or it may be attached to a sector and pinion and a dial and pointer added. An instrument of this type is shown in Fig. 7. The pressure developed by these instruments also obeys the ρv^2 law.

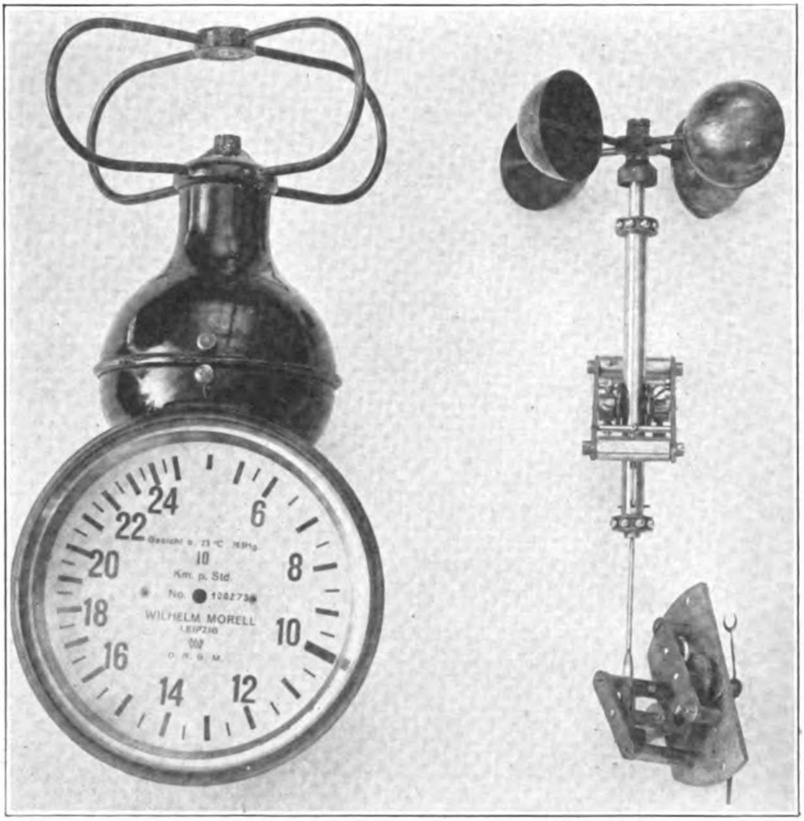

FIG. 8A. Morell Anemometer Air Speed Indicator

In instruments of the anemometer type the air speed is determined by the rate of revolution of a cup anemometer or air propeller. In most cases the rate of revolution is determined by attaching the rotating element to a centrifugal tachometer similar to that used in indicating the rate of revolution of the engine.

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(See below). These instruments are ordinarily located on one of the struts of the airplane and are read from this position by the pilot. Instruments of the anemometer type with electrical distant control have recently been developed in which the anemometer alone is located on the strut. Wires lead from a specially designed commutator operated by the anemometer to the indicator which is on the instrument board. Typical anemometer air

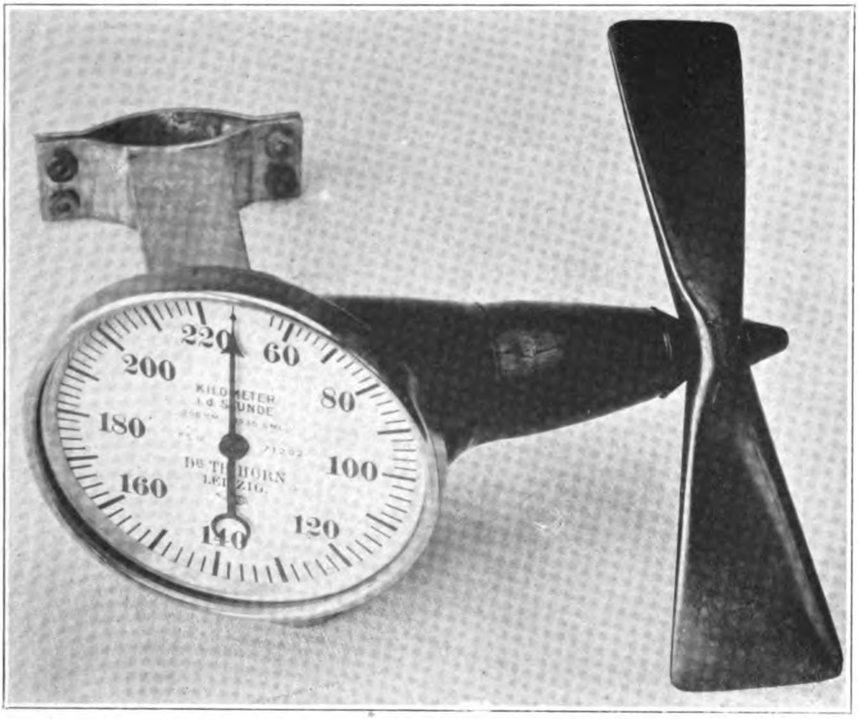

FIG. 8B. Horn Propellor Air Speed Indicator

speed indicators are shown in Fig. 8A-B. Hot wire anemometers in which the cooling effect of the air on an electrically heated wire grid have also been used to a limited extent but the device is complicated and not suited for ordinary use in determining the speed of the aircraft.

Ground Speed Indicators. The measurement of speed of aircraft relative to the ground is of importance in connection with

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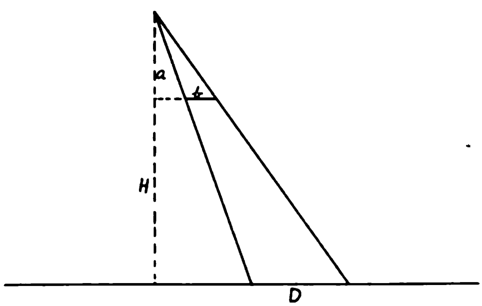
aircraft performance tests, long distance flying and military operations such as bombing. In case of aircraft performance tests the ground speed attained is ordinarily determined by flying the aircraft over measured courses or by sighting upon the aircraft from the ground with theodolites. These methods will not be considered in detail here, since we are primarily interested in the instruments carried by the aircraft itself. The methods of determining ground speed from the aircraft such as are used in long distance flying and in bombing are fundamentally either optical, dynamical, or electrical in principle. The actual instruments are still for the most part in an experimental state so only the methods of their operation will be considered here.

The simplest type of optical ground speed indicator depends upon determining with a stop watch the time for some object on the ground to pass between two sighting points in a horizontal line on the instrument. The ground speed can then be calculated from the separation of the two sighting points, the distance from the horizontal line defined by them to a third sighting point at the observer's eye, and the altitude of the aircraft. The principle may be demonstrated as follows:



Let a = distance from line b to the eyepiece.

H = the altitude.

b = distance between the two sighting points.

D = the distance traversed by the aircraft while the object on ground appears to move between the two sighting points.

t = time in seconds required.

S = speed of the aircraft.

Then

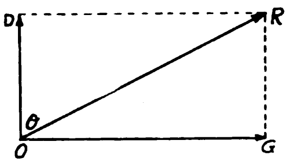
$$\frac{a}{H}=\frac{b}{D}$$

$$\therefore\ S=\frac{H}{a}\frac{b}{t}$$

$$=\text{const.}\ \frac{H}{t}$$

Another method is to use a rotating or reciprocating optical arrangement to neutralize the apparent motion of objects on the ground as seen through a telescope, or to cause a reference line in the telescopic field to move at the same rate as the image of the object on the ground. If then the rate at which the telescope or the image in the telescope field is moving is determined and the altitude of the aircraft is known the ground speed may be found. Several devices of this kind have been tried.

A modification of the last method is to introduce by means of a rotating telescope or similar device an artificial drift at right angles to the actual drift of the aircraft relative to the ground. From the direction of the resultant apparent drift and the magnitude of the artificial drift the ground speed can be computed. The principle may be illustrated as follows:



Let OG represent the ground speed of the aircraft the magnitude of which is to be found and the direction of which is shown by the use of a drift indicator, and OD the known artificial drift introduced at right angles to the ground speed OG by the rotating telescope or other device. Then OR will represent the resultant apparent drift as seen thru the rotating telescope and if the angle Θ between the artificial drift and the resultant apparent drift is measured the magnitude of the ground speed can be calculated by the relation

$$OG=OD\tan\Theta$$

Theoretically it would be possible to find the ground speed of an aircraft by determining the time integral of the accelerations to which it is subjected from the beginning of the flight. It has been proposed to do this by supporting a mass between springs

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so that it is free to move in a horizontal plane in a fore-and-aft direction. The displacement of the mass under these circumstances will be proportional to the acceleration of the aircraft. If then the time integral of this displacement can be obtained mechanically and shown on a direct reading dial, the ground speed at any given instant will be known. Actually the inherent friction of the integrating mechanism and the inevitable accumulation of errors in integration make the device impractical. It is also necessary that the mass move only in a horizontal plane to prevent accelerations of the mass due to gravity. This can be brought about apparently only by gyroscopic stabilization which means much added weight and complication. No practical instrument of this type has been made.

Directional radio telegraphy has recently presented another possibility for ground speed measurement. With a directional receiving apparatus, the position of the aircraft with reference to two sending stations of known distance apart may be determined at successive time intervals and from these observations the ground speed computed. This is at present the only practical method of determining the ground speed of aircraft when the ground cannot be seen.

Rate of Climb Indicators. Rate of climb indicators are used to determine the component in a vertical direction of the velocity of aircraft. Like statoscopes they usually depend for their operation on the expansion or contraction of a volume of air confined in a heat insulated container. This container is connected to the external air through a fine capillary tube. When the aircraft rises the pressure of the air in the container becomes greater than that of the surrounding atmosphere owing to the lag in the pressure equalization caused by the capillary tube. The magnitude of the excess pressure is a function of the rate of climb. If then means is provided for measuring the excess pressure this can be used to indicate the rate of climb. The method ordinarily adopted is to make one side of the container a flexible metal diaphragm connected to an indicating mechanism or to connect to the container a U-tube filled with colored liquid. An instrument of the former type is shown in Fig. 9. Motion of the flexible dia-

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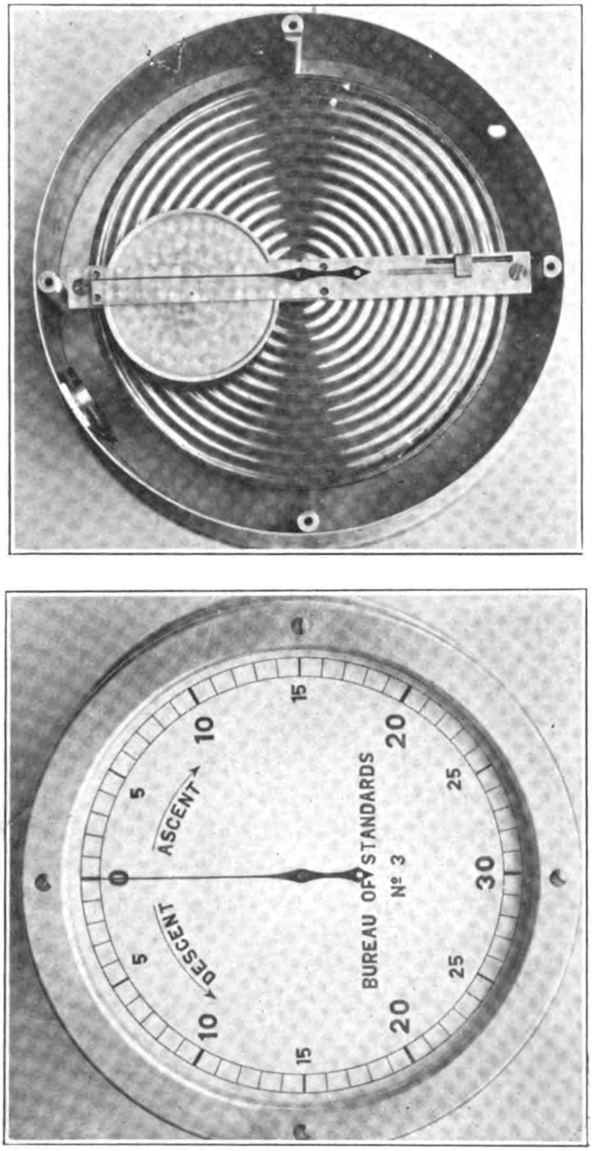

FIG. 9. Bureau of Standards Rate of Climb Indicator

phragm is multiplied by a system of aluminum pulleys and phosphor bronze strips and used to operate a pointer which moves over a dial graduated to indicate the rate of ascent or descent in feet per minute. An instrument of liquid manometer type is shown in Fig. 10. In this case the height of the liquid column is used as a measure of the vertical velocity.

ORIENTATION INSTRUMENTS

Compasses. The adaptation of the magnetic compass to use in aircraft presents serious difficulties because of the violent accelerations to which the instrument is subjected and also the unavoidable proximity of large moving masses of magnetic material in the aircraft. Liquid filled compasses are used almost exclusively because heavy damping is required. Aircraft compasses are in general quick-acting instruments with periods varying from 10 to 20 seconds. Compasses with longer periods are less disturbed by small transient accelerations but in general those with a period between the above mentioned limits are preferred especially when used in connection with turn indicators (see below) which have recently reached a practical state of development. Efforts have been made to overcome the unsteadiness and swirling of the liquid of airplane compasses by mounting the magnet and card in the center of a spherical bowl. Another method recently developed is to make the compass aperiodic by eliminating the ordinary card and mounting the needles on a light spider made of small straight wires projecting radially from the point of support in the damping liquid. Standard compasses of former types are shown in Fig. 11A-B and of the latter type in Fig. 12. Some are provided with both horizontal and vertical cards, others with inclined cards. The aperiodic compass dispenses with the card entirely and uses parallel wires on a rotatable bearing plate over the flat cover glass to sight on the needle.

The disturbing effect of masses of iron in the vicinity of the instrument board have led to an effort to develop distant reading compasses in which the compass proper can be located far away from the motor, for instance, in the fusilage, near the tail of the airplane, while the indicator is located on the instrument board.

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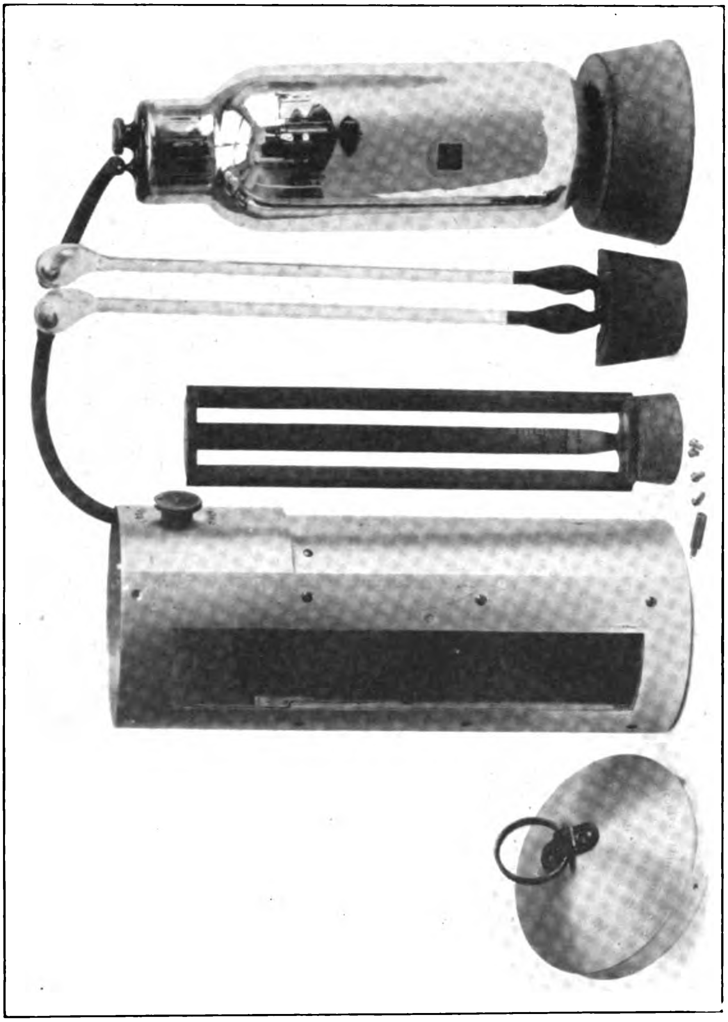

FIG. 10. Wright Rate of Climb Indicator

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One method which has been tried is to use the current developed by a coil of wire rotating in the earth's magnetic field. This can be used to indicate direction since the magnitude of the induced current is a function of the orientation of the coil with respect to the earth. This device is known as an earth inductor compass.

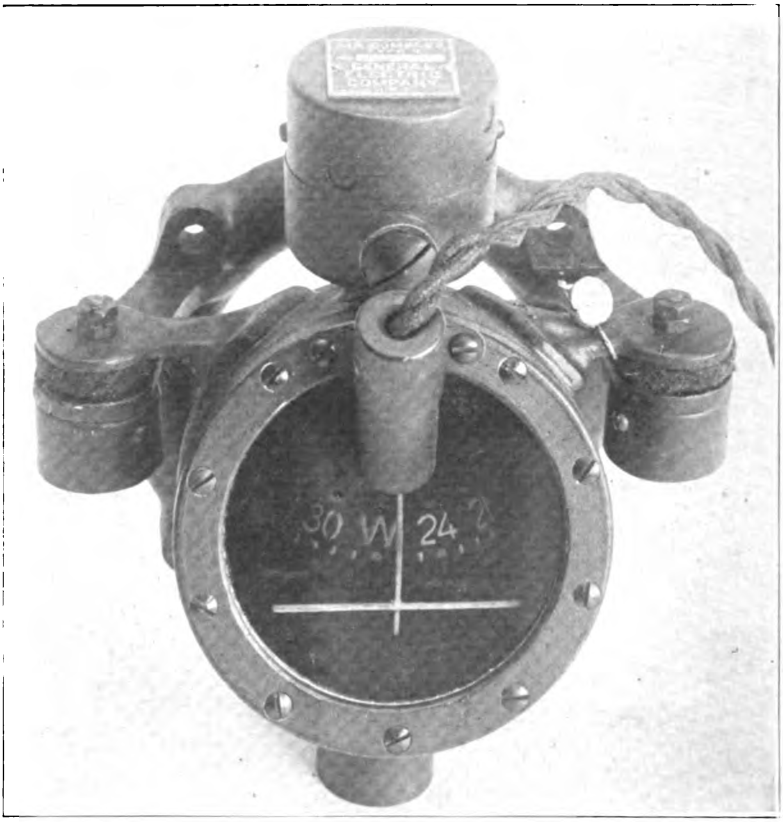

FIG. 11A. General Electric Company Compass

Another method takes advantage of the change of resistance of selenium when exposed to light by providing two selenium cells located at diametrically opposite points above the compass card. Below each cell is an incandescent lamp. The card shields the

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selenium cells to a greater or lesser extent according to the direction of the compass needle thereby changing the resistance of the selenium cells which constitute two arms of a Wheatstone bridge. This unbalances the bridge and indicates on a galvanometer on the instrument board the amount of displacement of the compass

FIG. 11B. Sperry Compass

card. This device is complicated and with its auxiliary attachments much heavier than an ordinary aircraft compass (See Fig. 13.)

Turn Indicators. Turn indicators are used to inform the aviator when he is deviating from a straight line course. The essential working element is a gyroscopic rotor which in accord-

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ance with the principle of gyroscopic action tends to maintain its direction in space when the airplane deviates. The resultant relative motion is made evident to the pilot by the motion of a pointer connected to the rotor thru a lever system. The rotor is ordinarily driven either by the impact of an air stream on the serrated edge of the rotor itself or by making the rotor an induction motor. In the air-driven type the air stream is maintained by a

FIG. 12. Campbell-Bennitt Aperiodic Compass

Venturi tube which exhausts the air from the case in which the rotor is enclosed. Small orifices are provided in the case opposite the serrated edge of the rotor. Thru these the air streams in from outside the case impinging on the wheel and causing it to rotate. The electrical type can be connected to the storage battery which is a part of the standard equipment of modern aircraft, or it can

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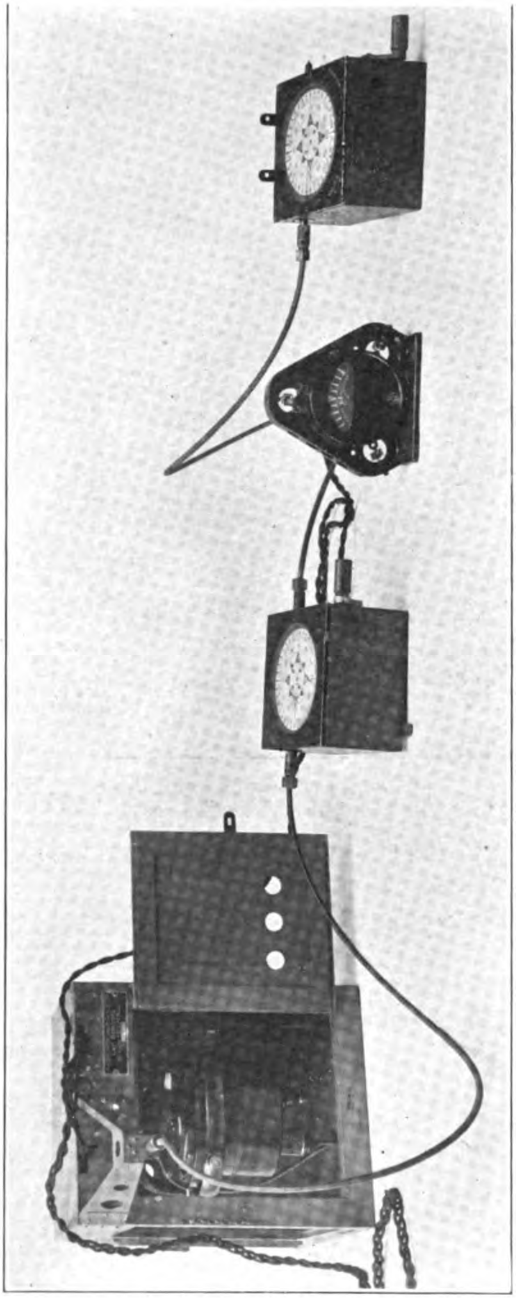

FIG. 13. Bamberg Distant Reading Compass with Indicator and Course Setters

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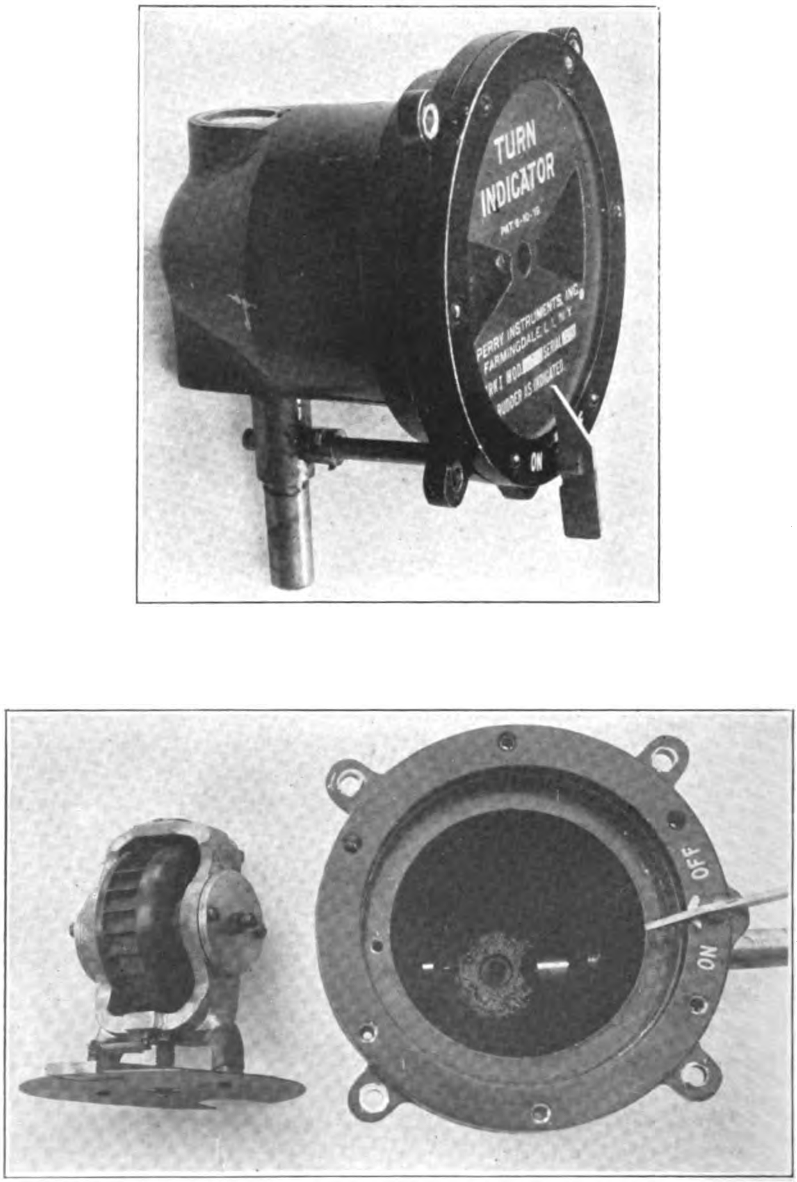

FIG. 14A. Sperry Air Driven Gyro-Turn Indicator

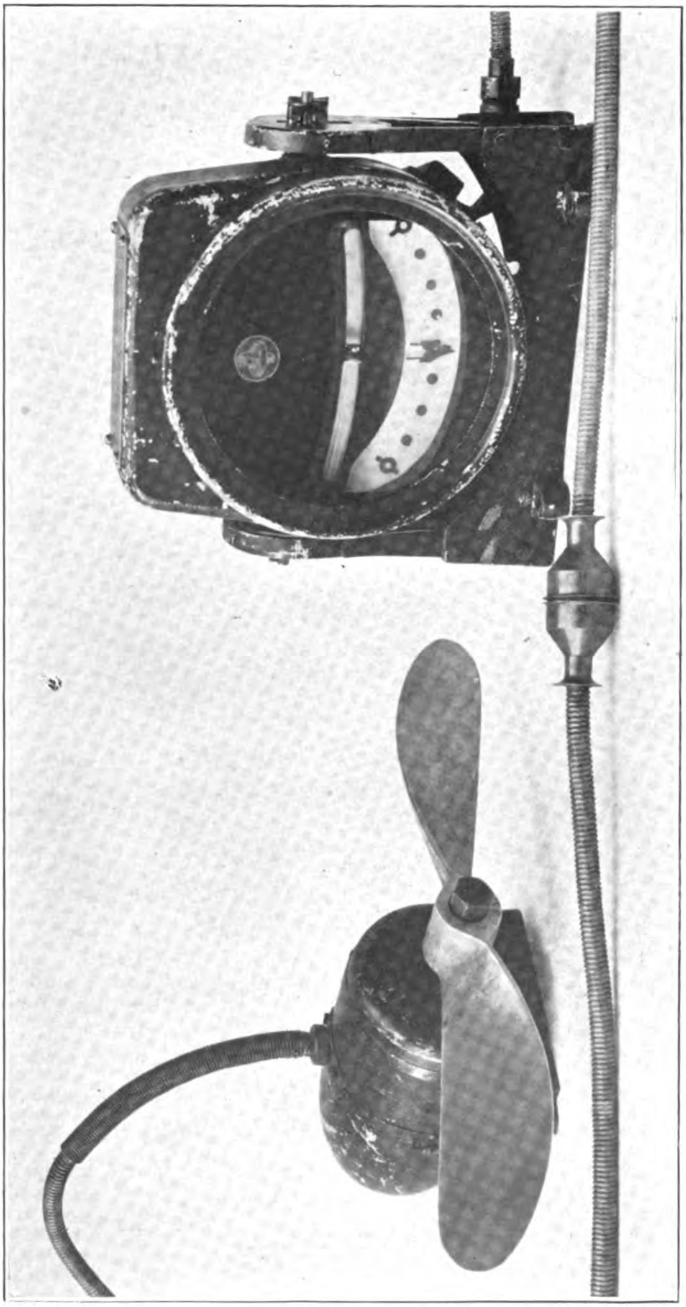

FIG. 14B. Drexler Electric Gyro Turn Indicator

be operated by a small auxiliary generator driven by a wind propeller. Instruments of both types have reached a practical stage of development and are an invaluable adjunct to the compass, particularly for use in cloud flying, since they can be made more sensitive to slight deviations than the compass and moreover function when the turn commences, at which time the compass is ordinarily temporarily useless because it is oscillating. Turn indicators of both types are shown in Fig. 14A-B.

Inclinometers. Inclinometers are used to indicate the inclination of aircraft with reference to the true vertical or to the resultant of gravity and forces of acceleration acting on the aircraft. There are two general types, those involving the principle of the liquid bubble level which indicate the aspect of the air plane with respect to resultant gravity and gyroscopic instruments which indicate the position of the airplane with reference to the true vertical. The former is much the simpler and more frequently used type. Representative inclinometers of liquid type are shown in Fig. 15A-F. These are designated by the name lateral and fore and aft inclinometers according as they refer to the condition of the airplane with reference to rolling and pitching. The liquid lateral inclinometer is essentially a curved glass tube filled with colored liquid in which a bubble forms. The displacement of the bubble indicates the inclination. The fore and aft inclinometer is the same in principle except that in this case it consists of a triangular shaped closed circuit of glass tubing partially filled with liquid. The liquid changes its level in the front arm of the circuit when the airplane pitches. See Fig. 15C. Liquid inclinometers of sector type are shown in Fig. 15D. In these a disk shaped receptacle with a circular glass face is half filled with colored liquid. The position of the surface of the liquid with reference to the normally horizontal diameter of the dial indicates the inclination of the aircraft. Liquid and air damped pendulum devices have also been used instead of instruments of the liquid bubble type. These are shown in Fig. 15E and F where 15E is a lateral inclinometer and 15F a fore-and-aft inclinometer.

An instrument of gyroscopic type is shown in Fig. 16. It is essentially a spinning top mounted on a pivot near its center of

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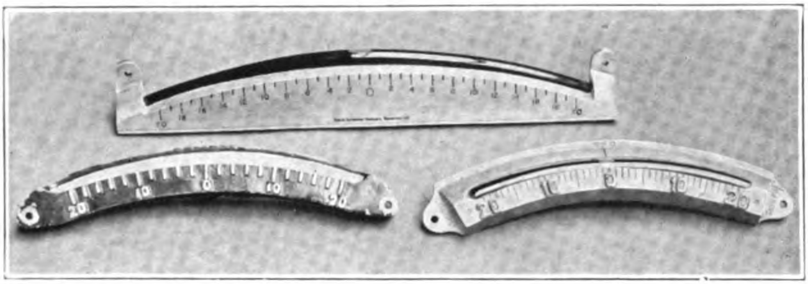

FIG. 15A. Liquid-Bubble Lateral Inclinometers

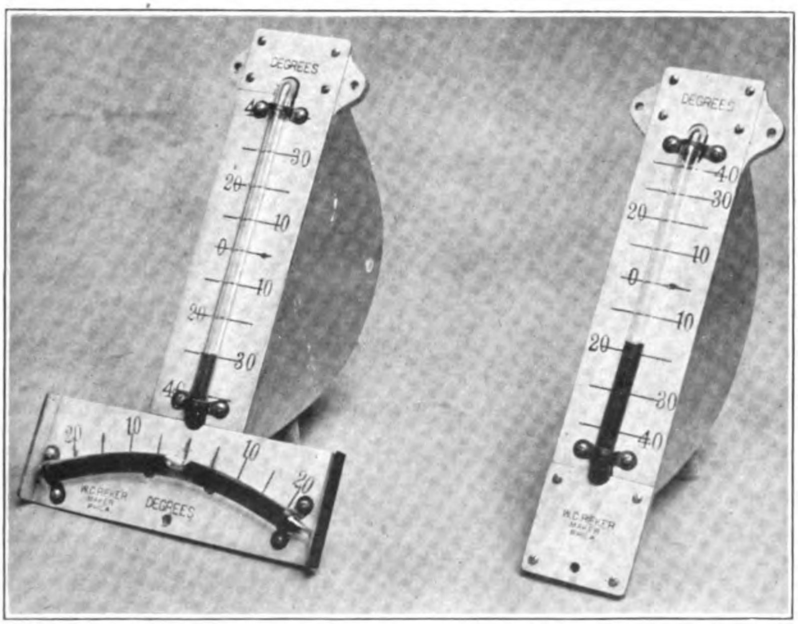

FIG. 15B. Ricker Liquid Fore-and-Aft Inclinometer and Combined Liquid Fore-and-Aft and Lateral Inclinometers

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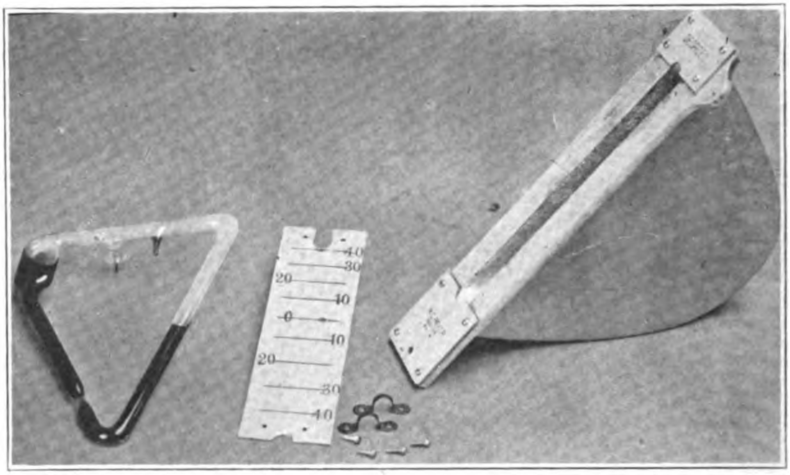

FIG. 15C. Ricker Liquid Fore-and-Aft Inclinometer—Disassembled

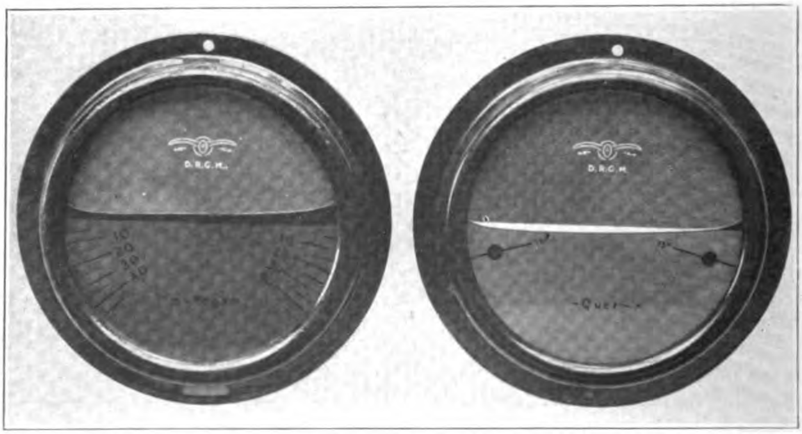

FIG. 15D. D.R.G.M. Sector Type Lateral Inclinometer

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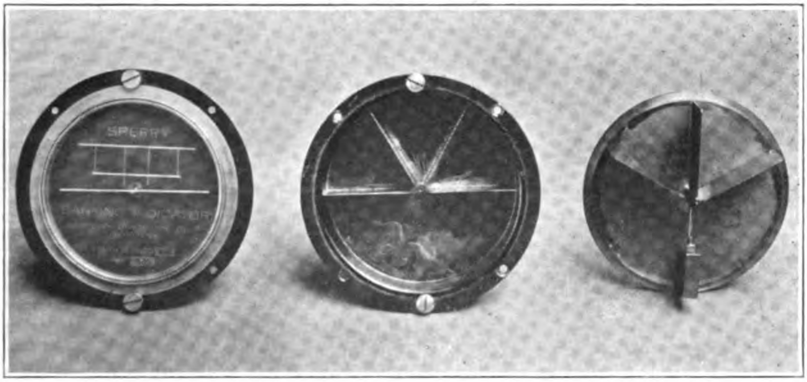

FIG. 15E. Sperry Pendulum Lateral Inclinometer

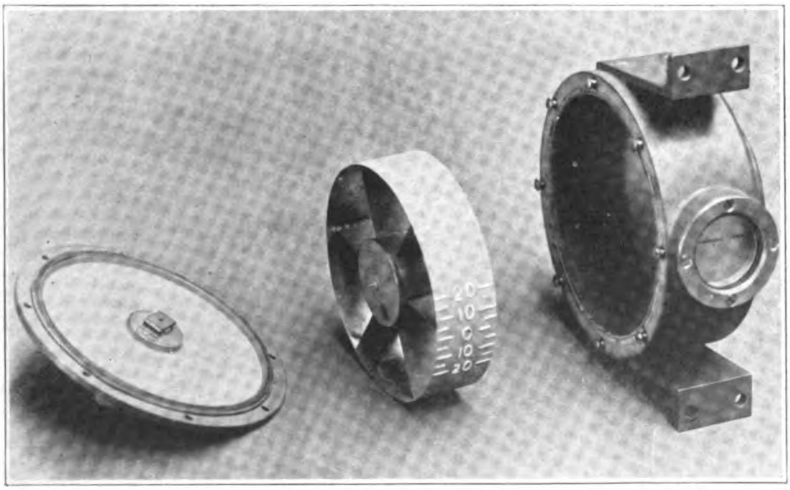

FIG. 15F. Sperry Pendulum Fore-and-Aft Inclinometer

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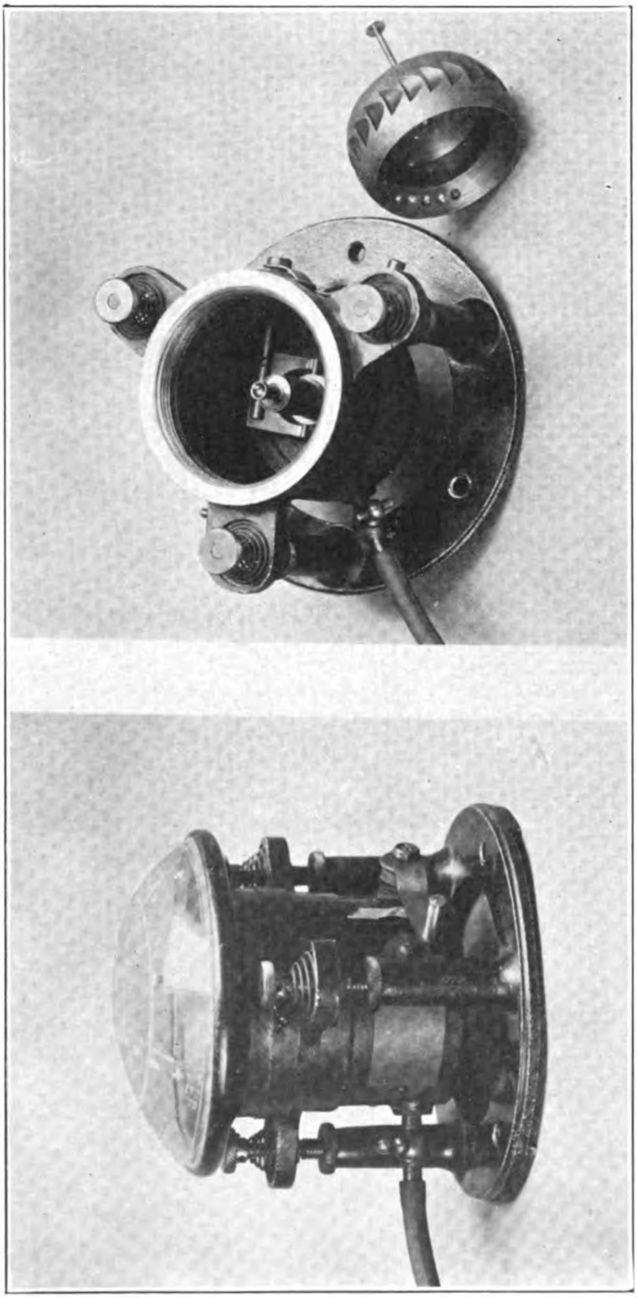

FIG. 16. Garnier Gyroscopic Top Inclinometer

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gravity. It is rotated rapidly by an air stream impinging upon the serrated edge of the top. The gyroscopic action of the top tends to maintain its position vertical when the airplane pitches or rolls. The amount of displacement either laterally or longitudinally is indicated by the position of the pin of the top relative to the spherical cover of the instrument which is graduated in degrees. The power to drive the top is supplied by a Venturi tube which exhausts the air from the case and rotates the top as in the air-driven turn indicator described above.

ENGINE INSTRUMENTS

Tachometers. Tachometers are used to indicate the rate of revolution of the crank or propeller shaft of aircraft engines. They are usually driven by a flexible shaft which runs from the engine to the instrument board where the tachometer itself is located. The two types most commonly used are the centrifugal and the chronometric.

The centrifugal tachometer is the same in principle as the familiar ball governor and depends upon the tendency of a mass to move away from the axis of rotation under the action of centrifugal force. This tendency is resisted by a spring. The amount of motion which is a measure of the rate of rotation, is applied through a transfer mechanism to the pointer which moves over a dial graduated in revolutions per minute. A centrifugal instrument of standard design is shown in Fig. 17. In some centrifugal tachometers the centrifugal element consists of two or more small weights connected to the shaft by links as in Fig. 17, and in others of a single inclined weight which tends to assume a horizontal position under the action of centrifugal force. Centrifugal instruments are much simpler in construction and more durable than the chronometric type but are not so accurate as the latter.

In chronometric tachometers the speed is measured by the amount of motion of a toothed rack or gear system in a measured interval of time, usually one or two seconds, which is determined by a clockwork escapement. This motion is communicated to the pointer which is deflected in the given time interval an amount depending upon the speed of rotation of the driving shaft. The

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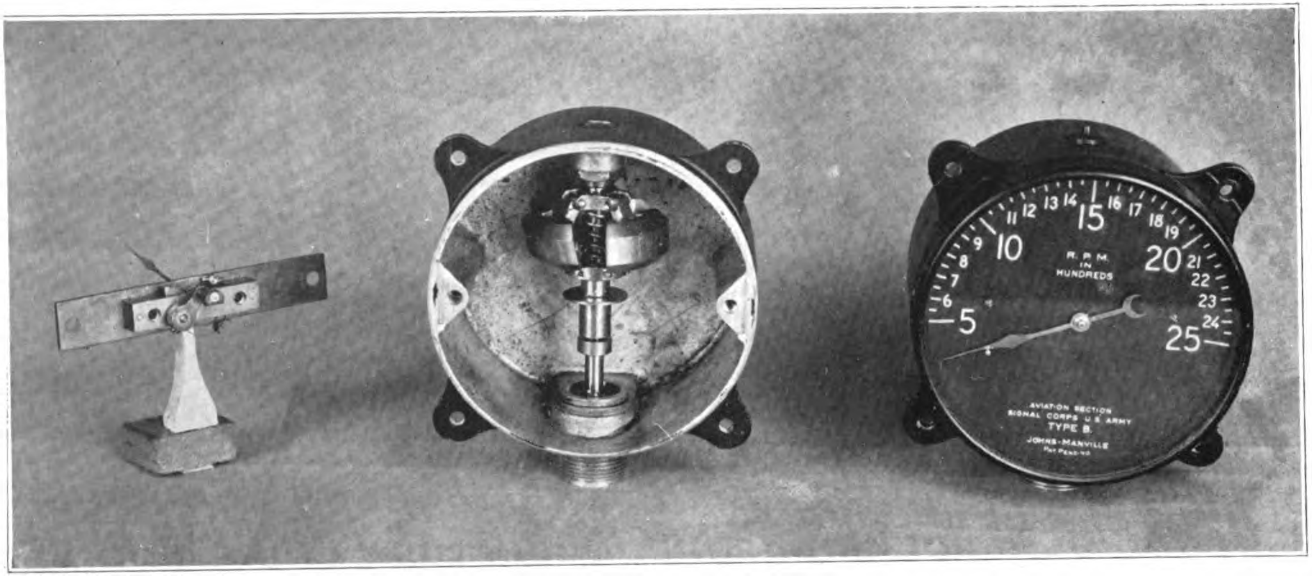

FIG. 17. Johns Manville Centrifugal Tachometer

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mechanism is so designed that the pointer is locked in position during each succeeding time interval while the toothed rack or gears are in action. At the end of each time interval the pointer is released and suddenly jumps to its new position which is determined by the rate of rotation of the engine shaft during the time interval just ending. The result is that the pointer of the instrument moves by discontinuous jumps instead of continuously, as in instruments of the centrifugal type. A representative tachometer of the chronometric type is shown in Fig. 18. As stated

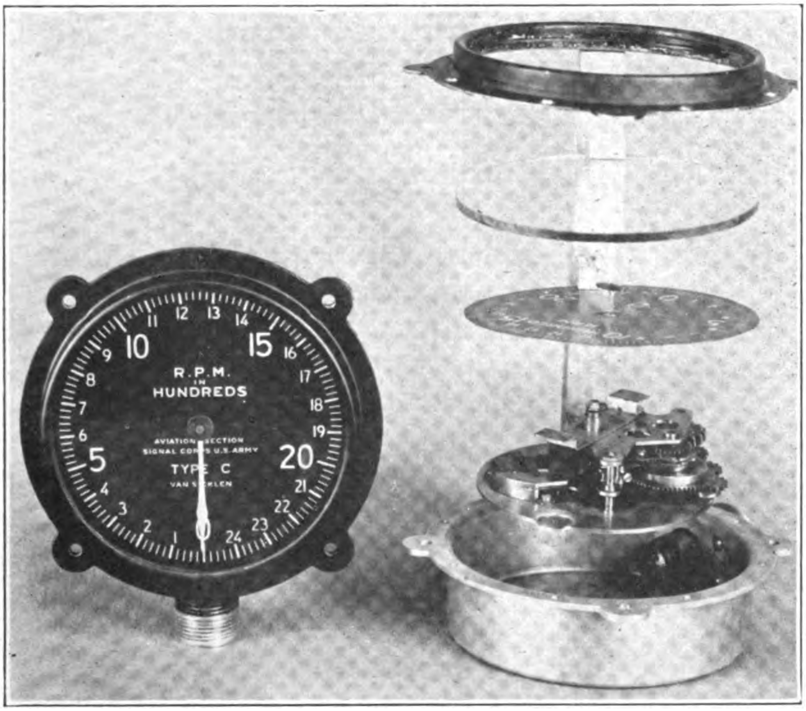

FIG. 18. Van Sicklen Chronometric Tachometer

above they are more accurate than the centrifugal tachometer but they involve complicated clockwork mechanism which easily gets out of order and is difficult to repair. However, a number of satisfactory instruments of this type have been made.

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Several other types of tachometers have been used to a limited extent. Among these may be mentioned magnetic and electro magnetic tachometers, air viscosity and air pump tachometers. In the magnetic tachometer a permanent magnet is rotated near a conducting disk thereby dragging the disk, by virtue of the induced eddy currents, in opposition to a resisting spring an amount depending upon the rate of rotation. A pointer attached to the disk moves over a scale graduated in miles per hour. An instrument of this type is shown in Fig. 19.

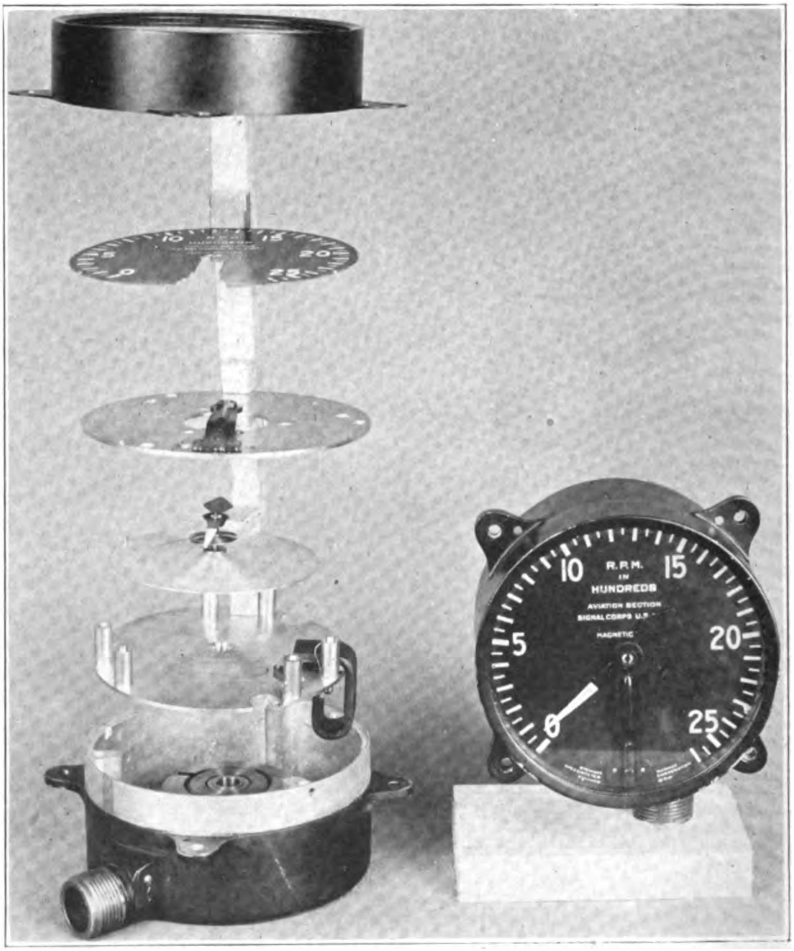

FIG. 19. Stewart-Warner Magnetic Tachometer

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In the electromagnetic type a small magneto is attached to the engine shaft and connected to a galvanometer on the instrument board. The voltage developed by the magneto and hence the deflection of the galvanometer is proportional to the rate of revolution of the engine. The galvanometer is graduated to read in revolutions per minute. Magnetic and electromagnetic tachometers show larger errors than the centrifugal and chronometric type instruments and do not maintain their calibration as well.

The air-drag or viscosity tachometer consists of two concentric cylinders separated by a thin film of air. One cylinder which is attached to the engine shaft tends to rotate the other by virtue of the viscous action of the air film between them. This tendency to rotation is opposed by a spring. A pointer is attached to the second cylinder and the amount of deflection is a measure of the rate of revolution of the first cylinder. The deflection in this case however is not proportional to the speed. The temperature errors of these instruments are large.

The air-pump type consists essentially of an air pump which forces air into a chamber provided with a leak orifice. The pressure developed in the chamber depends upon the rate at which the pump, which is connected to the driving shaft, rotates. In escaping from the chamber the air deflects a vane whose motion is opposed by a restraining spring. The amount of the deflection of the vane is thus a measure of the speed of revolution of the driving shaft. Air-pump tachometers are subject to large altitude errors caused by the change in air density.

Pressure Gages. Oil and air pressure gages are used in aircraft to indicate the air pressure in the gasoline tank and the oil pressure of the engine lubricating system. Both gages are ordinarily of the Bourdon type but of different range, air pressure gages having a range of approximately 0 to 5 lb. sq. in. and oil pressure gages from 0 to 100 lb. sq. in. A group of representative oil and air pressure gages is shown in Fig. 20. The essential working element is a Bourdon tube, one end of which is rigidly attached to the instrument case and the other to the indicating system, which is either a sector and pinion or a system of levers. With increase of internal pressure the Bourdon tube expands, thereby causing the

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pointer to move across the scale. In some of the instruments the pointer is concentric and in others eccentric, the advantage of the former being that it is given a much more open scale.

Gasoline Gages. Gasoline gages are used to indicate the depth of gasoline in the gasoline supply tank. The most common type consists of a float of cork, wood, or hollow metal which rests on the surface of the gasoline and which is connected to the indicating

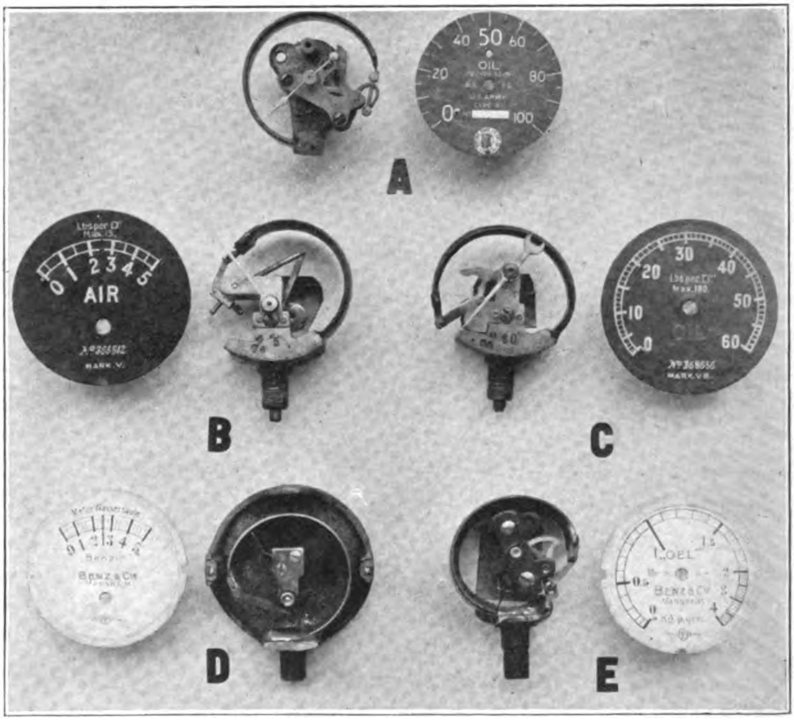

FIG. 20. Air and Oil Pressure Gages

mechanism by a metal rod or a flexible cord (See Fig. 21). Where it is possible to mount the indicator on the tank the float is allowed to travel up and down between two vertical fixed rods. A stiff twisted metal ribbon or inclined rod is rotated by the float as it changes its position, thereby operating the indicator which is mounted above the float and attached to the moving rod through

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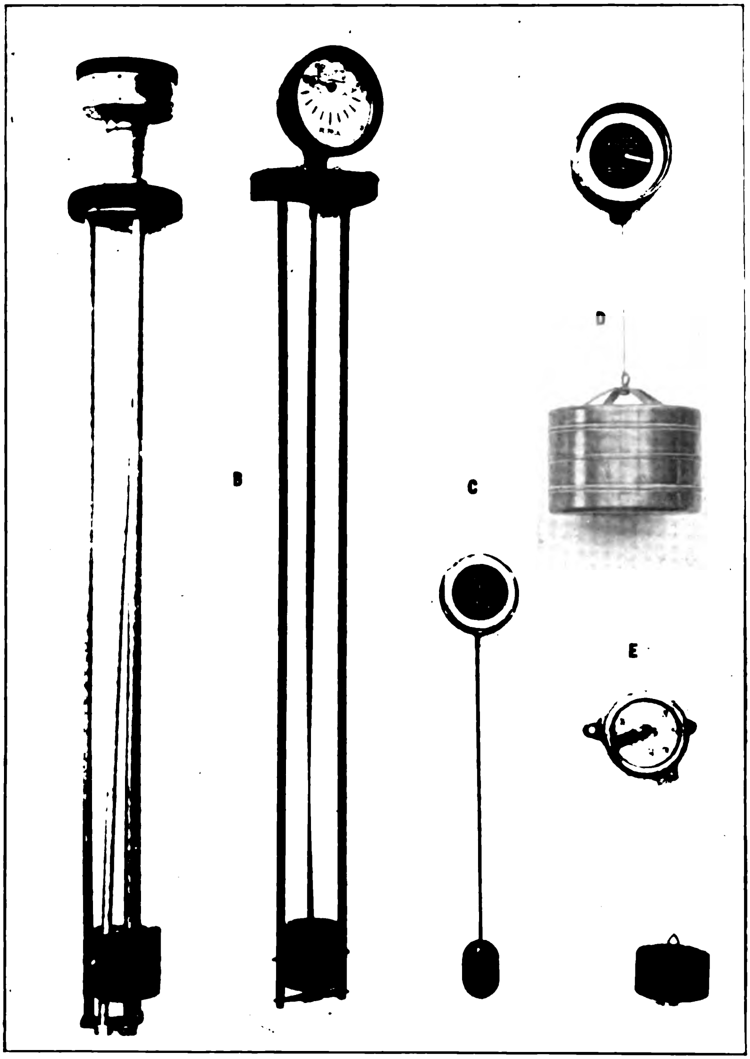

FIG. 21. Float Type Gasoline Gages

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a system of gears. A disadvantage of this type of gage is that the float is likely to stick between the guide rods if they get out of alignment. Another method is that shown in Fig. 21 C, in which the indicator is mounted at the center of the side of the tank facing the aviator. In this case the float is attached to a long metal rod and rotates about the indicator. At the end of the floating rod is a small magnet which drags the iron pointer about as the float follows the level of the gasoline. Between the magnet and the pointer there is a metal disk which protects the cover glass from the hydrostatic pressure of the gasoline.

Float gages in which the float is connected to the indicator by a light weight silk cord, are shown in Fig. 21 D and E. The float consists of an air-tight, hollow, brass cylinder which moves up and down in a metal tube which reaches from the top to the bottom of the tank and whose diameter is slightly greater than that of the float. When the indicator is located at a distance from the float the connecting cord passes through a metal tube with roller fittings at the angles. The gage itself consists essentially of a drum on which the cord winds and unwinds and a system of gears which connects the drum to the indicating pointer. The cord is always maintained taut by a coiled spring which is attached to the drum. The indicator shown in Fig. 21D has a magnetically controlled pointer similar to that described above. One shown in Fig. 21E is provided with a spiral scale and by means of a rack and pinion mechanism the tip of the pointer is made to follow the convolutions of the spiral as it rotates. This makes it possible to allow the pointer to make several complete revolutions without confusing the indications.

Another type of gasoline gage which has been used to a limited extent depends upon the hydrostatic pressure of the head of gasoline in the tank. This pressure is measured by an indicator with corrugated flexible metal diaphragm capsules similar to those of an airspeed indicator. The gage is operated by connecting the case of the indicator to the air space above the gasoline tank and the diaphragm capsule to a tube extending to the bottom of the tank. Air is caused to bubble through the tube either by the use of a handpump or automatically by the use of a power pump

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thereby impressing on the indicator a differential pressure equal to the head of gasoline. With this type of gage it is possible to have the indicator on the instrument board and connected to the gasoline tank by metal tubing. A disadvantage particularly from the military point of view, is that a rupture in the connecting tube is likely to cut off the fuel supply from the engine. A group of gages of this type is shown in Fig. 22.

Gasoline Flow Indicators. The rate of gasoline consumption in aircraft engines is found by the use of flow indicators. Typical instruments are shown in Fig. 23. In the one at the left of the figure a metal vane restrained by a coiled spring is deflected by the gasoline as it flows through the instrument. The gasoline flows past the vane in the space between the vane and the case. The case is provided with a cam surface which varies the space between the vane and the case as the vane rotates so that the deflection of the vane is made proportional to the rate of flow of gasoline. A pointer attached to the vane indicates the rate of flow in gallons per minute. In the indicator at the right the gasoline is forced out thru a slit in a vertical metal tube surrounded by a concentric glass tube. A small rider shown at the left floats on the gasoline. The height reached by the gasoline as it flows thru the slit and consequently the reading indicated by the rider is proportional to the rate of gasoline consumption.

Thermometers. Thermometers are used in aircraft to indicate the temperature of the radiator water and oil supply of the engine, the temperature of the atmosphere, and on lighter-than-air craft the temperature of the gas in the bags. The last two mentioned types are described below under Special Instruments. Thermometers for measuring the temperature of water and oil ordinarily consist of a metal bulb partially or completely filled with liquid which is located at the point whose temperature is to be determined and which is connected by means of a capillary tube to some form of pressure gage, usually of the Bourdon type, located on the instrument board. Two types of pressure thermometers are used. These are known as the vapor pressure and liquid filled type, according as they depend upon measuring the variation of the pressure of the vapor of a volatile liquid or the expansion of a

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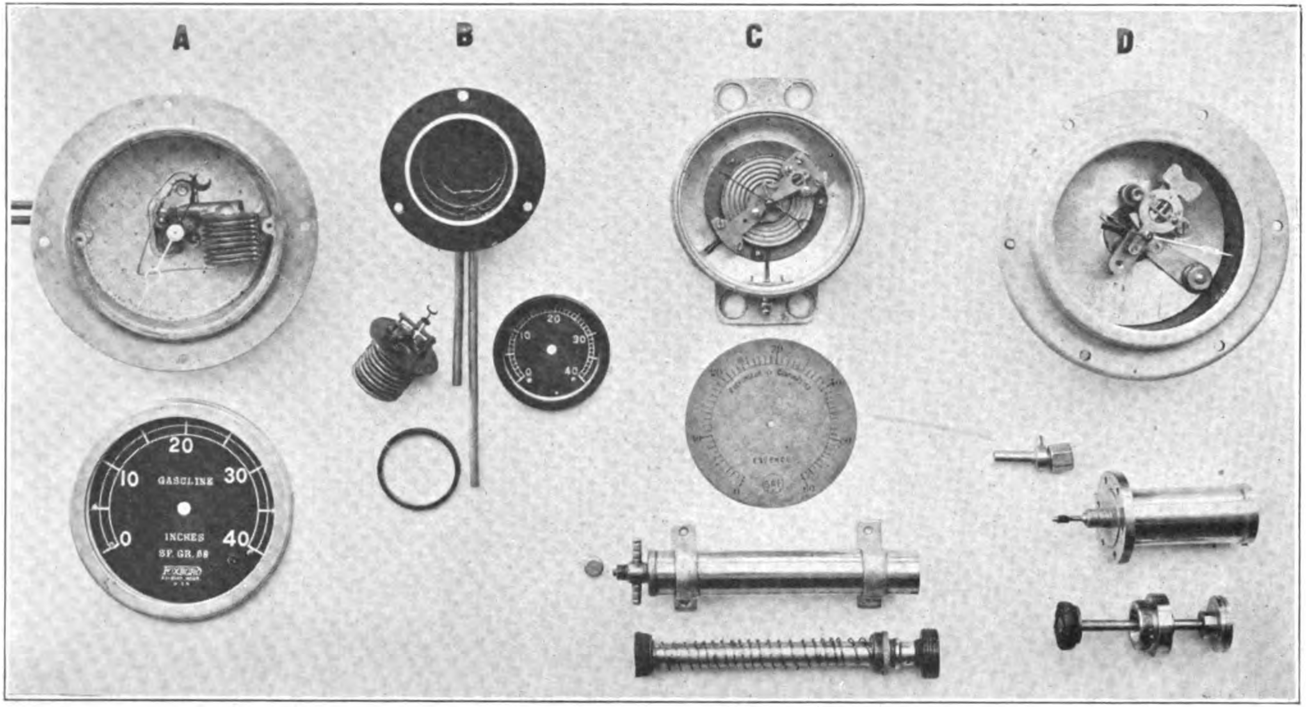

FIG. 22. Pressure Type Gasoline Gages

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liquid with change of temperature. In the former case the bulb is only partially filled with liquid and the connecting capillary tube and gage is filled with vapor, while in the latter case the entire system including the bulb, capillary tube and gage is completely filled with liquid under pressure. Ethyl ether and methyl chloride are the most frequently used as the volatile liquid. Ethyl alcohol is the liquid usually used in the liquid filled type. The vapor pressure type is affected by changes of altitude. The liquid filled type on the other hand, gives erroneous readings if the temperature of the gage and the capillary tube differ from that at which the instrument was calibrated. Typical thermometers of both types

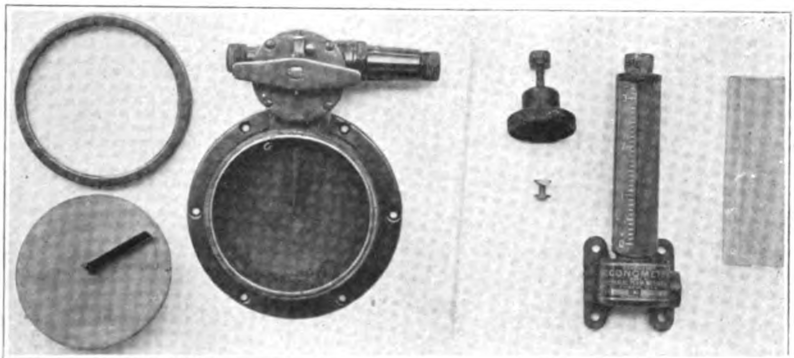

FIG. 23. Gasoline Flow Meters

are shown in Fig. 24. The vapor pressure instrument has an ordinary Bourdon tube which is connected to the indicating mechanism by a sector and pinion, such as that used in the pressure gages previously described. The liquid filled thermometer has a coiled Bourdon tube with several convolutions. It is also provided with a bimetallic temperature compensator which is connected between the Bourdon tube and the indicating mechanism.

NAVIGATING INSTRUMENTS

The use of aircraft for long distance flights over both land and water and for night flying has required the development of aerial navigating instruments. The methods applied are fundamentally

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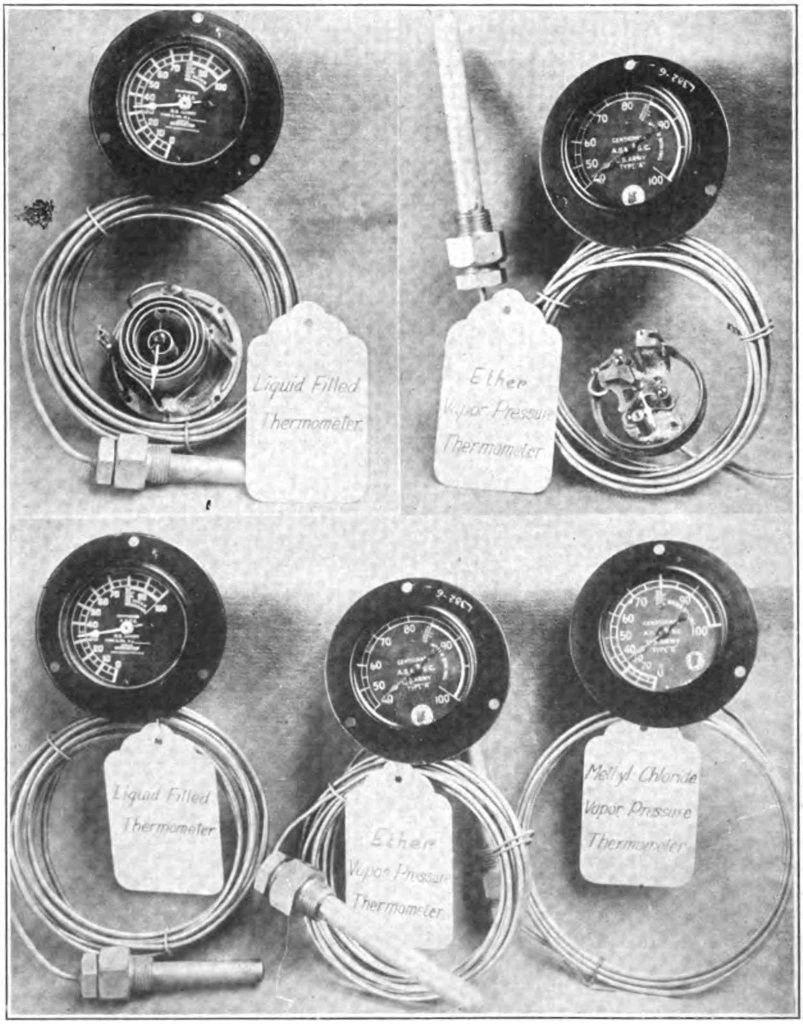

FIG. 24. Liquid and Vapor Pressure Thermometers

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the same as those used in the navigation of ships at sea, the most important difference being the necessity of adapting the instruments to the relatively swift and unstable aircraft and, also, the uncertainty introduced by swift air currents which change rapidly in magnitude and direction, not only with time but also with altitude. It has thus been found impracticable to chart air currents as is done in the case of ocean currents at sea.

The simplest method of air navigation is that in which maps of the territory traversed are used and the course is guided by following landmarks known to the pilot. More general methods which involve calculating the course of the aircraft are (1) dead reckoning in which position at a given time is calculated from a previous known position by determining the direction of flight and speed with reference to the earth, and (2) astonomical observations in which the altitude or azimuth of the sun or stars is measured and the position computed from the Greenwich siderial time, the equation of time and the position of the sun or stars. Still another method recently developed is the use of the radio direction finder in which the position is determined with reference to radio stations by the use of a radio direction finder on the aircraft.

Maps and Charts. When maps are used they are frequently mounted on rolls in a map case (See Fig. 25A) so that a number of maps can readily be made accessible. Sometimes the map is mounted on a board and protractors and parallels provided for convenience in locating directions and measuring distances. Such a device is shown in Fig. 25B.

Dead Reckoning Instruments. The factors involved in the method of dead reckoning are the direction of the aircraft as determined by a compass, the air speed, the ground speed, and the drift with reference to the earth. Instruments used in making the first three measurements have already been discussed in this paper. Drift indicators usually depend upon determining by sighting wires or parallel lines in the instrument or in the focal plane of a telescope the apparent direction of motion of objects on the ground or on the water or even of the waves of the sea whose motion is so slow compared with that of the aircraft that it can

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be neglected. Instruments of this type known as drift bearing plates are shown in Fig. 26. These consist of a rotatable graduated circle with diametral sighting wires which are turned parallel to the direction of drift. The vertical attachment is used in determining the ground speed and carries an eyepiece which is adjusted for altitude by the graduated scale on the attachment.

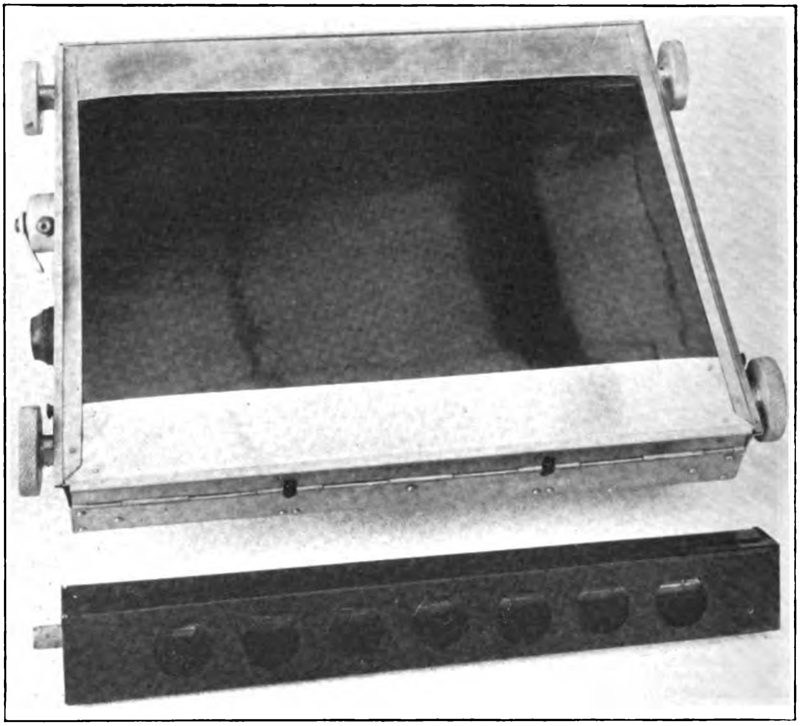

FIG. 25A. Map Case

The ground speed is determined by finding with a stop watch the time required for an object on the ground to pass between two ball sights on the instrument. A more complicated device with attachment for adjustment of the lubber-line of the compass is shown in Fig. 27. In this case the direction of drift is determined by adjusting a system of parallel lines in the focal plane of the

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telescope, shown at the left of the figure, to the direction of drift. This automatically changes the position of the lubber-line of the compass.

The drift indicator shown in Fig. 28 in addition to determining the direction of drift and the ground speed has adjustments which

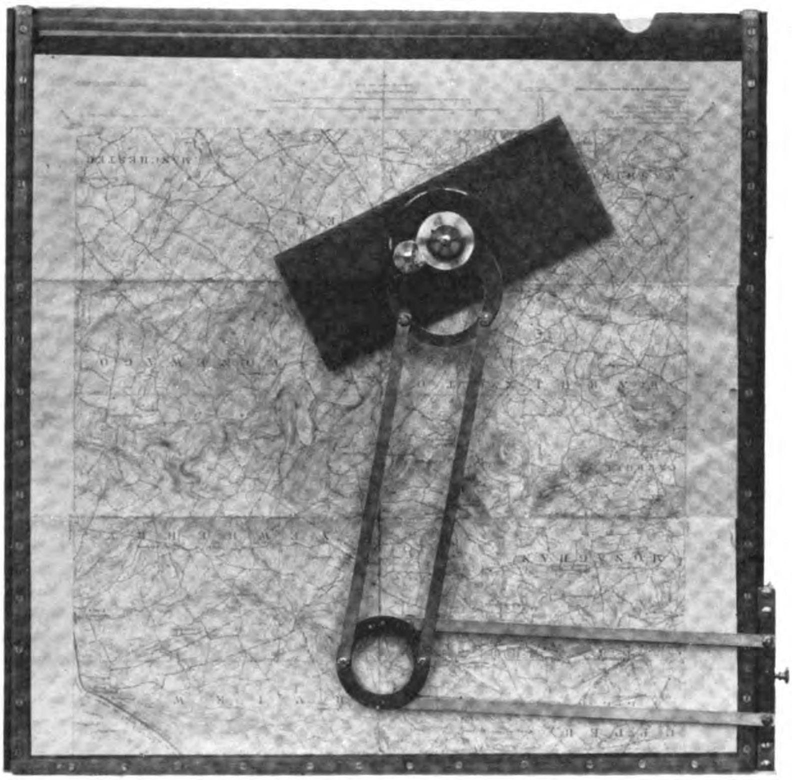

FIG. 25B. Bigsworth Chart Board

when the instrument is also set for the airspeed automatically determines the magnitude and direction of the prevailing wind, or as it is ordinarily expressed, solves the velocity triangle. The ground speed is determined as in the drift bearing plate just described by placing the eye at the eyepiece and noting the time

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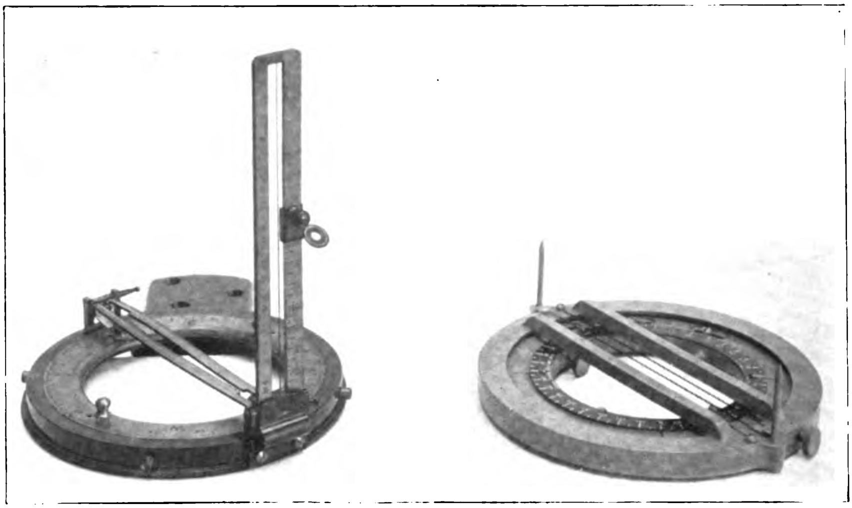

FIG. 26. Drift Bearing Plates

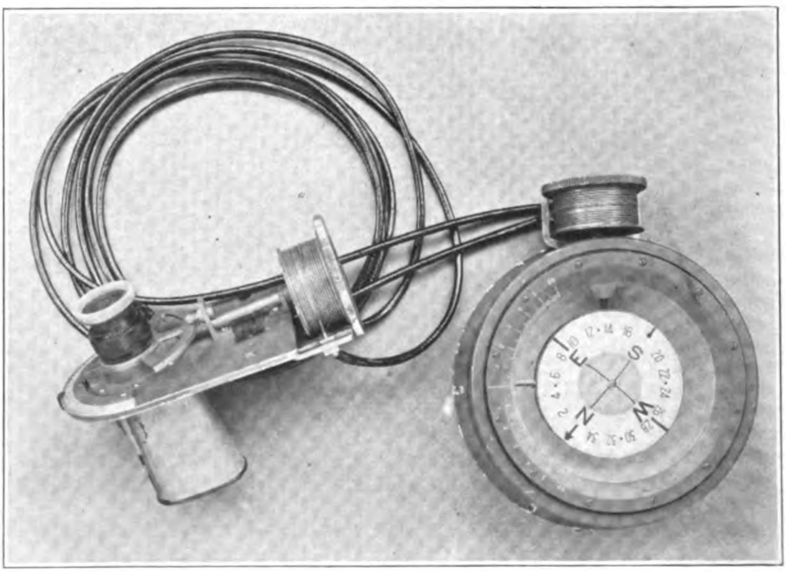

FIG. 27. Sperry Synchronized Drift Set

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required for an object on the ground to pass between two points on the sighting arm.

The device shown in Fig. 29 is a synchronized drift sight which determines the wind vector from the direction and magnitude of drift and the air speed. The repeater which is attached to the main instrument indicates the result to the pilot. The unique feature of this device is the method used to determine the drift. An object on the ground is followed by means of the telescope at the right of the figure and a series of points plotted on the paper by depressing the pencil which is maintained parallel to the

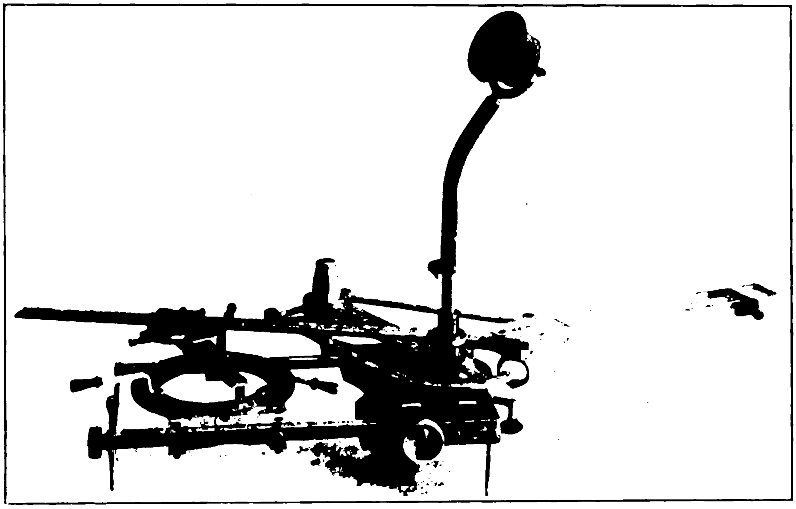

FIG. 28. Crocco Drift Indicator

telescope by a lever system. A series of points showing the direction of drift is thus obtained, defining a line in which the irregularities of the individual observations due to the rolling and pitching of the aircraft are eliminated.

A number of simple devices have been invented to aid in solving the velocity triangle. Three of these are shown in Fig. 30. They are used by setting the adjustable arms in the direction of the two known velocities of the velocity triangle, setting the adjustable sliders for the magnitude of these velocities and rotating the

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disk until the arrow on it is parallel to the line through the two sliders. The direction of the arrow is the direction of the third velocity component. Its magnitude is determined from the scale on the disk by the distance between the two adjustable sliders.

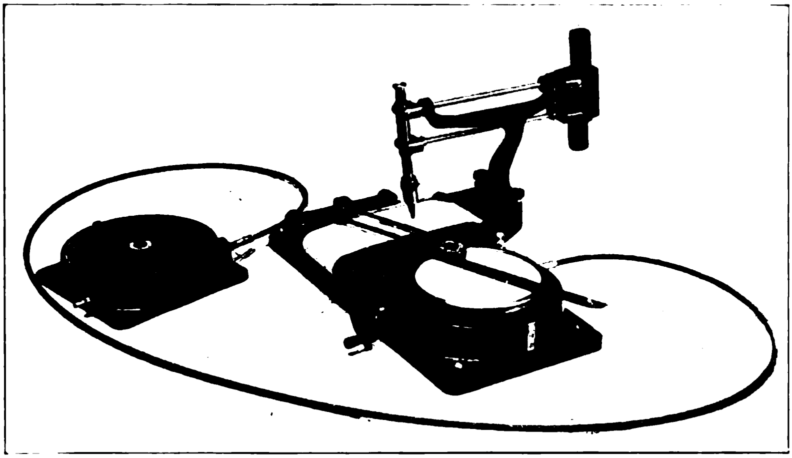

FIG. 29. Le Prieur Navigraph

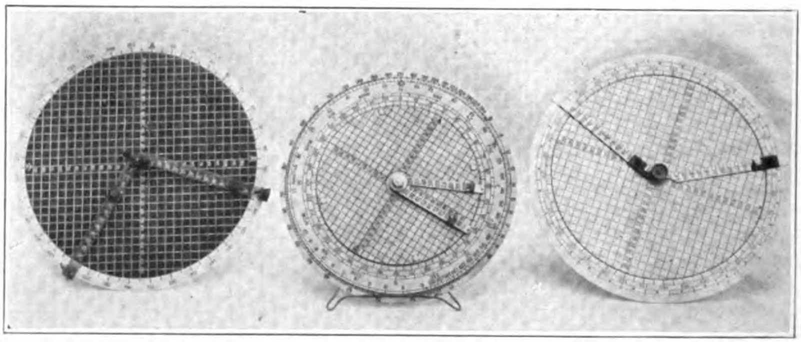

FIG. 30. Course and Distance Calculators

Astronomical Instruments. When astronomical methods are employed the requisite observations are almost always made with sextants which the observer uses to determine the altitude of the

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sun or some star. This enables him to calculate his position from the Greenwich solar time which is read from a chronometer, and the time equation and declination of the sun as determined from the Nautical Almanac. The sextants used (see Fig. 31A-D) differ from marine sextants principally in that an artificial horizon is used. In most cases this consists of a liquid bubble level which is so arranged that it can be seen in the optical field simultaneously with the sun or star on which the instrument is set. Sextants have also

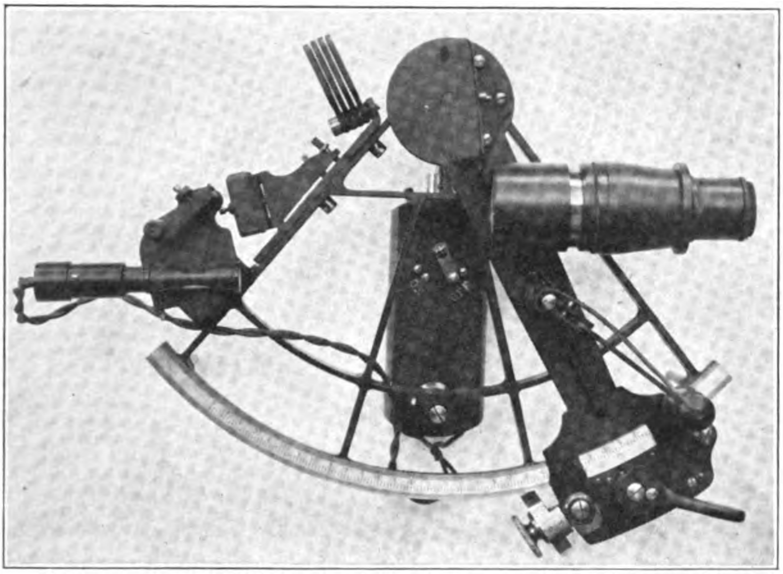

FIG. 31A. Byrd Bubble Sextant

been constructed in which pendulums have been used instead of a bubble level. Another method which has been tried is to establish an artificial horizon by mounting a mirror on the upper surface of a gyroscopic top and arranging the optical system so that the image of the sun or star as reflected from the mirror and also viewed directly are simultaneously seen by the observer. An auxiliary apparatus is required to drive the gyroscope. Usually the gyroscope is air driven, in which case the auxiliary apparatus

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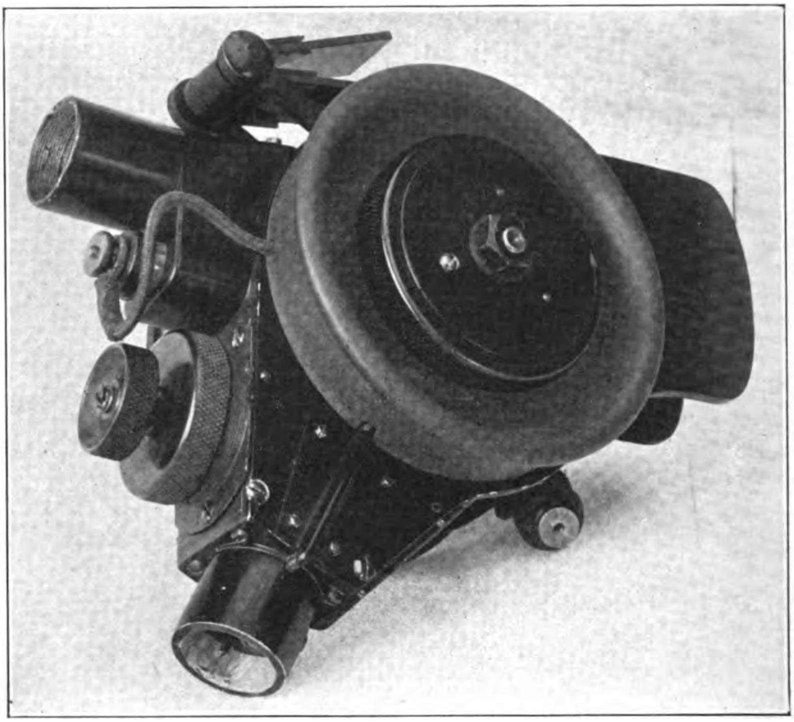

FIG. 31 B. Booth Bubble Sextant with Rotating Drum Scale

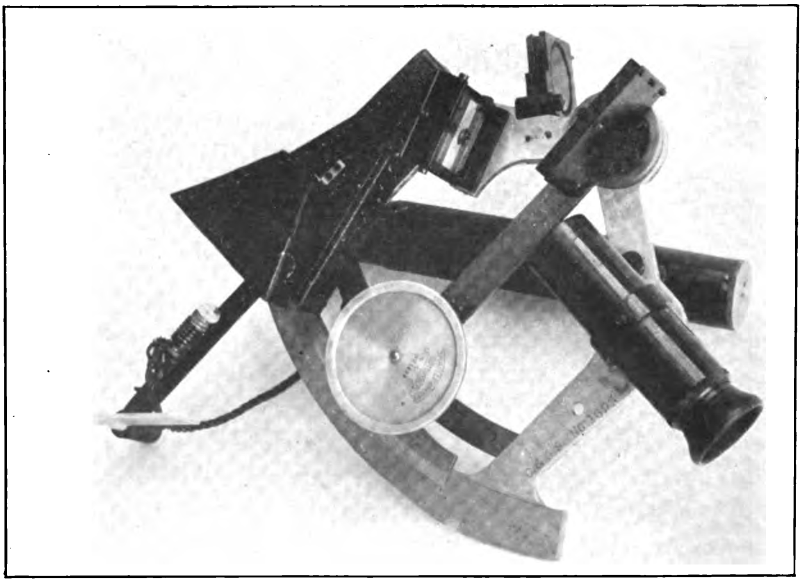

FIG. 31C. Schwartzschild Bubble Sextant

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is a pump, or Venturi tube. An instrument of this type is shown in Fig. 31 D. With good piloting and a skillful observer the error of bubble sextant observations ordinarily varies from 10 to 20 minutes.

Radio Direction Finder. The use of radio direction finders in aerial navigation is of recent origin and still in the experimental stage. These devices consist essentially of a radio receiving

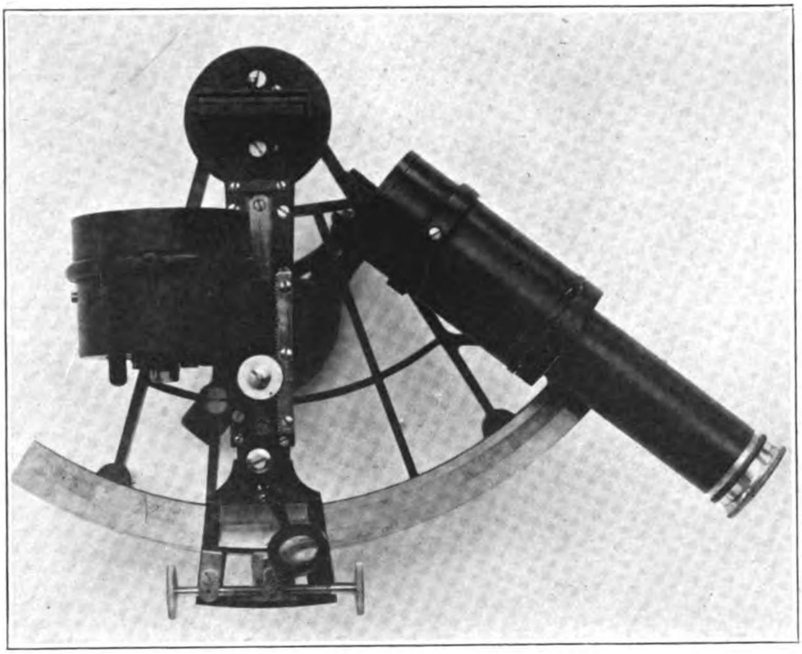

FIG. 31D. Derrien Gyroscopic Sextant

apparatus with a coil antenna which the pilot orients to determine the direction of the radio sending stations which are identified by the character of the signals sent. One important advantage of this method is that it can be used when both the earth and sky are obscured.

SPECIAL INSTRUMENTS AND ACCESSORIES

In this category may be included apparatus to supply oxygen to aviators at high altitudes, instruments used in airplane per-

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formance tests which in general are of the recording type, time pieces, and instruments pertaining particularly to the navigation of lighter-than-air craft such as manometers, ballast gages and hydrogen leak detectors.

Oxygen Instruments. The physical condition of aviators is seriously affected from lack of oxygen when altitudes above 15,000 feet are maintained for extended periods of time. This difficulty can be almost entirely overcome by supplying the aviator artificially with oxygen during flight. The oxygen is carried either in the form of compressed gas in steel cylinders or in liquid form in vacuum jacketed receptacles. A supply sufficient for a flight of two or more hours is ordinarily required. It has been found that four liters of oxygen per minute is not an excessive amount to supply in view of the aviator's physical activity and inevitable losses at the mask in breathing. The expression for the correct delivery at any altitude then becomes

$$V=4\left(1-\frac{P}{760}\right)$$

where V is the volume delivered in liters and P is the pressure of the atmosphere in millimeters of mercury.

An essential feature of the oxygen equipment is a device for controlling the amount of oxygen delivered to the aviator. In the earliest types this was simply a hand controlled valve attached to the oxygen supply tank which the aviator operated to deliver the gas according to his needs. In later forms an automatic barometric control is provided for regulating the amount of oxygen supply to the aviator according to his altitude. The instrument also has a pressure gage to indicate the pressure of the oxygen in the supply tank and a flow indicator to show when oxygen is being delivered by the apparatus. An automatic regulator of American design is shown in Fig. 32. It is provided with two pressure chambers; a high pressure chamber into which the oxygen flows from the supply tank to reduce the pressure from 150 atmospheres nearly to atmospheric pressure, and a low pressure chamber to control the delivery to the masks. The pressure in each of these chambers is controlled by the action of valves oper-

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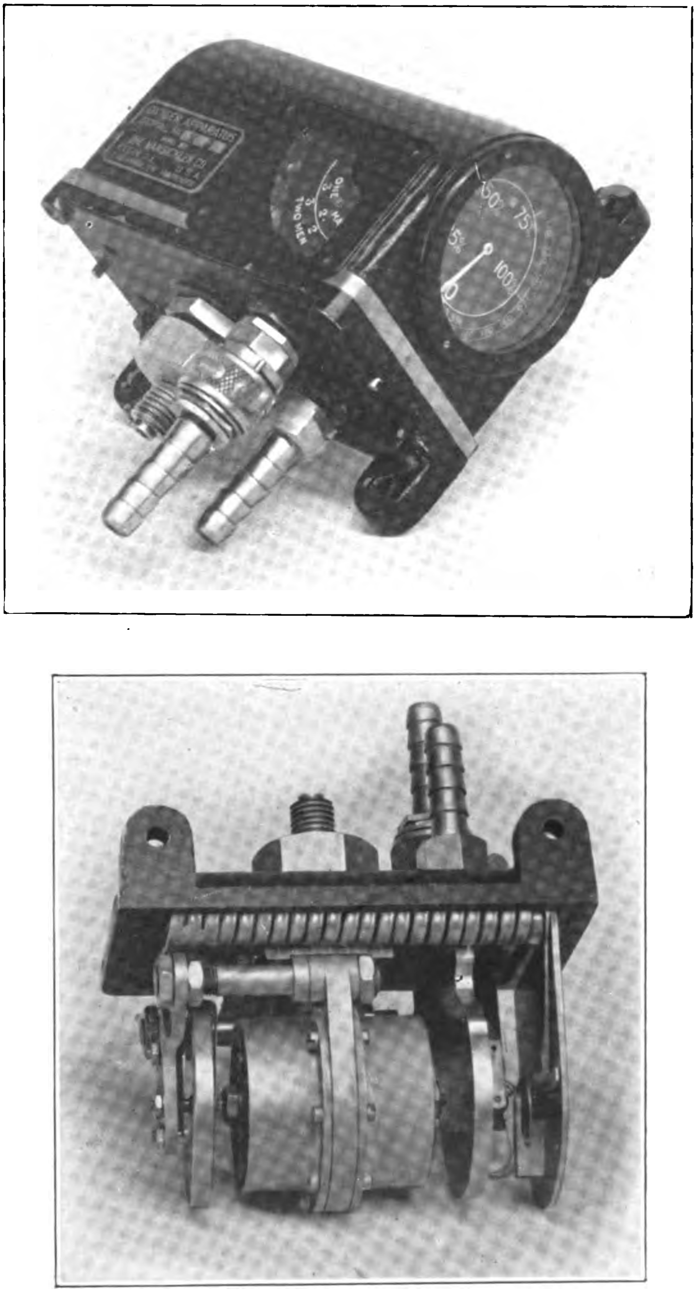

Fig. 32. Van Sicklen-Prouty Oxygen Regulator

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ated by corrugated metal diaphragms one of which forms one end of each chamber. The valve of the low pressure chamber is also acted upon by an aneroid capsule which expands with increasing altitude and thereby allows more oxygen to flow thru the apparatus, the amount increasing in accordance with the above mentioned altitude delivery formula. The helical coil gage shown in the figure at the right is used to indicate the pressure in the supply tank. The rate of flow is shown by the action of the pressure of the outgoing gas on another corrugated diaphragm capsule connected between the low pressure chamber and the masks.

Another instrument of the compressed oxygen type is shown in Fig. 33. It was originally of British design but was made in this county during the recent war. In this instrument the pressure of

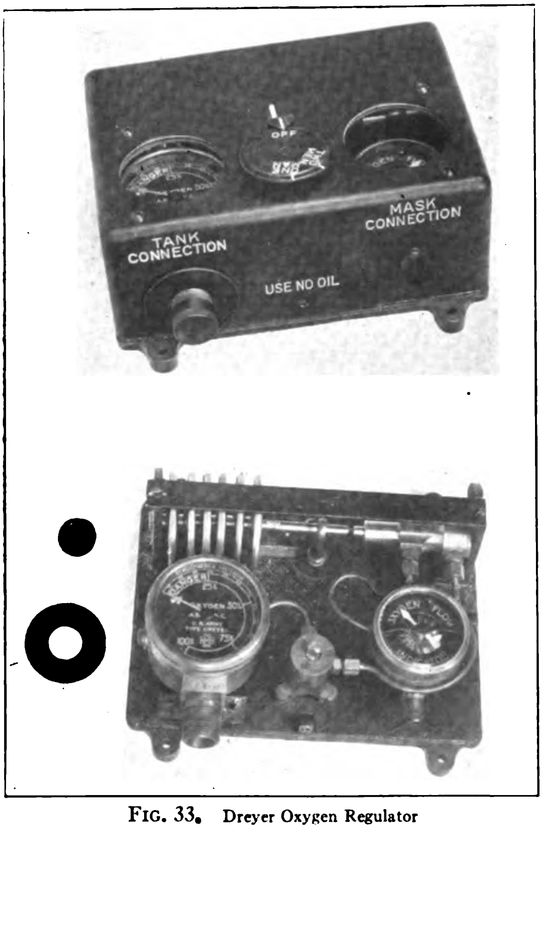

FIG. 33. Dreyer Oxygen Regulator

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the gas as supplied by the tank is reduced by passing into a chamber in which the pressure is controlled, as in the device described above, through the action of a valve connected to a corrugated metal diaphragm. This reduces the gas from a pressure of 150 atmospheres to a pressure of approximately one atmosphere above atmospheric pressure. From the reduced pressure chamber the gas passes to a piston valve controlled by a battery of aneroid capsules. With increase of altitude this battery of capsules expands under the action of internal springs thereby increasing the delivery of oxygen thru the piston valve. The amount delivered is determined by the size of the port in the piston valve which varies so as to deliver oxygen in accordance with the previously mentioned altitude-delivery formula. From the piston valve the oxygen passes thru a flow indicator, which is a sensitive anemometer on which the gas impinges as it passes to the masks. A Bourdon tube pressure gage to indicate the pressure of oxygen in the supply tank is also provided. The essential difference between this instrument and the one previously described is that in this device the supply of oxygen is increased by enlarging the port of the control valve while in the previous instrument the same result is effected by increasing the pressure under which the oxygen is forced from the reduced pressure chamber thru an outlet of constant size.

Where liquid oxygen is used the supply is ordinarily carried in double walled spherical copper vessels with an evacuated space between the inner and the outer walls and polished surfaces facing the evacuated space to prevent loss thru heat radiation. A typical instrument of this type is shown in Fig. 34. It has a capacity of from three to four liters of liquid oxygen which corresponds to from two-thousand to three-thousand liters of gas. The neck of the bottle is a long metal tube closed at the top and connected to a pressure gage and safety valve, shown on the instrument board, which are used to control the pressure of the gas which evaporates. This pressure forces the liquid oxygen out thru evaporating coils which surround the neck of the bottle. From these coils the gas passes thru flow indicators on the control board and thence to the masks. Liquid oxygen apparatus has the advantage of being

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considerably lighter and more compact than the complete equipment necessary when compressed oxygen is used. On the other hand there is an inevitable loss of gas due to evaporation when

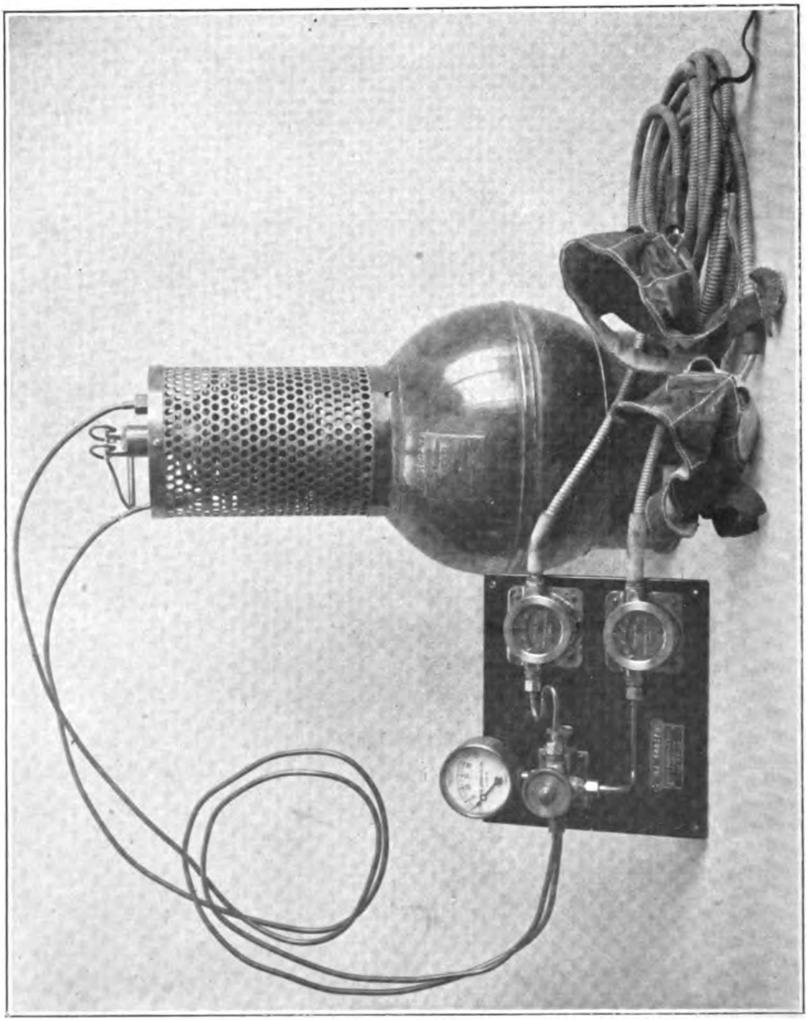

FIG. 34. Liquid Oxygen Apparatus

the supply is not actually being breathed. On this account it is necessary that the apparatus be filled within a relatively short

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time before using and that a liquifying plant be available in the vicinity.

Recording Instruments. For experimental work and airplane performance tests permanent records of altitude, air speed, rate of revolution of the engine, rate of ascent or descent, temperature, and humidity are sometimes required. To this end special instruments have been designed which are the same in principle as indicating instruments of the corresponding type previously described but which are provided with recording attachments. Typical instruments of this class are described below.

The air speed recorder shown in Fig. 35 is of the Pitot-Venturi type. It was designed for use with the nozzle shown in Fig. 6B-F and is provided with two pressure chambers, one of which is connected to the Pitot head and the other to the Venturi head of the nozzle. Rubberized silk diaphragms constructed like bellows are used. Under the action of the differential pressure these operate the lever system which controls the recording pen. The excursion of the diaphragms is resisted by a coiled spring.

A recording tachometer is shown in Fig. 36. In this instrument a long pointer with attached pen is substituted for the indicating element of a Tel chronometric tachometer. The pen rests on a circular chart graduated in revolutions per minute which is rotated uniformly by the clockwork mechanism shown in the right-hand figure.

Rate of climb recorders have been made which are the same in principle as the rate of climb indicators previously described except that the excursion of the diaphragm is made to deflect a small mirror from which a beam of light is reflected onto a photographic film moved by clockwork. In this way a permanent record of the rate of ascent and descent of the airplane is obtained. The photographic record is used instead of a pen and ink recording device because the force available for operating the recording mechanism is too small to give satisfactory results with a pen and ink recorder of the ordinary type.

An instrument which gives records of temperature and humidity and in addition the barometric pressure is shown in Fig. 37. The lower pen is operated by the aneroid capsules of an ordinary

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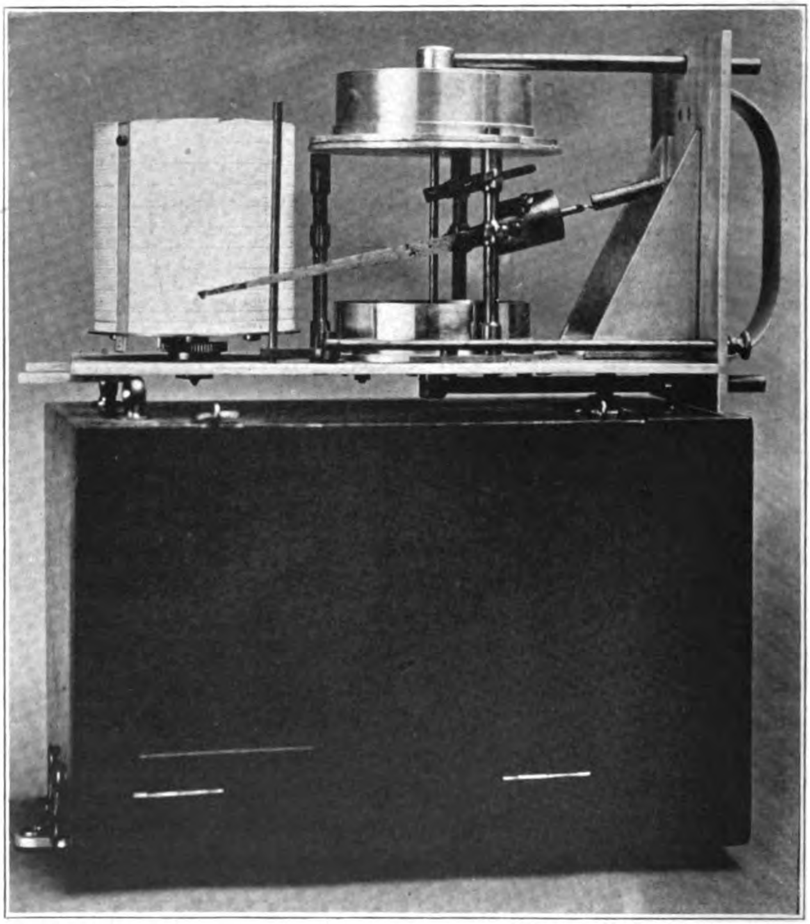

FIG. 35. Toussaint-Lepère Recording Air Speed Indicator

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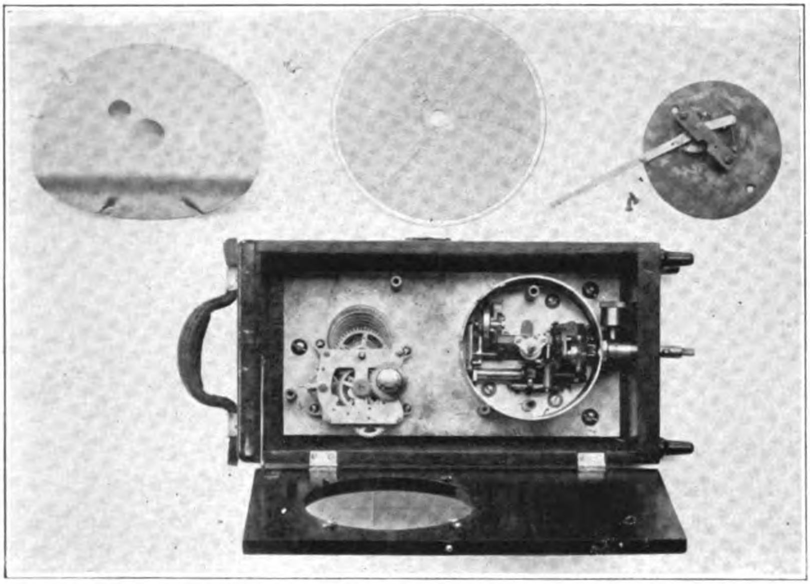

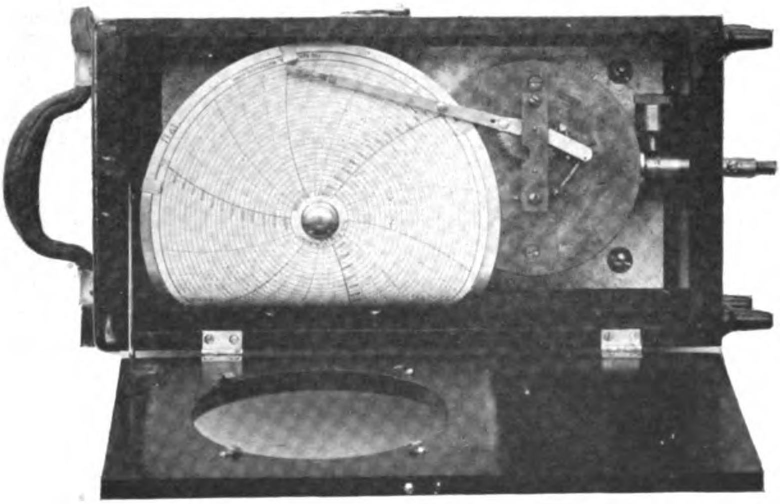

FIG. 36. Bristol Recording Tachometer

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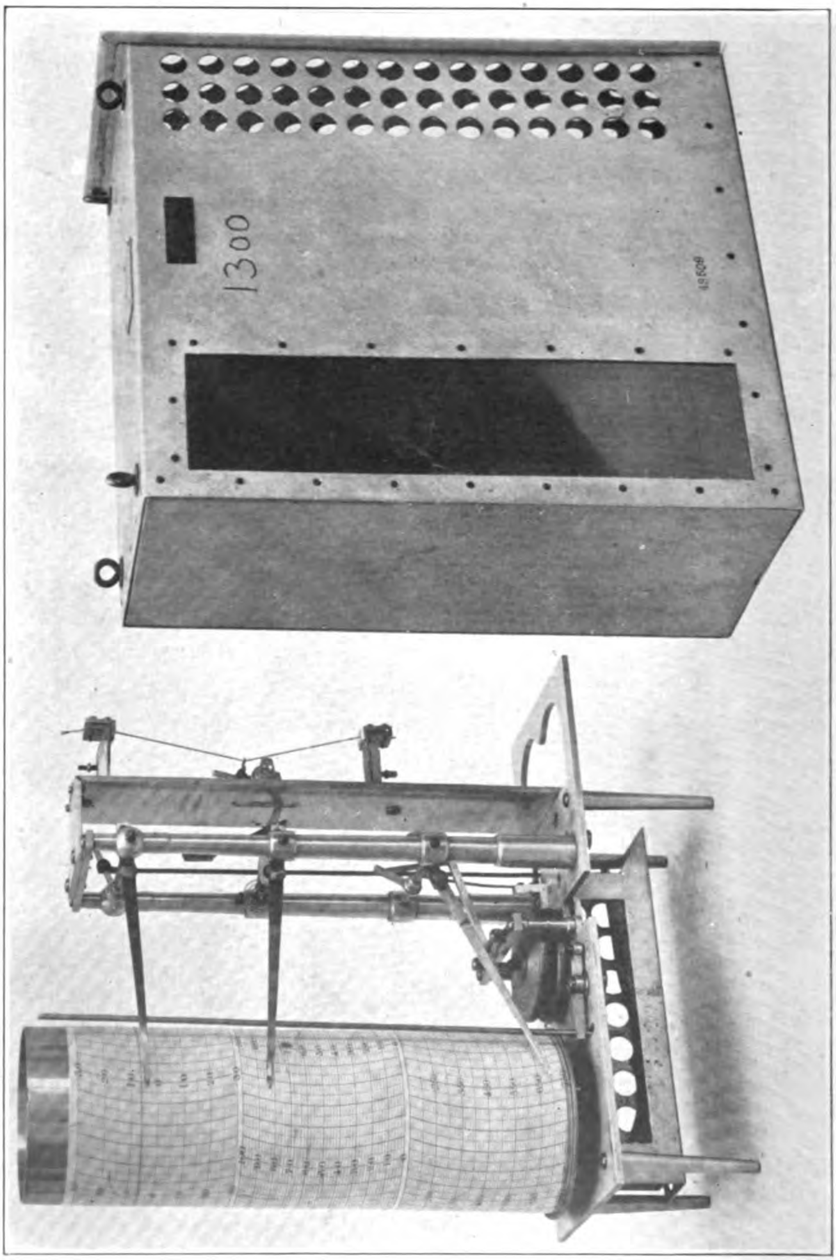

FIG. 37. Richard Baro-thermo-hygrograph

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barograph and gives a record of the atmospheric pressure. The middle pen is controlled by the bundle of human hairs shown at the right of the figure and gives a record of the relative humidity, in consequence of the expansion and contraction of the fibres with changes of the moisture content of the atmosphere. The top pen is connected by a lever system to a bimetallic strip shown beneath the instrument and gives a record of the temperature thruout the flight.

Strut and Gas Temperature Thermometers. In performance tests on airplanes it is necessary to determine the temperature of the surrounding air at intervals during flight. This is effected by strapping a pentane or other liquid type thermometer, which can be read at a distance, to one of the struts of the airplane, or of fastening the bulb of a liquid filled or vapor pressure type thermometer to the strut and locating the indicator on the instrument board. Instruments of both types are shown in Fig. 38. They are called strut thermometers.

Electrical resistance thermometers are also used in aircraft, more commonly on lighter-than-air craft to determine the temperature of the atmosphere and the gas. These consist of resistance coils of fine wire, located at the point whose temperature is to be determined, which are connected to an indicator graduated directly in degrees. An ohmmeter serves as an indicating element. Several resistance elements may be located at different parts of the aircraft and the temperature of each part determined in succession by making suitable connections at the indicator.

Time Pieces. A clock or watch is part of the standard equipment of most aircraft. Any reliable make of clock mechanism can be used for the purpose provided it is sufficiently rugged to withstand the shocks of landing and the inevitable vibration experienced in aircraft. Chronometers of precision are ordinarily not required. The only clocks peculiar to aircraft are reversing stop watches used in bombing. These are so constructed that when the stem is pressed for the second time the pointer starts to move back to zero instead of stopping as in a stop watch of the usual type.

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Manometers and Hydrogen Leak Detectors. Among the special instruments pertaining particularly to the control of lighter-than-air craft may be mentioned water ballast gages, manometers and hydrogen leak detectors.

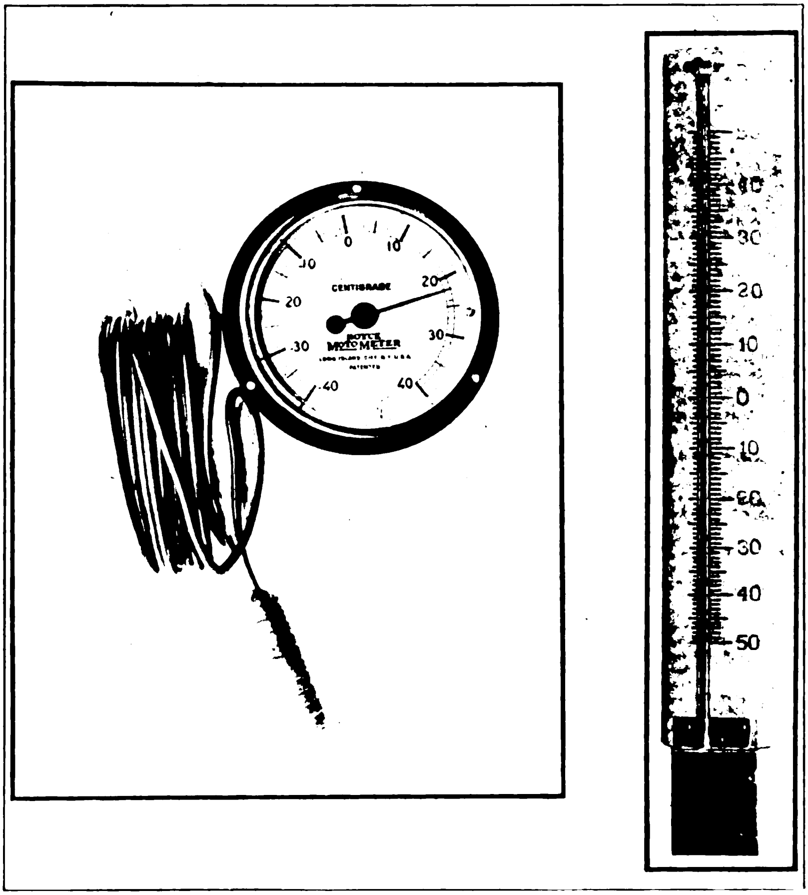

FIG. 38. Strut Thermometers

A typical water ballast gage is shown in Fig. 39. It consists of a corrugated metal diaphragm capsule enclosed in an airtight case and is used to indicate the pressure of the head of ballast water. The dial is graduated in inches of water.

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A manometer to show the pressure of the gas in the bags of lighter-than-air craft is shown in Fig. 40. It is provided with a thin colon leather diaphragm which indicates the difference in pressure between the gas in the bags and the external air through the intermediary action of a lever system which operates a pointer which sweeps over a vertical linear scale. The distinctive characteristic of this type of pressure gage is the extreme sensitiveness required. In the one shown three inches of water gives full scale deflection.

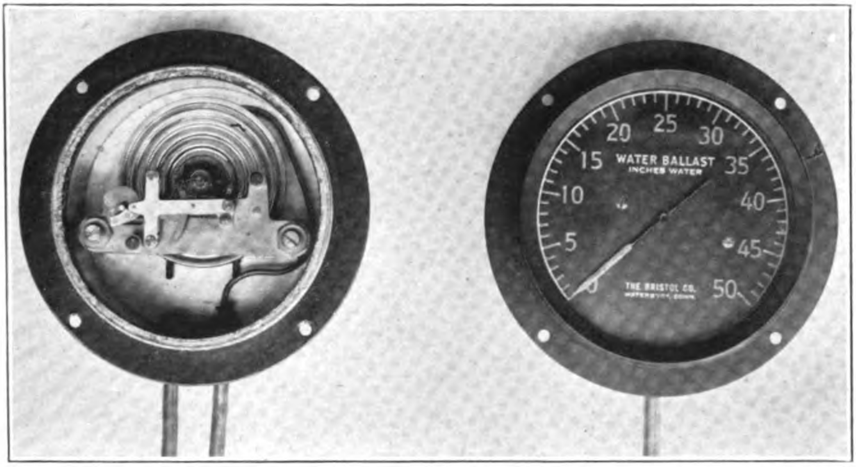

FIG. 39. Bristol Water Ballast Gage

A hydrogen leak detector is shown in Fig. 41. This instrument is used to indicate when gas is escaping from the bags of lighter-than-air craft. It is provided with a disk shaped air chamber the back face of which is made of semi-permeable porcelain. The front face is provided with a flexible corrugated metal diaphragm. When the porcelain face is placed near a leak, owing to the difference in the rate of diffusion of hydrogen and air, the hydrogen diffuses through the porcelain into the closed chamber more rapidly than the air diffuses out thereby increasing the pressure in the chamber which causes the diaphragm to expand. This motion is used to operate the indicating mechanism.

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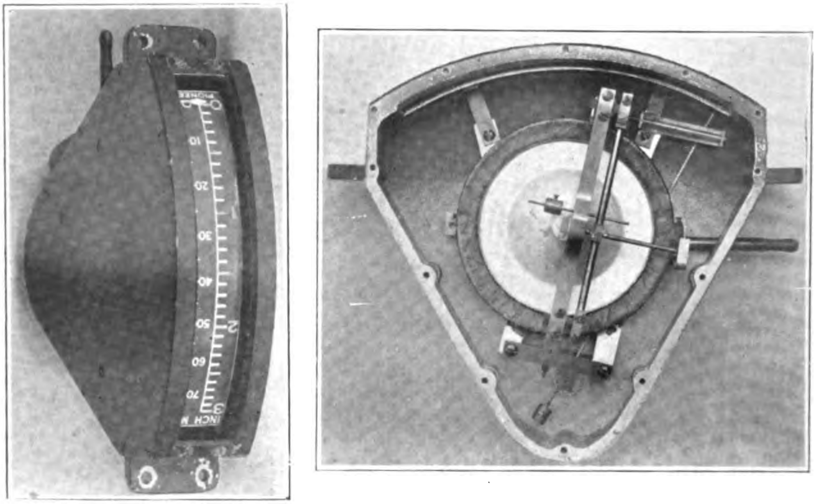

FIG. 40. Pioneer Gas Pressure Manometer

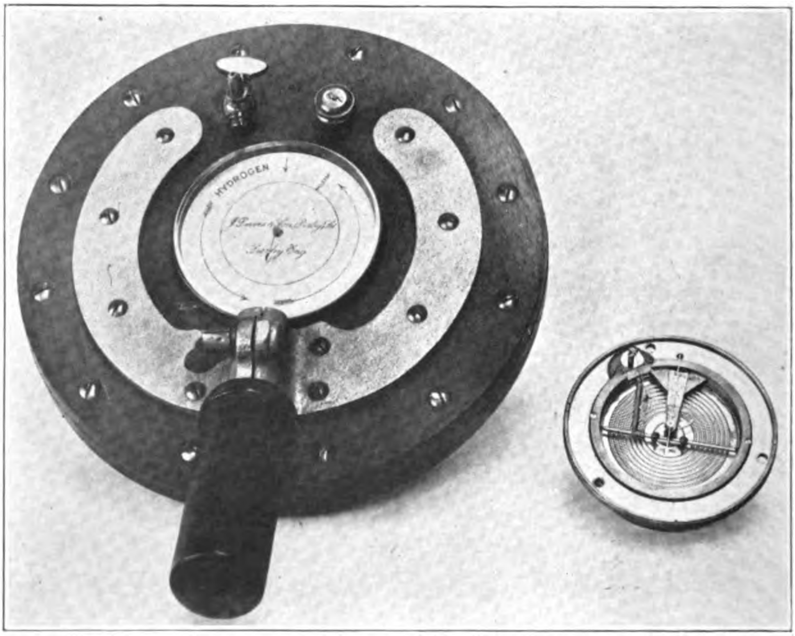

FIG. 41. Davis Hydrogen Leak Detector

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CONCLUSION

The recent origin of aeronautic instruments is emphasized by the fact that practically all of the instruments described in this paper have been invented or adapted to the special needs of aircraft within the past decade. The equipment of early airplanes and dirigibles was extremely meager and in many cases entirely lacking, the pilot depending on his individual skill and experience in the maneuvering of his craft, but with the increase in size and complexity of aircraft the need of instruments became apparent and has stimulated a rapid development. Improvements in existing instruments and the development of new types are continually being made as the rapid growth of aviation creates new needs.

NATIONAL BUREAU OF STANDARDS,
DEPARTMENT OF COMMERCE
WASHINGTON, D. C.
JULY 29, 1922.

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BOOK REVIEW

RESEARCH IN INDUSTRY. By A. P. M. Fleming, C. B. E., M. Sc., M. I. E. E. and J. G. Pearce, B. Sc., A. M. I. E. E. Published by Sir Isaac Pitman and Sons, (New York), XVI+244 pages. $4.00 net.

This book is written primarily as a guide to industrial concerns in establishing and developing research laboratories. The authors point out that "ethical and moral progress is closely related to material progress" and that the latter, in turn, "depends essentially upon the acquisition, development and application of new knowledge." Then follows a discussion of the nature of research and a classification of the agencies engaging in research. Considerable attention is given to the factors which must be considered in planning a research laboratory, in selecting workers, relative number of investigators and assistants, costs and size of building needed. Exteriors and floor plans of a number of typical existing research laboratories are given. These include such well known laboratories as those of the Metropolitan-Vickers Electrical Company in England, the A. D. Little Company in Boston, and the Westinghouse Electric and Manufacturing Company in Pittsburgh. One of the most valuable chapters is one on "The Research Worker" in which is discussed the training and characteristics of the investigator, opportunities which he should have, and his general relation to the organization of which he is a part. Not the least important part of the book is a very comprehensive, classified bibliography, containing some 250 titles.

While the book is written primarily from the British standpoint, it should nevertheless prove valuable to any industrial organization confronted with research problems.

F. K. R.

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Journal
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Review of Scientific Instruments

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A PHOTO-ELECTRIC THEORY OF COLOR VISION

BY
JANET H. CLARK

INTRODUCTION

The principal theories of color vision so far advanced, the Young-Helmholz, Hering, and Ladd-Franklin theories, have all assumed photochemical changes in the retina as the basis of vision. This is probably due to the fact that, at the time these theories were formulated, the only known results of the absorption of light energy by matter were the production of heat and the production of photochemical changes. A great deal is now known about another reaction between matter and light, namely the emission of electrons on the absorption of light energy. This phenomenon is called the photo-electric effect. There is the normal effect, seen, usually, only with ultraviolet light, in which the number of electrons emitted increases rapidly with decreasing wave-length of the exciting light. Some substances, the alkali metals, show the selective effect in which the photo-electric current has a marked maximum at some wave-length in the visible. (See Fig. 1.) This selective effect is due to light that is polarized in the E|| plane. The maximum in the curve is, however, still marked when unpolarized light is used as is shown by the curve for rubidium in Fig. 2. The chief facts of photo-electricity are these:—(1)Uniform maximum velocity of electrons for light of a

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given wave-length, (2) velocity of electrons independent of intensity of light, (3) number of electrons increases with intensity of light, (4) velocity of electrons increases with decreasing wave-length, (5) in the normal effect the number of electrons increases with decreasing wave-length, (6) with increasing wave-length the effect eventually becomes zero.

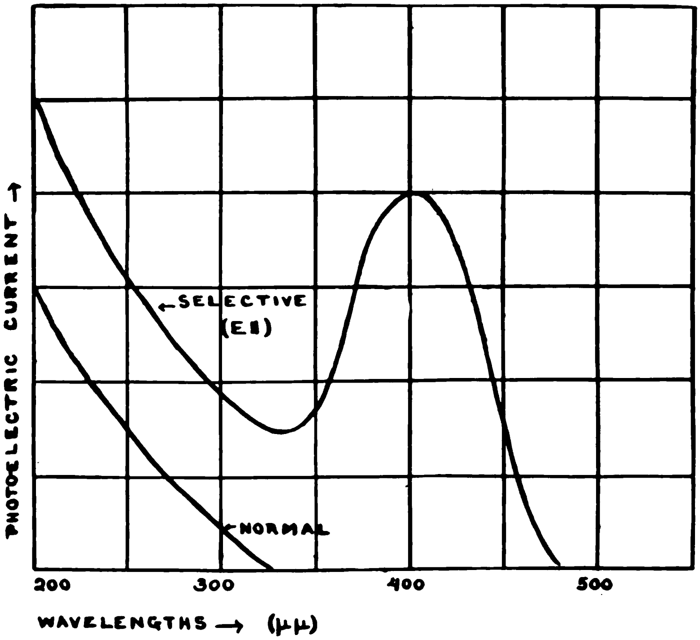

FIG. 1. Typical curves showing the normal and selective photo-electric effects.

Photo-electrons can escape from the surface of an illuminated substance only when the light is absorbed in a surface layer. If it penetrates to any depth, the emitted electrons do not reach the surface and the absorption of light results in an increased conductivity. The electrons may eventually become attached to other atoms or groups of atoms and this redistribution of valency electrons usually results in photo-chemical reactions. I think it can be safely stated that the foundation of all photo-chemical reactions is the emission of photo-electrons on the absorp-

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tion of light. This statement is supported by Lewis[1] who says: "At the present time there appears to be a somewhat general tendency to regard photo-chemical reactions as primarily due to a photo-electric effect. The close relationship between photo-electric conductivity and photo-chemical sensitiveness confirms the theory that a loosening of electrons is the immediate cause of photochemical reactions."

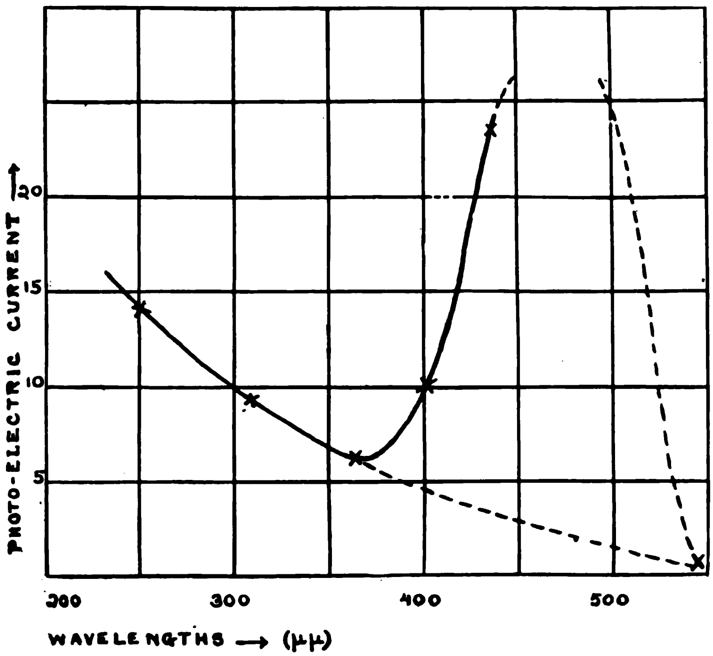

FIG. 2. The photo-electric current from rubidium with unpolarized light, showing the selective and normal photo-electric effects superposed—(from Hughes "Photo-Electricity").

It seems certain then that the photo-chemical theories of vision must eventually be supplanted by a photo-electric one. Several photo-electric theories have been proposed, for the most part by physicists, and have met with little or no encouragement from the physiologists. The most noteworthy of these theories is by Joly.[2]

[1] Lewis, W. C. M., "A System of Physical Chemistry," Vol. 3, p. 134, London; 1918.

[2] Joly, J., Phil. Mag., *41*, p. 289; 1921. Proc. Roy. Soc., B. *92*, p. 219; 1921.

Joly assumes that rhodopsin is the one photo-active substance in the retina and exists in the rods, between the rods and cones, and between the cones in the fovea. It is generally believed that there is no rhodopsin in the fovea, so Joly's fundamental assumption will undoubtedly be denied. However, that does not invalidate his further reasoning. The rhodopsin may be regarded merely ly as a sensitizer for dim lights, playing no part in color vision, and still one may assume that an unknown photo-electric substance lies between the cones. Joly further assumes that there is but one nerve fibril leading from the rod and nine leading from the cone, these nine being grouped in three bundles of 2, 3, 4 fibrils respectively. These bundles are capable of responding respectively to stimulation by electrons emitted from the photosensitive substance around the cones by red, green, and blue light. The energy carried by these electrons is shown to be, from the quantum relation, in the ratio 2:3:4, so that the 2 fibril bundle responds to electrons emitted by red light, the 3 fibril bundle to green light, the 4 fibril bundle to blue light. Stimulation of one fibril in the rod or nine fibrils in the cone gives a sensation of white light.

After reading Joly's theory with great interest, I looked up the histological evidence on the subject and found that Greef,[3] in his exhaustive work on the histology of the eye, mentions no neurofibrils in the rods and cones and doubts the existence of an axial filament in their outer segments. Cajal[4] found no neurofibrils in the rods and cones or in the outer nuclear layer. According to him the first neurofibrils occur in the outer plexiform layer. However, Leboucq[5] gives a detailed description of a system of intracytoplasmic filaments arising near the centrosome at the base of the external segment, spreading out to form the ellipsoid, then condensing into a single filament in the rods and into a bundle of three or four filaments in the cones. So the histological evidence seems to bear out Joly's hypothesis.

[3] Greef, Graefe-Saemisch Handbuch d. gesamten Augenheilkunde, 2 Aufl. vol. 1, ch. 5; 1899.

[4] Cajal, S. R., Internat. Monatschr. f. Anat. und Phys., *21*, p. 369; 1904.

[5] Leboucq, G., Arch. d'Anatomie Microscopique, *10*, p. 555; 1908–09.

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The Young-Helmholz theory upheld the doctrine of specific nerve energy, assuming one type of fiber capable of carrying one type of nerve impulse, so that the ending in the brain alone would be responsible for differences in sensation. The Hering and Ladd-Franklin theories assumed that the same nerve fiber can carry two or more types of nerve impulse. Joly's theory assumes that each cone contains three different types of nerve fiber, each capable of carrying only one type of nerve impulse. On this hypothesis he offers an explanation of how one cone can respond to three different stimuli. Nothing is said however of the nature of the nerve impulse itself and there is no explanation of why the sensation of white can be aroused either by stimulation of the one fibril in the rod or of the whole nine fibrils in the cone.

In view of my own difficulties with Joly's very suggestive and interesting theory, I wish to propose another photo-electric theory of color vision which probably has as many difficulties in the path of its acceptance as his, but which I hope will also be suggestive of future developments in the search for a thoroughly tenable and satisfactory photo-electric theory of vision.

THEORY OF COLOR VISION

The following are the fundamental hypotheses made in the development of the theory:

1. Vision is produced by the emission of photo-electrons from a light sensitive substance occurring in both rods and cones. This substance shows the selective photo-electric effect with a maximum corresponding to the wave-length of maximum luminosity in bright light.

2. Quantitative differences (i.e., differences in luminosity) depend on the number of electrons emitted. Qualitative differences (differences in color) depend on the velocity of the emitted electrons, it being a definite fact that each wave-length causes the emission of electrons with a characteristic velocity.

3. Chromatic vision is possible only in the cones. Rhodopsin, being found in the rods alone, is therefore concerned only with achromatic vision. It acts as a sensitizer to dim lights and, in the presence of the sensitizer, the maximum of the curve of photo-electric sensitivity is shifted to wave-length 535mμ.

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In considering the structure of the retina (Fig. 3) it is evident that there is one clear anatomical difference between the rods and cones. Each cone is connected directly to the brain through one nerve fiber, whereas several rods are connected through one fiber. Therefore, if by any means a characteristic disturbance were set up in a cone, it would reach the brain with its character unchanged. But characteristic disturbances starting from several rods and traveling along the same fiber to the brain would undoubtedly reach the brain as a confused disturbance. From the anatomical picture of the retina one would, therefore, expect the cone alone to be capable of chromatic vision.

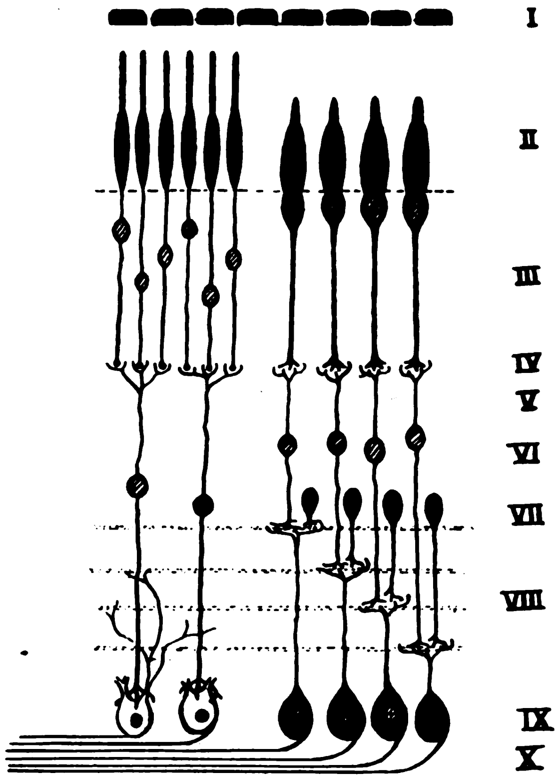

FIG. 3. Schema of the structure of the human retina—I Pigment layer; II rod and cone layer; III outer nuclear layer; IV external plexiform layer; V layer of horizontal cells; VI layer of bipolar cells (inner nuclear); VII layer of amacrinal cells; VIII inner plexiform layer; IX ganglion cell layer; X nerve fiber layer.

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A disturbance characteristic of a certain wave-length of light might be set up in the following way. When monochromatic light falls upon the rods and cones they emit electrons with a maximum velocity characteristic of the wave-length of the exciting light. These electrons would move out a certain distance, which Joly (2) calculates to be 5×10^{-7} cm for yellow light. These electrons may be supposed to attach themselves to the molecules of the dielectric medium around the cone and come to rest at a certain distance from the cone, this distance depending on their mean velocity, and therefore on the wave-length of the exciting light. They would, therefore, form negatively charged layers around the rods and cones, which are left positively charged. For this conception I am indebted to Joly's theory of the latent image in a photographic plate (see H. Stanley Allen's Photo-electricity).[6] The layer of negatively charged electrons and the positively charged cone or rod will form the plates of a condenser. When the charge built up is great enough there will be a discharge and a high frequency alternating current will pass to the brain. If the resistance of the circuit is small the current would be oscillatory if great it would be a single pulse. It is immaterial whether there would be oscillations or a single pulse, the important point is that the high frequency current produced by the discharge has a distinct character. The frequency is given by the formula

$$f = \text{frequency} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

where C is the capacity of the condenser, R the resistance of the circuit, and L the inductance. C, the capacity of the condenser is different for each wave-length of light, since for each mean velocity of electrons emitted there would be a different distance between the charged layers (condenser plates). Therefore the frequency of the current sent to the brain would be different and specific for each wave-length of exciting light. If now we think of each illuminated rod and cone as a condenser system connected to the brain through a number of spark gaps (synapses), the cone

[6] Allen, H. Stanley, "Photo-electricity," London; 1913.

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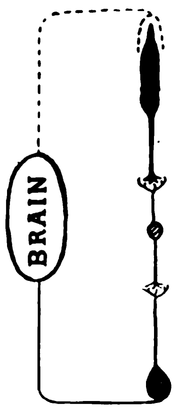
system will be similar to the diagram in Fig. 4 and the rod system to the diagram in Fig. 5.

If the condenser in a system like Fig. 4B is charged, *a* negative and *b* positive, there will come a point where a discharge will pass across the spark gaps and the condenser will discharge giving a high frequency current. In the cone, Fig. 4A, since each cone



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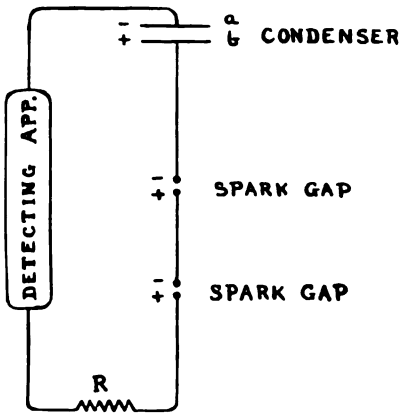

B

FIG. 4. (A) Cone circuit to the brain.
(B) High frequency circuit containing one condenser, analogous to the cone, and spark gaps, analogous to the synapses in the cone circuit.

is connected directly to the brain through one nerve fiber, the oscillations will reach the brain unchanged for the whole system in Fig. 4B would break down at once sending its characteristic frequency to the detecting instrument. In the rods, where several are connected to the brain through one fiber, conditions are similar to Fig. 5B. In this system condenser I will discharge when the potential difference between its plates is sufficient to break down the resistance of spark gaps 1 and 4, condenser II when it can break down 2 and 4, condenser III when it can break down 3 and 4. Therefore the discharges from the three rods will not pass simultaneously to the brain but will occur irregularly, producing an irregular current made up of the three overlapping frequencies.

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Therefore, although one and the same photo-electric substance may be supposed to exist in both rods and cones, their method of connection with the brain makes color vision possible in the cones and only achromatic vision possible in the rods. The rhodopsin may be looked upon as an auxiliary photo-electric substance, existing only in the rods, and active as a sensitizer so that, in its presence, the photo-electricity of the rods is increased and vision

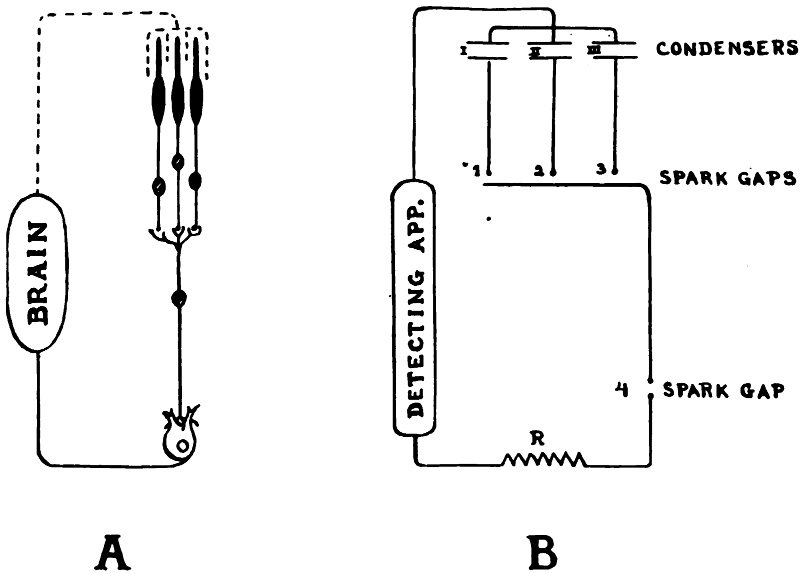

FIG. 5. (A) Rod circuit to the brain.
(B) High frequency circuit containing three condensers in parallel, analogous to the rods, and spark gaps analogous to the synapses in the rod circuit.

in dim lights is made possible. In support of the view that rhodopsin acts by virtue of its photo-electricity, it may be said that Parsons[7] calls attention to the fact that, although rhodopsin is very sensitive to light, it is extremely resistant to the chemical action of strong oxidizing and reducing agents.

This theory as outlined above has, I believe, this strong point. It suggests a mechanism whereby different wave-forms characteristic of monochromatic light may be started in a single cone and

[7] Parsons, J. H., "An Introduction to the Theory of Color Vision," p. 12, Cambridge; 1915.

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conducted to the brain along one nerve fiber. Also it definitely suggests that nerve impulses consist of high frequency alternating currents, which may be a series of damped oscillations or, if the resistance is great enough, a single pulse. The time necessary for the propagation of a nervous impulse would, therefore, consist of two parts (1) the time necessary to charge a condenser system sufficiently to break down the resistance of the spark gaps in the circuit, (2) the time for the propagation of an electric oscillation along a nerve. The velocity of an electric wave may be nearly anything, since it depends on the resistance, capacity, and inductance of the system in which it travels and on the sensitivity of the receiving instrument. Short waves, along wires where the resistance is negligible, travel with the velocity of light, but the velocity of electric oscillations along a nerve fiber would be of a different order of magnitude, so that there is no reason why the nervous impulse should not be an electric oscillation or pulse in spite of its relatively slow velocity of transmission. One obvious objection to the theory is this. Since the characteristic frequency of the alternating current set up by the absorption of the monochromatic light is $f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$ it is necessary to assume that the resistance of the circuit is the same for every cone in the retina in order that the frequency may vary only with the capacity (i.e. with the distance between the layer of electrons and the cone). It is not necessary to assume that the resistance is the same in different individuals. The frequency excited by the absorption of yellow light may $=f_1=\frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1}-\frac{R_1^2}{4L_1^2}}$ for one individual and may $=f_2=\frac{1}{2\pi}\sqrt{\frac{1}{L_2C_2}-\frac{R_2^2}{4L_2^2}}$ for another. So long as it is a definite and characteristic frequency in each individual he will interpret the stimulus as yellow light. It is, however, necessary that the resistance in the path from every cone to the brain should be the same in any one individual, and this may be an unjustified assumption.

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Granting this assumption however, this theory is able to explain how different qualities of sensation can be stimulated in one nerve fiber by different wavelengths of light. When it is applied to explain the observed facts of color fusion, color blindness, and after images it is somewhat less successful, but since no theory has been entirely successful in explaining all phenomena, it fares perhaps no worse than the rest.

POSITIVE AFTER IMAGES

The positive after image is produced by a short exposure with subsequent occlusion of light. It is easily explained by assuming that cones which are charged by the stimulation of light, but have not quite reached the point of discharging, discharge after the occlusion of light.

NEGATIVE AFTER IMAGES

After stimulation of the retina with strictly monochromatic light the electrons would practically all occupy one layer. It seems safe to assume that there are a limited number of these stable positions possible. If the electrons are stimulated by different monochromatic lights there might be a fairly large number of stable positions possible. If white light is used, giving electrons of many different speeds, probably only a few stable positions would be simultaneously possible, I am inclined to think that it would simplify the whole conception to assume that when electrons of various speeds are emitted they can only occupy two stable positions at a time, there being a number of these coupled layers possible. A certain distance between these stable layers would account for the frequency difference between complementary colors. Then, on stimulation with yellow light containing some white light, most of the electrons would fall into the yellow layer. Those coming off with a slower velocity would be slowed down by repulsion of the yellow layer and fall back into the cone. Those emitted with a velocity great enough to pass through the yellow layer would, after passing through, be accelerated by the repulsion of this layer and take up a position corresponding to the blue layer. A simultaneous discharge from these two layers would give a sensation of yellow mixed with a small amount of its com-

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plementary color and the effect would be that of yellow mixed with some white. Owing to the scattered light in the instrument, even spectral colors contain a certain amount of white light.

If it were possible to suppose that one layer might discharge without the other, we might get a sensation of yellow from the discharge of the yellow layer leaving the blue one partly charged. Subsequent stimulation by white would charge up this blue layer further and a discharge would give the negative after image. This does not seem likely and it seems more possible to suppose that, after the discharge of the heavily charged yellow layer, that particular layer would represent a condition of temporary instability so that electrons emitted on subsequent stimulation with white would none of them fall in the yellow layer, but would all fall in its coupled or complementary layer, which would, on discharge, give the negative after image.

Color Fusion

If the retina is stimulated simultaneously by two complementary colors the emitted electrons will fall into two stable layers, and a discharge from the two rings will give a sensation of white. If however the colors are closer in the spectrum than complementary ones, if for instance the retina is stimulated simultaneously by red and yellow, the electrons will fall into one intermediate layer, their two separate layers not being simultaneously stable, and on discharge will give the sensation of orange.

Color Blindness

To explain color blindness one could assume that in some eyes certain stable positions are not possible for the emitted electrons. That red should be most frequently absent is natural since the layer lying nearest to the cone would be the most unstable. The coupled layer for green might be supposed to drop out along with the red. This would amount to assuming a congenital defect in the medium between the cones. A congenital defect in the cones themselves might be a more attractive hypothesis. If, in certain eyes, the photo-active substance in the cones gave a photo-electric curve in which the maximum and the long wave-length limit were both shifted to shorter wave-lengths, the result would be red-

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blindness and a shift in the luminosity curve towards the blue. Since the red-blind eye does show a shift in its luminosity curve towards shorter wave-lengths, it seems quite possible that the various types of abnormal color vision are due to abnormalities in the photo-electric emission of the light active substance in the cone.

COLOR FIELDS

On the foregoing theory one would expect color vision all over the retina (i.e., wherever cones were present). As the cones become fewer and fewer towards the periphery it would take greater and greater intensities of light to stimulate color vision as the stimulus passes outwards from the fovea. It has recently been shown (Ferree and Rand[8]) that the far periphery of the retina is not blind to red, blue and yellow. With stimuli of sufficient intensity the limits of red, blue and yellow coincide with the limits of white light vision.

EXPERIMENTAL EVIDENCE

Poole[9] has recently investigated the retina for photo-electric activity, and, finding none, considers Joly's theory and all other photo-electric theories of vision untenable. However, in the retina, the escape of electrons under light is not in the surface layer of nerve fibers but down in the rod and cone layer, so that emission of electrons from the surface would not be expected. Evidence should be looked for in a change in conductivity of the retina under light, which would be expected if electrons are emitted from the rods and cones. This is precisely what is shown by the experiments of Einthoven, Sheard, Bovie, etc. (Sheard "Physiological Optics"),[10] in which the current of injury, obtained by connecting the cornea and the cut end of the optic nerve to a galvanometer, is found to undergo a positive variation on exposure to light.

CONCLUSION

The theory as stated above seems to explain the main facts of color fusion, after images, and color blindness reasonably well,

[8] Ferree, C. E. and G. Rand, *Amer. Jour. Physiol. Optics*, *1*, p. 185; 1920.
[9] Poole, J. H. J., *Phil. Mag.*, *41*, p. 347; 1921.
[10] Sheard, C., "Physiological Optics," Chicago; 1918.

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and it does so by means of only one photo-active substance which is undoubtedly a much to be desired simplification.

It undoubtedly explains more satisfactorily than any other theory how different qualities of sensation can be stimulated through one nerve fiber by different wave-lengths of light.

The entire truth in regard to color vision is certainly far from being known as yet but, feeling confident that the correct theory, when found, will be a photo-electric one, I suggest the above outlined theory hoping that it will lead to investigation, criticism, and further speculation towards that end.

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EARLY PIONEERS IN PHYSIOLOGICAL OPTICS

BY
JAMES P. C. SOUTHALL

Among the ancient Greek philosophers, notably Pythagoras, Empedocles, and Aristotle who flourished in the sixth, fifth and fourth centuries, respectively, before the Christian Era, various theories of vision, all more or less vague and confused, were propounded. According to certain of these conceptions an object was visible by virtue of emanations which it discharged into the eye; whereas another view was that the emanations proceeded from the eye to the object whence they were reflected back into the eye. Whether these emanations were incorporeal or not was a subject of speculation. Pythagoras and his disciples taught that the colour of a body was partly objective and partly subjective, being a concurrent effect of certain peculiarities in the object and corresponding disturbances which these produced in the mechanism of the eye itself; but apparently they were not able to differentiate clearly between the external and internal causes and effects. The only point on which all agreed was that light proceeded in straight lines or rays, and hence the Greek geometers were able to explain the phenomena of shadows and to develop the science and art of perspective representation.

But until the epoch of the great Arabian philosopher Alhazen (*c.* 1100 A. D.) it is hardly possible to speak of a science of vision. The influence of his notions which were singularly clear and correct continues to the present time. In the first book of his celebrated treatise "Opticæ Thesaurus" (which is the title of the Latin translation made long afterwards) Alhazen gives an anatomical description of the eye which is perhaps the earliest detailed account of the structure of that organ. The names which he applied to the various parts of the eye are practically the same as those by which they are called today. He distinguished three transparent fluids or humours, namely, ***humor***

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aqueus, *humor crystallinus*, and *humor vitreus;* and four membranes, namely, *tunica adhærens*, *cornea*, *uvea*, and *tunica reti similis*. Single binocular vision he explained as being due to the fact that the images formed in the two eyes were somehow united and made to overlap in the common optic nerve so as to produce an unique composite impression in the brain itself.

In the historical development of optical science it is difficult to estimate the place of Roger Bacon (1214–1294), because from Bacon's own writings one never can tell whether the extraordinary optical contrivances to which he alludes were actual inventions or merely the exuberant fancies of a very rare scientific imagination and insight into natural phenomena, as seems far more probable. In one place, for example, Bacon tells us of the possibility of constructing an airship (*instrumenta volandi*) whereby a human being sitting in the midst thereof and revolving a mechanism which was attached to artificial wings could ascend from the earth like a bird; with as much assurance as if he himself had actually made such a machine and tested it. Apparently he was acquainted with the use of spectacles which were probably invented about his time, but almost certainly not by Bacon himself.

A very celebrated book which went through many editions and was translated in various European languages was the "Magia naturalis" written by Johann Baptista Porta (1538–1615), who was the inventor of the *camera obscura* which is described in this work.[1] After explaining the principle of the simple pinhole camera, the author proceeds with a certain air of mystery to reveal something marvellous, and tells how when a convex glass lens is inserted in the aperture of the instrument the image depicted on the screen will be so much brighter and sharper than before that it is possible to distinguish the very countenances of the passers-by in the street outside. Porta attached much impor-

[1] The title of the edition of 1591 is: "Jo. Baptistæ Portæ Neapolitani Magiæ Naturalis Libri Viginti, Ab ipso quidem authore ante biennium adaucti, nunc vero ab infinitis, quibus editio illa scatebat mendis, optime repurgati; in quibus scientiarum Naturalium divitiæ & deliciæ demonstrantur. Accesit Index, rem omnem dilucidà repræsentans, copiosissimus. Francofurti, Apud Andreæ Wecheli heredes, Claudium Marnium & Joann. Aubrium. MDXCI."

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tance to the manifold applications of this invention; and then he adds significantly: "There can be no doubt that the human eye is such a *camera obscura* into which the light comes from outside; the pupil acts like the aperture in the window-shutter, and the crystalline lens performs the office of the white wall or screen" on which the image is cast. The analogy here suggested is indeed very striking, but it is hard to see how, knowing the action of lenses as he did, Porta could have supposed that the ocular image was completed in or on the crystalline lens.[2]

In another one of his books, namely, "De refractione,"[3] published in 1593, Porta again compared the eye with a *camera obscura;* and also called attention to the contraction of the pupil in bright light and to its dilatation in feeble illumination. However, this important observation had been mentioned by Leonardo da Vinci (1452–1519) apparently as a well ascertained fact in his time. A certain Fabricius ab Aquapendente, who was very nearly contemporary with Porta, relates that Fra Paoli Sarpi had communicated to him the fact that both pupils are affected simultaneously in the same way depending on the degree of illumination. Concerning singleness of binocular vision, Porta inferred that only one eye was used at a time and that, somewhat after the manner of birds, the right eye was employed when the object was a little to the right and the left eye when it was to the left.

The subject of colour was another matter which greatly interested Porta and which he says he had studied with much curiosity for more than forty years, although it was so intricate and difficult as to be almost beyond mortal ken. No wonder the poets had called the many-coloured rainbow the daughter of Thauma! But Porta believed he had at last found some clues to this mystery also.

[2] According to Porta the seat of vision had to be between the pupil and the centre of the eye where the visual rays from the various points of the object intersect, if the object was to be seen erect; and consequently he inferred that the seat of vision was in the crystalline lens, which, as he says, is "extra oculi centrum."

[3] "Joan. Baptistæ Portæ Neap. de Refractione, Optices Parte Libri Novem. Ex officina Horatij Salviani. Neapoli. Apud Jo. Jacobum Carlinum & Antonium Pacem, 1593."

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Among the pioneers in physiological optics the name of Franciscus Maurolycus (1494–1577) deserves a place chiefly on account of one of his numerous writings called "Photisimi de lumine et umbra" published near the end of his life in 1575. It is only a small volume but it contains several discoveries which are important in the theory of optics. Incidentally, Maurolycus seems to have been the first to show that a ray of light issues from a transparent isotropic plate bounded by plane parallel faces in the same direction as it was going before it entered. He gave also an explanation of the circular arc of the rainbow and distinguished in it four principal colours which he called *croceus* (yellowish like saffron), *viridis* (green), *cœruleus* (sky-blue), and *purpureus* (purple), together with three other colours which he regards as transitions or *connexiones*. It is usually stated that Newton was the first to distinguish seven colours in the prismatic spectrum.

Knowing only the general effect of glass lenses as convergent or divergent, Maurolycus deserves much credit for being the first to explain the action of the crystalline lens in the human eye by analogy with artificial lenses; and although he assumed (like Porta) that the image in the eye must be erect since that was the way the object appeared, he was at least nearer the truth in insisting that the rays were not converged in the crystalline lens itself, but beyond it. From the fact that a far-sighted eye requires a convex correction-glass, Maurolycus argued that the curvature of the crystalline lens in an eye of this kind was not sufficient, whereas in a near-sighted eye it was too great.[4]

But it was Kepler (1571–1630) who, far in advance of his predecessors, began to formulate clear and precise notions as to the *modus operandi* of the eye and the science of vision. In his famous treatise known as the "Supplements to Vitellio,"[5] which in spite of

[4] Another work on optics by this same author written in 1553 but apparently published many years after his death has the following title: "R. D. Francisci Maurolyci Abbatis Mersanensis mathematici celeberrimi Diaphanorum partes seu libri tres; in quorum primo, de perspicuis corporibus; in secundo, de Iride, in tertio: de organi visualis structura, & conspiciliorum formis, agitur. Lugduni Apud Ludovicum Hurillion, MDCXIII, cum privilegio."

[5] "Ad Vitellionem paralipomena; quibus astronomiae pars optica traditur, etc." Frankfurt, 1604. (The writer has recently seen an extract from a letter by Dr. M. von Rohr stating that he is preparing a German translation of this noteworthy volume.)

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certain manifest faults and obscurities is a work of great originality and importance, Kepler gives a minute description of the anatomical structure of the eye. The *cornea* is the transparent and more convex portion of the tough outer coating of the eye called from its hardness the *sclerodes tunica*. The anterior side of the *sclerodes* and even the *cornea* itself are contained in a very thin transparent membrane called the *tunica adnata* or *adhærens*. The posterior dark-coloured side of the second coating of the eye with its numerous arteries and blood-vessels constitutes the *choroides tunica*, while the anterior part on the outside forms the *iris* diaphragm with its central round window or pupil. The third coating is the *retina* on which the images of external objects are focused. The *humor crystallinus*, contained between the *humor aqueus* in the anterior and the *humor vitreus* in the posterior chamber of the eye, is suspended in a transparent capsule called *aranea* (*arachnoides*) *tunica*, whereas the vitreous humour is enclosed in the *hyaloides tunica*. No special capsule is provided for the aqueous humour which is comprised between the *cornea* and the *aranea* and the *processus ciliares*. These latter proceed from the *uvea* on the posterior side of the membrane whose front surface is the *iris* and form a kind of radial collar surrounding the crystalline lens.

Kepler is very explicit about the retina (*retiformis tunica*) as the surface on which the optical image in the eye is projected. This "reddish white" surface he supposes to be a portion of the inside of a hollow sphere, which acts like a screen (*papyri vice*) for receiving the image and partly surrounds the crystalline lens. Thus he says that the pupil is the common base of cones of incident rays emanating from the various points of the object, and that these rays emerge finally from the crystalline lens converted by refraction into bundles whose vertices lie all on the retina where consequently a small inverted image resembling the object is depicted point by point. So firmly was Kepler convinced of the essential correctness of this theory that he affirmed that if the other membranes of the eye could be removed so as to expose the transparent retina from behind, an observer might see traced on it a minute image of the external field of view. The experiment

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which is here suggested was actually made some years later by Kepler's contemporary, Christoph Scheiner, who cut a hole in the upper sclerotic coating of a freshly enucleated eye and then removed all the opaque portions whereby he was enabled to perceive the image of a luminous object focused on the retina. These experiments were made first with the eyes of sheep and oxen, but later in 1625 in Rome Scheiner performed the same experiment with a human eye.[6]

Kepler's later book on "Dioptrics,"[7] which is as distinguished for its logic and clearness as the "Supplements to Vitellio" is lacking in these respects, likewise contains many acute and original speculations in regard to the process of vision. By this time Kepler was so certain that it was necessary for a sharp image to be focused on the retina in order for the eye to see distinctly, that he inferred that the posterior surface of the crystalline lens must be hyperbolic in form instead of spherical; although unless the rays inside the lens were parallel to the axis it is difficult to see what particular advantage is to be gained by an hyperboloidal rear surface. But it is worth noting here that Kepler anticipated Descartes in conceiving of an aspherical lens for uniting a bundle of rays exactly in one point.[8]

In Prop. LXI of the Dioptrics Kepler asserts that vision is a sensation stimulated in the visual substance ("Sehgeist" or "plena spiritus") which he conceives to form a sort of sensitive layer over the surface of the retina. The coloured delineation of the external world which is traced on the retina by the action of the optical system of the eye on the rays of light which come into it is, according to Kepler, not a mere superficial effect like that of light playing over the surface of an ordinary screen or wall, but it is a selective and penetrating influence which produces some change or photochemical action (as we would say) in the visual

[6] See Caspar Schott's "Magia universalis naturæ et artis" (Würzburg, 1657), p. 87. Also, Dr. Joseph Priestley's "Geschichte und gegenwaertiger Zustand der Optik," Klügel's German edition (Leipzig, 1776), p. 93.

[7] "Johannis Kepleris, Sæ. Cæ. Mtis. mathematici Dioptrice. Augustæ Vind., 1611" (small quarto, 79 pages). See also "Johannes Keplers Mathematikers Sr. Kaiserlichen Majestaet, Dioptrik," Nr. 144, Ostwalds Klassiker, Leipzig, 1904.

[8] See "Dioptrik," Props. LIX, LX, pp. 27, 28.

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substance contained in the retina. Kepler says he was led to infer this from the nature of light itself which, when it is powerful and concentrated, has so-called focal properties and may even cause combustion; and so he argues that between the comparatively minute quantities of light which in the eye are directed towards the delicate fine-grained structure of the retina there is the same connection in kind as exists outside the eye when a beam of sunlight is focused on the ordinary gross matter of an inflammable body. In support of this conception he alludes to the case where the eye has been fatigued and strained by being exposed to a bright illumination, sometimes to such an extent that even when the source of excitation has been removed a strong after-image of it persists for quite a time; indicating that the impression made on the retina must have been a curiously deep-seated effect. These remarkable opinions which anticipated Boll's discovery of the so-called visual purple by more than two centuries and a half bear testimony to Kepler's original and remarkable genius.

But the impression produced on the retina is by no means the whole of the visual act, and Kepler explains that this sensation has now to be transmitted to the brain in an unbroken current where it is communicated to the centre of consciousness. However, owing to the imperfect state of the knowledge of the anatomy of the nerves and the brain in his day, Kepler was not able to form an entirely correct picture of the actual route of transmission from the organ of vision to the visual centre of the brain. He wonders also if perhaps the subtle visual material which he assumes to be spread over the retina is itself conveyed along the optic nerve from the eye to the brain.

On the other hand, Kepler was obviously at a loss in trying to explain why objects appear to be erect although their images on the retina are inverted. He touches on this question in the "Supplements to Vitellio" and uses some metaphysical language about "how the passive must lie opposite to the active," and adds that the movements of the eye enable us to distinguish the top of an object from the bottom, since the eyes must be directed upwards towards a lofty monument or downwards in gazing at the sea from a high cliff.

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Without being aware of the previous speculations and hypotheses of Maurolycus concerning near-sight and far-sight, Kepler finds the explanation of these defects in the fact that the length of the eye is either too great or two small (axial ametropia); as Scheiner believed also. In a near-sighted (myopic) eye the rays are focused in the vitreous humour and produce blur-circles on the retina; whereas in a far-sighted eye the rays are not bent enough and are focused therefore beyond the retina, producing blur-circles on it as before. Kepler guesses that in either case, assuming the eye to be otherwise normal, the trouble is due to the mode of life and occupations of the individual and not to any natural or inherent defect. Elderly folks are usually far-sighted or presbyopic because they are feebler than in youth and because it is easier and more natural to maintain the axes of the two eyes parallel than to turn the eyes inwards in order to see near objects distinctly.

According to Kepler, a perfectly healthy eye has the faculty of seeing equally well both near and far. On the other hand, when a person is quite unable to see anything distinctly, there is some pathological or diseased condition of the eye which may be partial or total blindness (Prop. LXIV of the Dioptrics). He supposes that accommodation is produced by varying the interval in the vitreous humour between the crystalline lens and the retina until the image is sharply focused on the latter, whether the object in question be far or near. Just as it is possible to vary involuntarily the size of the pupil depending on the degree of illumination, Kepler believed also that the eye could be stretched equatorially so as to shorten the eyeball and reduce the interval between lens and retina; or, conversely, that the eyeball could be elongated by equatorial contraction so as to increase this interval. The seat of this mechanism was supposed to be located in the web-like structure which supports the crystalline lens and connects it with the uvea by the dark radial branches called the ciliary processes, which appear to be formed like a comb to enable each muscle to act by itself. When all these muscles contract together and are shortened, the pupil becomes smaller and at the same time the lateral parts of the eye are drawn inwards while the eye-

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ball as a whole is elongated. The reverse process occurs when the ciliary processes are lengthened.

Concerning binocular vision, Kepler says (Dioptrics, Prop. LXII) that when the retinas of the two eyes are stimulated in equal fashion, the two images are fused and the object is seen single; whereas under unequal stimulation fusion does not take place and the object appears double. Again, in the "Paralipomena," Kepler explains that it is by an unconscious process of triangulation in which the distance between the two eyes is the constant and essential factor that we are enabled to form more or less imperfect estimates of distance; and he even suggests that for very short distances a similar method may serve in the case of monocular vision, the fundamental base-line being the diameter of the pupil itself. However, this latter observation would appear to be rather far-fetched, not merely on account of the comparatively small dimensions of the pupil but particularly also because these dimensions are variable.

Another remarkable man of this epoch whose name has been mentioned already and who ranks alongside of Kepler as one of the founders of physiological optics was Christoph Scheiner (1573–1650). A professor in one of the colleges of the Society of Jesus in Germany, he rose to high eminence in the councils of that great religious order. While he was still only a student of mathematics at Ingoldstadt, he read with eagerness the Sidereal Messenger in which Galileo announced those wonderful astronomical discoveries that excited such a fierce controversy through all the learned world of that day. Partly owing to his naturally prudent temperament but perhaps more to his religious training and associations, Scheiner did not espouse the Galilean philosophy and points of view with the same ardour as his countryman Kepler; and yet he was one of the first to extend those new methods in physics and astronomy, with so much industry and ability that his name will always be called in company with the most illustrious men of that notable era in the history of the intellectual development of Europe. Strange to say, his remarkable achievements have remained comparatively unknown.

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When Galileo began to publish his discoveries in 1610, there were only a few telescopes available in Germany. Kepler himself had not been able to get a glimpse of the "Medicean Stars" until August 30, 1610. But Scheiner immediately procured or made for himself a number of these new instruments, and in his first scientific announcement he speaks of having as many as eight "tubes" of different proportions. Kepler described in his Dioptrics the optical construction of the astronomical telescope which is today called by his name, but as his peculiar genius lay rather in conceiving than in executing it is doubtful whether he ever saw an actual instrument of this type. Scheiner, on the other hand, possessed unusual mechanical ability and ingenuity, and undoubtedly it was he who constructed the first astronomical telescope composed of a combination of two convex lenses, probably about 1614;[9] although Rheita in a book entitled "Oculus Enoch et Eliæ" (1645) claims to have been the first to have made a telescope according to Kepler's description. Scheiner also constructed for Duke Maximilian of Tirol a so-called terrestrial telescope composed of three convex glasses which gave a magnified erect image of distant objects.

Scheiner was indeed a prodigious and indefatigable worker and his writings cover a wide range of subjects. Several years ago Dr. von Rohr to whom modern optical science is indebted for so many valuable contributions and extensions of our knowledge, edited and published a German translation of selected portions of Scheiner's very original and noteworthy treatise entitled "Oculus sive fundamentum opticum,"[10] the first edition of which was issued at Innsbruck in 1619. A third edition was published in London in 1652 after Scheiner's death. It is only possible to give here an exceedingly brief and necessarily very inadequate summary of the general character and main conclusions of this volume which, composed more than three centuries ago, was perhaps the first formal treatise on physiological optics. The entire work is

[9] See Scheiner's "Rosa Ursina," Bracciani, 1626–1630.

[10] Ausgewählte Stücke aus Christoph Scheiners Augenbuch; übersetzt u. erläutert von M. von Rohr: "*Zft. f. ophthalm. Optik*, 7, pp. 35–44; 53–64; 76–91; 101–113; 121–133; 1919.

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divided into three principal parts, the first of which contains a kind of *résumé* of what was known at that time concerning the eye and vision, more particularly as to the anatomy of the eye with an account of various related experiments; the second part is concerned with the manner of the refraction of light and the procedure of the visual rays in the eye itself; while the last part treats particularly of vision and the appearances of objects. The most striking characteristic is the consistent use of the method of observation and experiment which Scheiner employs to explain and support his hypotheses and opinions.

He succeeded in obtaining at least an approximate measurement of the curvature of the cornea by observing the image of a window or other suitable object reflected therein and comparing this appearance with the similar image seen in a small glass sphere placed as near the eye as possible on the temporal side. A number of little glass marbles or bulbs of various sizes enables the experimenter to select by trial that one which shows an image of the object as nearly as possible equal to that which is reflected in the cornea and thus to compare the curvature of the latter with the known curvature of the test globe.

That Scheiner was an accurate and painstaking observer is abundantly manifest. He noted the minute forward displacement of the pupil in the act of accommodation; together with the fact that accommodation is invariably accompanied by a modification of the diameter of the pupil. As to the ciliary processes, he supposes that these muscles are capable of tension and relaxation perhaps for the purpose of producing a slight alteration in the total length of the eyeball in the process of accommodation. With singular acumen he suggests that there may be likewise a concomitant change in the form of the crystalline lens itself This seems to have been the first conjecture of the essential factor in accommodation as Thomas Young endeavored to prove long afterwards (1793). Certainly, there is no allusion to it in Kepler's writings.

In order to explain the procedure of the visual rays inside the eye and how the image is formed on the sensitive retina, Scheiner devised numerous simple and ingenious experiments, models of

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their kind, some of which indeed have become classic. He was particularly fond of stenopæic tests of various kinds; one of which, usually referred to as "Scheiner's Experiment," is still to be found in modern text-books on physiological optics. In another experiment he shows how distant objects will generally be seen double when they are viewed through two pinhole apertures in a cardboard held close to the eye, provided the interval between the holes is less than the diameter of the pupil. In fact, there will be as many images as there are pinholes comprised within an area of the card not greater than that covered by the pupil of the eye. By the help of these simple devices he was able to trace the general procedure of the visual rays without knowing the precise law of refraction.

Entoptic phenomena and likewise the curious effects of irradiation receive much attention in Scheiner's book. He endeavors to explain why a piece of white-hot iron appears to be about three times as large as when it is cool; which leads Dr. von Rohr to comment that Leonardo da Vinci had also observed that part of an iron bar heated to incandescence appeared thicker than the cold portion.

Scheiner devised also a simple experiment for locating approximately the position of the pivot or centre of rotation of the eye. Finally, he was at some pains to determine the refractive powers of the various ocular media, reaching the conclusion that the aqueous humour and the crystalline lens act nearly as if they were composed of water and glass, respectively; whereas he inferred that the refractive power of the vitreous humour was intermediate between those of the other two.

The process of vision as expounded by Descartes (1596–1650) seems to be based on the views of Kepler and Scheiner with whose writings he was undoubtedly familiar, although he makes little, if any, allusion to either of these authors. For an extremely interesting account of Descartes' opinions on this subject the reader is referred to the first section of a paper by J. W. French entitled "The unaided eye," published in the *Transactions of the Optical Society* (London), XX, pp. 209–236 (1919); which contains a translation of Descartes' description of the human eye taken

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"from the work entitled *Renati Descartes Opera Philosophica* dated 1656," together with *facsimile* reproductions of two of the original diagrams.

When Christiaan Huygens (1629–1695) began his famous work on Dioptrics in 1653, which was never completed and which was published first in more or less fragmentary form a few years after his death and again recently in a complete annotated edition in both Latin and French,[11] the keenest scientific minds of that day were concentrated on the phenomena of light and vision. The law of refraction had at last been discovered. Huygens' contributions in the domain of physiological optics are perhaps not generally known, especially as they are to be found scattered here and there in the main body of the "Dioptrica," but in the aggregate they form a considerable mass of solid achievement in this field. Huygens indeed touched nothing which he did not adorn, and he was never satisfied until he had gotten to the bottom of the matter. His admiration of the marvellous mechanical construction of the eye and its adaptation to human needs frequently breaks forth in some of the most eloquent and famous passages of this work. In the first part of the Dioptrics[12] and afterwards in the article "On the eye and vision,"[13] Huygens gives a description of a "simplified eye" formed of two concentric hemispheres one of which has a radius three times as great as that of the other. They are placed in juxtaposition with their flat surfaces in contact. The curved surface of the smaller hemisphere represents the cornea, while that of the other portion represents the retina or rather the choroid which Huygens was inclined to regard as the seat of vision.[14] The entire cavity inside the two superposed hemispheres is supposed to be filled with an aqueous

[11] "Christani Hugenii Zelemii, dum viveret, Toparchae Opuscula Postuma, quae continent Dioptricam. Commentarios de Vitris Figurandis. Dissertationem de Corona & Parheliis. Tractatum de Motu. De Vi Centrifuga. Descriptionem Automati Planetarii. Lugduni Batavorum, Apud Cornelium Boutesteyn, 1703."

"Œuvres complètes de Christiaan Huygens publiées par la Société Hollandaise des Sciences. Tome treizième. Dioptrique. 1653; 1666; 1685–1692. La Haye, Martinus Nijhoff, 1916."

[12] Œuvres complètes, Tome XIII, pp. 128–134.

[13] *Loc. cit.*, pp. 787–802.

[14] *Loc. cit.*, p. 795.

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humour of refractive index $\frac{4}{3}$; the actual dimensions being of no particular consequence but the relative dimensions such that rays of light proceeding from a distant object-point and undergoing refraction at the transparent surface of the smaller hemisphere will be accurately focused on the inner surface of the larger hemisphere. In order to get rid of the faults of spherical aberration as much as possible, Huygens provides also something in the nature of a pupil by making the plane surfaces of the two hemispheres opaque where they are in contact except for a small circular area right around their common centre. Huygens' simplified eye, as above described, bears a striking resemblance to the so-called "reduced eye" conceived long afterwards by J. B. Listing.[15] In both cases the optical system of the eye is replaced by a single spherical refracting surface with its centre at the nodal point of the eye, the relative index of refraction being $\frac{4}{3}$. But while the geometrical similarity of the two conceptions is well-nigh perfect the purposes for which they were devised were quite different. Listing's "reduced eye" was intended merely as an aid to oculists and ophthalmologists for the simplification of their calculations and constructions; but Huygens' aim was primarily to study the essential *modus operandi* of the actual eye with its pupil in place; and afterwards he is careful to point out how inferior this crude contrivance is to the marvellously delicate and adapted mechanism of the living eye. Subsequently, in 1691, when, partly in consequence of certain careful ophthalmic measurements made by the physician Pecquet in 1667 in his presence, Huygens was more familiar with the actual dimensions of the human eye, he described a "schematic eye" which more nearly represented the optical system of the actual eye,[16] but which was considerably more complicated than his original "simplified eye."

As to the mechanism of accommodation, Huygens supposed that this power might be produced by a forward movement of the crystalline lens or by an increase in the convexity of the lens (as Scheiner had imagined) or by a combination of both of these

[15] Beitrag zur physiologische Optik (Göttingen, 1845), pp. 16–18.

[16] *Loc. cit.*, pp. 800–802.

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causes.[17] At the time of Pecquet's measurements referred to above he was struck with the flexibility of the crystalline lens and how its form changes under the pressure of the fingers,[18] and hence he concluded that the act of accommodation was effected by a change of curvature of the lens. Subsequently, however, in 1670 he returned again to the explanation of the forward movement of the lens without change of form.[19]

Another thing which constantly excited Huygens' admiration was the contrivance of the pupil of the eye which remained always circular in form whether it was big or little.

He was an empiricist concerning the old question as to how we see objects erect when their images on the retina are upside down. Our judgments of appearances, he says, are largely a matter of habit, and even if everything had to be viewed through a glass which inverted them, we should still speak of seeing things erect and should experience no difficulty in pointing to the top or bottom of an object.[20]

Huygens found the ideas of myopia and presbyopia in Kepler's writings, and he formulated very definite rules for determining the powers of lenses for the correction of these defects of vision.[21] It is interesting to compare his way of treating these problems with that of Barrow (1630–1677) in his lectures on Optics.[22]

Concerning emmetropia also Huygens says "there is nothing in nature which shows the geometry of the Creator more than the eyes" and it is indeed "admirable that the surfaces of the cornea and the crystalline lens are just of that degree of convexity to bring parallel incident rays to a focus in the fundus of the choroid. Perhaps it is different in the case of little children who may be able to adjust their eyes in some manner; but that is also no less marvellous."[23]

[17] *Loc. cit.*, p. 133.

[18] *Loc. cit.*, p. 789.

[19] *Loc. cit.*, p. 794.

[20] *Loc. cit.*, p. 745; also p. 829.

[21] *Loc. cit.*, pp. 134–138.

[22] "Lectiones XVIII Cantabrigiæ in Scholis publicis habitæ in quibus opticorum phenomenωn genuinae rationes investigantur, ac exponuntur. Etc. Ab Isaäco Barrow. Londini, MDCLXIX." Pp. 102, 103.

[23] *Loc. cit.*, p. 756.

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Notwithstanding that there is an image formed in each eye, Huygens tells us that Nature has taken particular pains to prevent us from seeing double; for each point in the fundus of one eye has its "corresponding point" in that of the other eye, and it is only when an object-point is imaged in a pair of corresponding points in the two eyes that it is seen single as it really is.[24] These corresponding points according to him lie both on the same side of the optic axes and similarly placed with respect to the two optic nerves; and hence it is plain that a distant object will be seen double when the eyes are converged on a nearer one, and *vice versa*. Huygens' theory of corresponding points on the retinas of the two eyes is indeed as complete as that to be found in modern text-books. Robert Smith (1738) in his "Compleat System of Opticks" entertains exactly the same views, and he observes also that Leonardo da Vinci had explained how the most accurate painting can never give a plastic impression of relief as perfect as that which is obtained from looking at real objects, because the images of a solid object in the two eyes are necessarily somewhat different. If Huygens had only recognized this fact clearly, he might have anticipated Wheatstone (1838) in the invention of the stereoscope.

The secure foundations of the science of physiological optics were laid principally by the genius and penetration of Kepler, Scheiner, and Huygens. It was chiefly through their labours in this field that their great successors, Young (1773–1829), Donders (1818–1889), and Helmholtz (1821–1895),—to mention only a few of the most illustrious names—were enabled to develop and extend the theories of light and vision in all their manifold ramifications with other branches of human knowledge.

DEPARTMENT OF PHYSICS
COLUMBIA UNIVERSITY
NEW YORK, N. Y.
JUNE 1, 1922.

[24] *Loc. cit.*, p. 796.

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INSTRUMENT SECTION

THE MECHANICS OF OPTICAL POLISHING

BY
Elihu Thomson

The problem of how it is that, for example, a glass surface which has been smoothed or finely ground can, by proper means, be polished not only so as to be invisible ordinarily, but so that under the severest tests it shows no diffusion of light (as of the sun's rays falling on it) has at times engaged the attention of the ablest physicists. The late Lord Rayleigh studied the matter and his paper[1] on the subject is well known. He properly explains the polishing process on the principle of removal, by a process similar to grinding, of the high points of the surface, and progressively so until the whole ground surface has been cut away, but the cutting is by an action so fine that the grain produced is beyond the power of resolution by a microscope or other powerful optical means.

It is the purpose here to show that while this view is measurably correct it does not go far enough, and that the polishing is a unique mechanical process; a self regulated planing down of the surface to a real level without even the finest scratches or other character which would lead to diffusion of any light falling on the surface.

Some have most erroneously tried to explain the result of the process, by assuming that the glass has, during the polishing, actually flowed; or that there was some peculiar plastic condition brought about which allowed the glass surface being polished, to take on the characteristics of a liquid surface. There is no need for such hypotheses and no validity in such assumptions. This will be made clear.

In burnishing of plastic metals by a hard burnisher, there is, of course, such flow, but with hard, brittle, non-malleable materials like glass, the process is decidedly not like burnishing.

[1] Lord Rayleigh, Proc. Roy. Inst. Gr. Britain, March, 1901; Trans. Opt. Soc. *19*, October, 1917.

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Glass may receive an optical polish in either the wet or dry way. Other materials of a brittle, non-malleable nature are dealt with similarly; such are quartz, agate, calcspar (Iceland), and many jewels and minerals.

In the manufacture of plate glass, the ground surfaces (the last, or smoothing stage, being often called *mud ground*) are not worked by grinding to so fine a grain of surface as in the better class of accurate optical work, and the polishing is done by runners of felt charged with rouge (crocus) and water moved over the plate by machinery. The result is that the surface obtained is not an optical one; it has a smoothness and polish similar thereto, but is wavy throughout, as can easily be discerned by a skilled eye in regarding the reflection of an edge from such surface; and, of course, by other simple tests. It is neither optical in the large or in small elements of surface. The yielding felt runners have swept out indiscriminately the hollows, small and large, and have not held the surface to a definite figure. Similar yielding polishers are used in finishing the very irregular surfaces of cut glass. The cheaper kind of lenses, where accuracy of figure is not needed, are often cloth polished, a process which, if carefully conducted, gives a result intermediate between the plate glass surface and the true optical surface, such as is obtained by a pitch polisher with rouge and water. The considerations as to the true nature, the mechanics, of the polishing process are applicable to all such cases, but will be given in connection with the pitch polishing, most usual in good work. They apply, too, to the case of dry or paper polishing with paper faced tools charged with tripoli (diatomaceous earth) a method of polishing which has been used to some extent in France for medium grade lenses.

In rouge polishing with pitch for a carrier, as is usual, the surface of the pitch is moulded to fit the glass and is divided (usually) into small square facets by grooving. It is worked over the glass, or the glass worked upon it by movement in all directions or such innumerable paths are given that no definite course is repeated. This is essential to the best result.

The conditions as found, in successful work, are as follows. The rouge, though very hard, is friable and breaks down to a very

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fine powder. Too hard (non-friable) rouge will tend to fine scratches. These scratches are not like grinding or crushing, but are smooth bottomed grooves, discoverable by a magnifier.

The pitch is at all times yielding. It is made so by tempering and testing. If too hard, it tends to cause fine scratches, all over the surface which is being polished. These, with very hard pitch, may resemble grinding, but ordinarily they show no crushing, but are smooth cuts.

In grinding, on the contrary, the surface is crushed, while in polishing it is clean cut. Smooth cutting is the rule. The polishing is indeed a kind of planing process; the particles of rouge set themselves into the pitch surface and cut smoothly; they do not roll or grind. There are millions of fine planing or cutting edges at work fixed in position by becoming, at least temporarily, embedded in the pitch surface, which readily yields to receive them. They make smooth cuts as can readily be seen by examination of the scratches when the pitch is overhard or the rouge too hard and nonfriable. Good rouge is friable without apparent limit, and rouge washed out of a used polisher may be so fine as to float for days in colloidal solution.

All the above considerations are fairly well known and recognized, but there is one additional condition, or circumstance, which, so far as the author knows, is worthy of record, no attention having been hitherto drawn to it.

It is this: by the very nature of the case the particles which are doing the cutting in polishing are all *automatically adjusted*, in successful work, to cut to the *same depth* during any stroke. The yielding nature of the pitch surface not only ensures this, but makes it a necessary consequence, for any particle of rouge riding higher than another is at once depressed to the proper level by sinking into the pitch surface. The innumerable cutting edges of all the particles reach a common level, and with motion of the polisher in all directions, and cutting smooth (no crushing or grinding) the result cannot fail to be what it is, an optical surface without grain or irregularity. The rouge is friable without limit, so that the polishing particles may, in the process, become finer and finer. With felt, cloth, or paper as a carrier for the

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polishing powder, the effect is much the same; the particles are held to position when cutting, as planing tools. Even fine washed carborundum will polish glass if held in the surface of soft wood or cork, and the author has even produced a fair polish on a glass lens by a soft metal tool charged with fine carborundum. In such case, the polishing takes place in a few seconds, but the technical difficulties are very great. In dry polishing, a sheet of paper is pasted down on the surface of the polishing tool, and a special high grade pure paper, rather heavy and uncalendered, is used. This is charged by gently rubbing its surface with a lump of fine tripoli selected for the purpose, the fine silicious skeletons composing which constitute the polishing powder. The first application to the smoothed surface, as of a lens, which surface has the fine grain usual in such a case, is to show innumerable fine scratches, criss-crossing in every direction. They are, however, smooth scratches. As the work goes on, the tripoli works down to finer and finer conditions, while the polishing comes up gradually, no new application of the powder being required after the start. It is manifest that here too is the condition of smooth cutting and particularly a self adjustment of cutting depth, owing to the yielding character of the paper surface, so that at the end all the cutting is done in one surface of movement. It is believed that this dry paper process is much less used than formerly. It cannot be expected to yield the high accuracy that may be obtained with the wet pitch.

It is thought that in pointing out the mechanics of the polishing process, and more especially the smooth cutting and self-adjustment of cutting particles above described, the interesting process of the production of an optical surface may be relieved of something of the mystery which has been its accompaniment.

The author has drawn upon an experience of more than fifty years in occasional working of optical surfaces on glass of many kinds and on media, such as crystal quartz and fused quartz, Iceland spar and others.

The amount of material removed from the surface under treatment, is, of course, seen to be almost infinitesimally small per stroke, and it is only by the long continuance of this action that

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at last there is a sufficient removal to secure an optical surface. Time is saved by carrying the fine grinding or smoothing as far as possible before applying the polisher. As Rayleigh has stated, and it is of course the common experience, polishing begins on the highest or most elevated parts of the surface, seen only under a magnifier, and these are removed while the polished spots widen out, and, if the surface has been well prepared, or *bottomed*, as it is termed, spread to include the whole surface. If the surface has not been well bottomed, there will remain pits which the slow planing action of the polisher is incompetent to remove in reasonable time, and if the polishing is continued too long, the surface is more than likely to have lost its truth, or has been seriously deformed. This, however, depends on the polisher itself keeping its form. Too soft pitch is a guard against polisher scratches from particles of grit, but not conducive to accuracy. Accuracy can be helped by remolding the polisher at intervals by slight warming of its surface and application to a true surface of the same character as that being produced, while moistening the said surface to prevent adhesion.

No matter what degree of smoothness has been attained in polishing, the continued smooth removal of the glass surface goes on as long as there is rouge, pitch and water applied; a fact which is, of course, taken advantage of in parabolizing a concave astronomical glass mirror.

GENERAL ELECTRIC CO.
LYNN, MASS.
JULY 20, 1922

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A VARIABLE RESISTOR OF LOW VALUE[1]

BY
C. N. HICKMAN

In making electrical measurements there is often a great need for a continuously variable resistor of low value. Variable resistors consisting of a copper wire sliding in an insulating tube containing mercury[2] have been used in the Resistance Measurements Section of the Bureau of Standards for several years. The Inductance and Capacitance Section has constructed a number of them with a surrounding copper tube on which the current returns, thus decreasing the inductance of the circuit.

These instruments are very useful for varying the resistance of a circuit provided the current is small. However, they cannot be used when the value of the resistance must be known with a high degree of accuracy on account of the high temperature coefficient of mercury. Also, where careful measurements are being made, the continuous change of resistance due to heating makes a mercury resistor unsuitable for carrying large currents.

The resistor described in this paper is a modification of these instruments which reduces the temperature coefficient to a negligible quantity by using manganin instead of mercury. This instrument is equipped with a calibrated scale from which the resistance may be read accurately. Within reasonable limits the resistance is independent of the current.

Fig. 1 shows a sketch of the instrument. A copper and a manganin wire of the same diameter are welded together and slide on the inside of a thin glass tube G_2. This glass tube is surrounded by a thin copper tube T. All of this apparatus is inside of another glass tube G_1. The glass tubes are provided with reservoirs

[1] Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] F. Wenner, Phy. Rev. *32*, p. 614, 1911. This Bulletin Vol. 8, p. 584. Sci. Paper No. 181.

J. H. Dellinger, Phy. Rev. *33*, p. 215; 1911.

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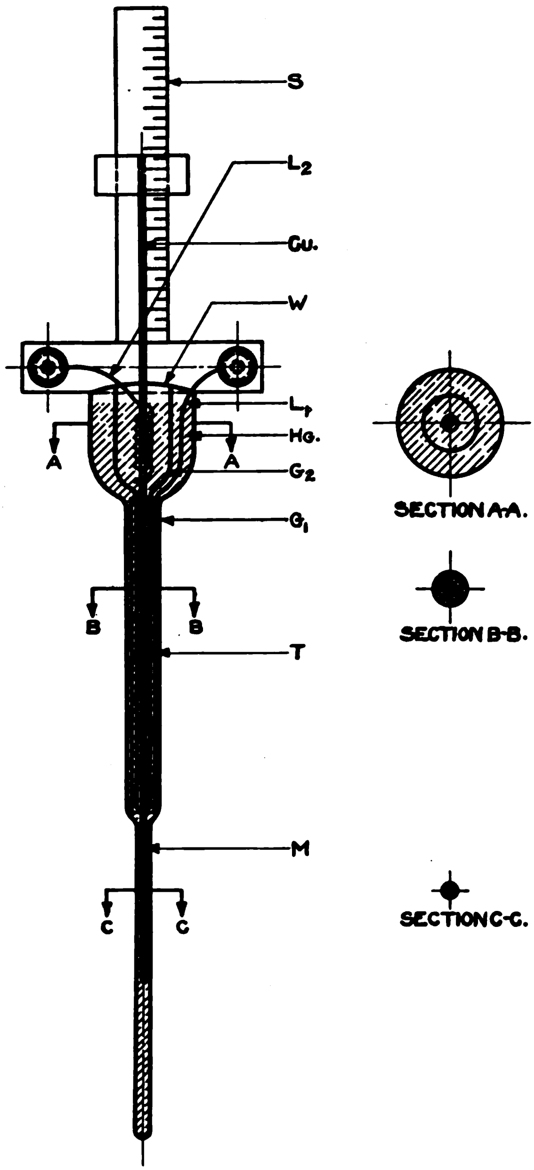

FIG. 1. Manganin Slide Resistor

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through which the leads enter (Sec. AA). The entire system of tubes is filled with mercury. Thus the current flows in on lead L_1, down the copper tube T, through a thin film of mercury to the copper or manganin wire, and up to the inside reservoir, then through another thin film of mercury to L_2.

When the slide is down, the copper and manganin junction is at the lower end of the copper tube. If the slide is raised on the scale S, a portion of the manganin wire will replace an equal length of the copper wire. Thus resistance may be added continuously until the slide is at the top of the scale S, in which position the maximum resistance is obtained.

A resistor of this type having a range of one one-hundredth of an ohm has been used at the Bureau of Standards for about three years. The diameters of the wires are each about 0.260 cm. Their lengths are each approximately 15 cm.

The total inductance of the instrument is only about 0.023 microhenry. Since the copper and manganin wires each have the same diameter, there would be no change of inductance as the slide is moved were it not for the fact that the manganin, having a higher resistance than the copper, causes a little more current to flow through the thin film of surrounding mercury. The maximum change in inductance resulting from this change in current distribution is less than 0.001 microhenry.

The resistor has been calibrated a number of times during the last three years with currents of from 0.001 ampere to 5 amperes, and the calibrations are consistent to 0.0001 ohm. The calibration curve is a straight line, the calibrated points seldom being off the line more than a few hundred-thousandths of an ohm.

SUMMARY

A continuously variable resistor which is suitable for carrying a large current and which permits the use of an accurately calibrated scale is described. The variation of the resistance is accomplished by substituting a manganin wire for a copper wire of the same length and diameter.

Readings may easily be repeated to one ten-thousandth of an ohm on a resistor having a range of .01 ohm and the instrument

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remains constant to this degree of accuracy. The variation of inductance as the resistance is changed is negligible, being less than 0.001 microhenry for the total range of the resistor.

BUREAU OF STANDARDS
WASHINGTON, D. C.
JULY 25, 1922

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A ROTARY SLIDE-WIRE FOR PRODUCING UNIFORM VARIATION IN POTENTIAL DIFFERENCE[1]

BY
S. J. MAUCHLY

One of the requirements in the calibration of certain atmospheric-electric apparatus is a device for varying the potential difference between the two members of a condenser at a constant and definitely known rate.[2] A special form of "rotary potentiometer," which was devised for this purpose and constructed in the instrument-shop of the Department of Terrestrial Magnetism, is described here in the hope that it may be found applicable to other uses.

The description will be facilitated by a preliminary statement of the principal requirements to be satisfied. Briefly, these are as follows: (*a*) A slide-wire of sufficiently high resistivity and low thermal expansion to allow the use of the required terminal potential differences without objectionable expansion and other thermal effects. (*b*) A supporting cylinder which, in addition to satisfactory insulation, has approximately the same thermal expansion as the wire. (*c*) A traveling contactor which maintains good electrical contact; likewise, suitable brushes at the ends of the cylinder. (*d*) A driving mechanism for turning the cylinder uniformly at desired speeds. (*e*) Provisions for altering the speed of the cylinder. (*f*) Means for accurately determining the speed of rotation of the cylinder. (*g*) Convenient regulation of the potential difference applied to the slide-wire. (*h*) Such co-ordination of the various details that the rate of change in the potential difference between the traveling contactor and either terminal of

[1] Published by permission of the Director, Department of Terrestrial Magnetism.

[2] See, for example, "An Apparatus for Automatically Recording the Electrical Conductivity of the Air" by W. F. G. Swann, in "Annual Report of the Director of the Department of Terrestrial Magnetism" for the year 1917, Yearbook of the Carnegie Institution of Washington, 1917, p. 279. As indicated in the paper referred to, a preliminary form of the apparatus here described was used by Swann in 1917.

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the slide-wire may be known and kept constant to about one part in 250 for observation periods of at least 5 minutes.

Fig. 1 shows a general view of the apparatus and its control equipment. (Only the upper half of the switchboard is involved

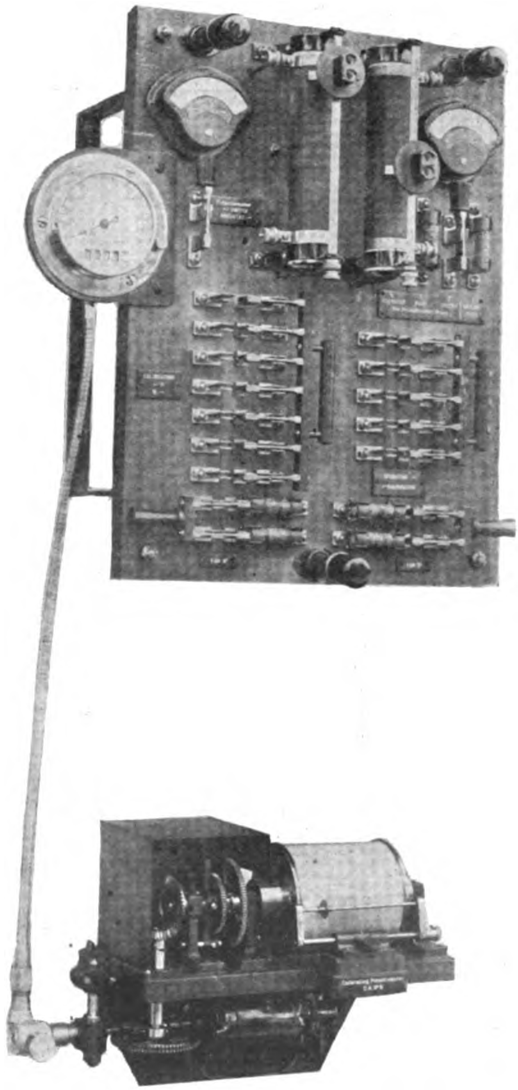

FIG. 1. General View of Constant-rate Slide-wire and Control Equipment

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in the control of the slide-wire; the lower half is related to apparatus associated with the slide-wire for calibration and does not concern us here.) Fig. 2 is a schematic diagram of the slide-wire connections, exclusive of motor control, while the drawings reproduced in Fig. 3 give plan, and front and side elevations, drawn to scale, showing the more important details of construction. The

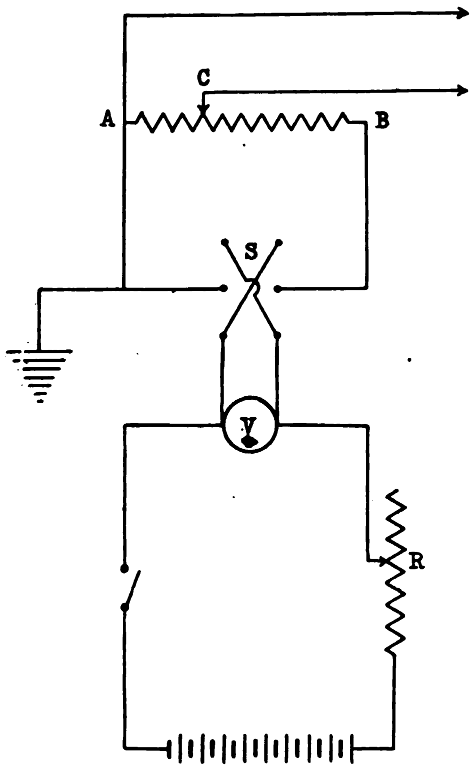

FIG. 2. Schematic Diagram of Slide-Wire Connections

photograph in Fig. 4 shows the apparatus as seen from above, with protecting cover removed, and gives certain mechanical details not shown elsewhere. As shown in Figs. 1 and 3, the present apparatus was constructed for wall mounting, but this is merely a detail of adaptation to surroundings. In what follows attention will be limited mainly to matters not brought out by the

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figures, or which are of first importance for the proper functioning of the apparatus.

The slide-wire *AB* of Fig. 2, consists of about 25 meters of No. 25 (B. and S. gauge) annealed chromel wire. This is wound under

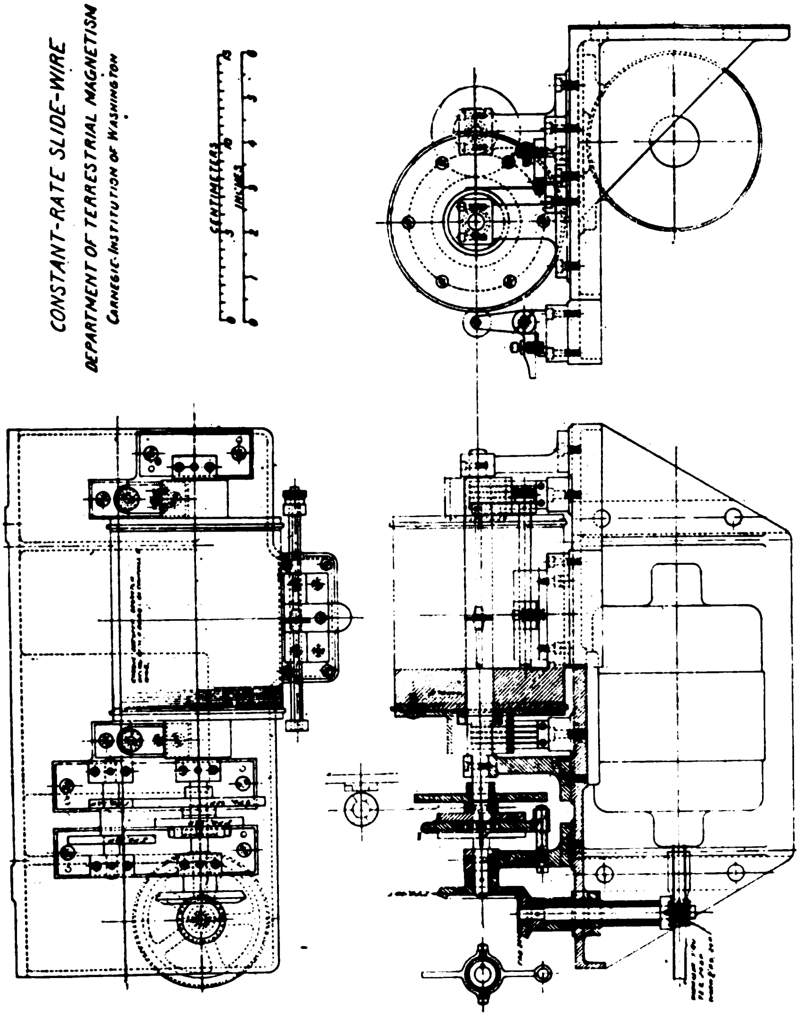

FIG. 3. Plan and Elevations, Constant-Rate-Slide-Wire

moderate tension, in a screw-cut spiral groove of 80 turns, on an accurately-turned white marble cylinder whose diameter and length are each approximately 10 cm. The spiral groove has a

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pitch of about 1.25 mm (20 threads per inch) and is cut to a depth of 0.15 mm with a round nose tool of 0.225 mm radius. The resistance of the wire is about 150 ohms, and heating effects are negligible for applied voltages of the order of 30 volts. Even with 36 volts there is no perceptible loosening of the wire although there is noticeable warming. The marble cylinder was first bored to receive its shaft. After being rigidly mounted and

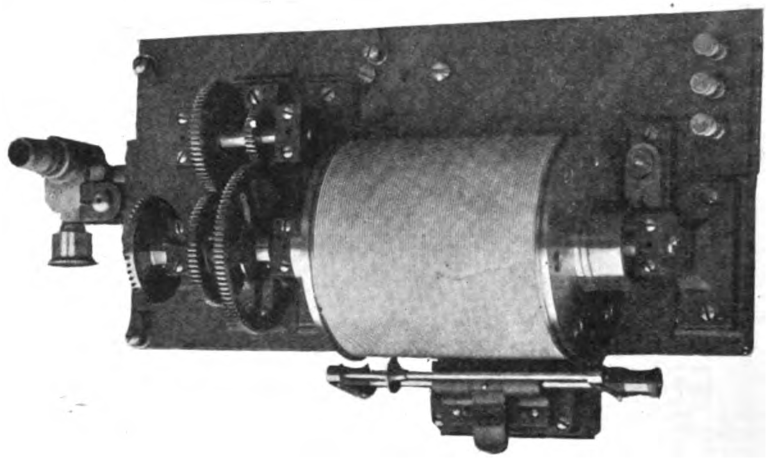

Fig. 4. View of Slide-Wire from Above

keyed to the shaft it was turned into final shape, on its own shaft, by means of a diamond tool. The cylinder is faced at each end by a combination face-plate and slip-ring, secured to the cylinder by screws tapped into the marble. As shown in Fig. 3 (front elevation) the clearances between the slip-rings and the lock-nuts of the shaft are such as to provide ample insulation. In order to eliminate all uncertainty as to the effective length of the slide-wire, the first two turns, at either end, are securely soldered at one point to the adjacent face-plate. Careful determinations, made after the wire was wound, show the resistance per turn to be practically uniform for the entire length of the cylinder.

Each of the brushes connecting the slip-rings to the binding-post terminals, consists of a single piece of phosphor bronze (No. 28, B. and S. gauge), slotted for the greater part of its length into five 3-millimeter strips. The brushes and binding posts are

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mounted on hard rubber blocks of which the details are shown in Fig. 3.

The traveling contactor, represented by C in Fig. 2, consists of a brass wheel 15 mm. in diameter with a 60° V-shaped groove, about 0.2 mm deep, for engaging the wire. It is mounted on a steel axis which is parallel to the axis of the cylinder and along which it slides as it follows the wire spiral. The axis of the contactor forms part of a rigidly-hinged frame which is insulated from the base by a slab of hard rubber. Figs. 1 and 4 show the general construction of the contactor and its supporting frame. Firm contact with the wire is maintained by means of a stiff spring, and a simple thumb lever at the middle of the hinged frame, enables one to raise the contactor for shifting its position to any part of the cylinder. The location of the contact spring, under the thumb lever, is best shown in Fig. 3, side elevation. With moderate care to keep contact surfaces clean, both the contactor and brushes have given very satisfactory service.

The cylinder is operated by a 1/40 h.p. direct-current, shunt-wound motor with a normal speed of 1800 r.p.m. Suitable speeds for the cylinder are secured by means of a steel worm, attached to the motor shaft, and a double train of cut brass gears. The two reductions in speed thus obtained are 144 to 1 and 432 to 1, respectively, and are controlled by a shift-lever. Further changes of speed and all close adjustments are made by means of a rheostat in series with the armature. In the present form of the apparatus the time required for the contactor to travel the entire length of the slide-wire may be varied from 5 minutes to 25 minutes. The motor is energized by a storage battery in order to avoid the irregular fluctuations of commercial circuits. There is, however, the usual slow initial increase in speed so that the best conditions for constant speed are not reached until after the motor has been running for a time. This slow change of speed is, however, easily controlled by the rheostat and causes no trouble.

A tachometer of a type used for indicating speeds of air-plane propellers is attached to the motor shaft, as shown in Fig. 1. By this means the actual speed of the cylinder is readily determined

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to one part in 300, or even better, and the sensitivity of the tachometer is such that, by aid of the control rheostat, the speed may also be kept constant to about one part in 300.

The rheostat indicated by *R*, in Fig. 2, is used for the control and adjustment of the over-all potential difference. With this in mind, one might be led to question the need of providing more than one speed for the operation of the cylinder, since the applied potential difference may be reduced at will. However, it is preferable to obtain the lower rates of change in potential difference by reducing the speed of the cylinder in order to keep the readings of the voltmeter well removed from the lower part of its range and to reduce the importance of contact effects. The use of the pole-changing switch *S* (Fig. 2) is obvious from the diagram.

The apparatus described has been in use for nearly a year and has been found well adapted to the work for which it was designed. With the present form it is possible to secure variation in potential difference, uniform to 1 part in 250, for values ranging from several millivolts per second to about 0.1 volt per second, with the absolute values known to somewhat higher accuracy. This, naturally, does not indicate the limit of merit to be secured by this type of apparatus. With greater care in the selection of the resistor, special treatment of the marble, greater attention to contacts and motor control, both the constancy and the accuracy could be considerably increased if needed.

The author is under obligation to Mr. C. Huff for the design of many of the details and for the preparation of the drawings; the apparatus was constructed under his general supervision, in the instrument-shop of the Department, with the assistance of Mr. W. F. Steiner. Valuable assistance was also received from Mr. O. H. Gish in the determination of the attainable accuracy and degree of constancy.

Department of Terrestrial Magnetism
Carnegie Institution of Washington
Washington, D. C.

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A SIMPLE DIRECT-READING POTENTIOMETER FOR STANDARD CELL COMPARISONS

BY
Marion Eppley and William R. Gray

Of the many types of potentiometer that have been described[1] there are but two which are available for the direct comparison of the electromotive forces of standard cells. They are the "slide-wire" type, and the "Feussner" type.

The "slide-wire" type is the better of the two theoretically, on account of the absence of contact resistances in the measuring circuit. However, it possesses an uncertainty due to the inability of adjusting a slide-wire to uniformity of resistance over its entire length. Also; it is impracticable to manufacture an instrument of this design reading to more than five figures. In the range required for standard cell measurements, this makes the fifth decimal place an estimated one. This place can be estimated to about 20 microvolts upon high-grade instruments. Thus, their precision is approximately .002%, while their accuracy is usually guaranteed to .02% at their full scale reading.

The "Feussner" potentiometer is expensive to make owing to the great number of coils and the need of their being accurately adjusted to each other to insure a measuring current of constant value. The contacts also must be most carefully designed, made, and cared for, if the intended accuracy of the instrument is to be maintained. White[2] states that an error of ten microvolts or more may result from lack of care of the contacts in the Feussner potentiometer as made by Otto Wolff, a maker noted for his elaborate and smooth-working switches. The Feussner potentiometer could be made with an infinite number of dials in so far as theoretical

[1] White, J. Amer. Chem. Soc., *36*, p. 1868–1875; Laws, Electrical Measurements, p. 273–288; Watson, Practical Physics, p. 495–498; Griffiths, Methods of Measuring Temperature, p. 60–66.

[2] White, Ref. 1, p. 1871.

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considerations are concerned. From the practical standpoint, six figures are all that can be secured owing to the total resistance being fixed at its upper limit by galvanometer sensitivity and insulation leakage, and at its lower limit by the resistance of the dial switches.

There are ways[3] in which the residual emf of two opposed standard cells can be measured to ten microvolts or better without any very great accuracy in the resistance and ammeter forming the set-up. Difference methods, however, do not give final results without calculation, an objection when a large number of routine measurements must be made.

If an accuracy of ten microvolts or better is desired, without the need of calculation, some such arrangement as that described below is necessary. In this design the two potential-point feature of the slide-wire type of potentiometer is retained with its advantage of absence of contact resistances in the measuring circuit. The uncertainty due to lack of uniformity in the slide-wire is eliminated by the use of coils throughout. The use of coils is made possible by limiting the range of the instrument to that required for comparing standard cells. No originality is claimed, except perhaps for the self-checking feature.

DESCRIPTION: The diagram gives all salient features of the potentiometer circuit. The reproduction of the photograph of the top shows the arrangement of dials and switches. The self-checking device is perhaps not so evident, for in adjusting the measuring current no coils are used other than those of the potentiometer circuit proper.

Each stud is drilled, on a radius greater than that of its switch-arm, with a hole reamed to receive a plug. A flexible connector with plug S_1 is attached through the standard cell to one pole of a double-throw switch. Another flexible cord with plug S_2 is attached directly to the other of this pair of poles. The opposite two poles of the double-throw switch and its two middle poles are connected as is usual in self-checking potentiometers. When the two plugs are inserted in the holes in the two studs corresponding to the

[3] Lindeck and Rothe, Zeitschr. f. Instrumentenkunde, *20*, p. 293, 1900.

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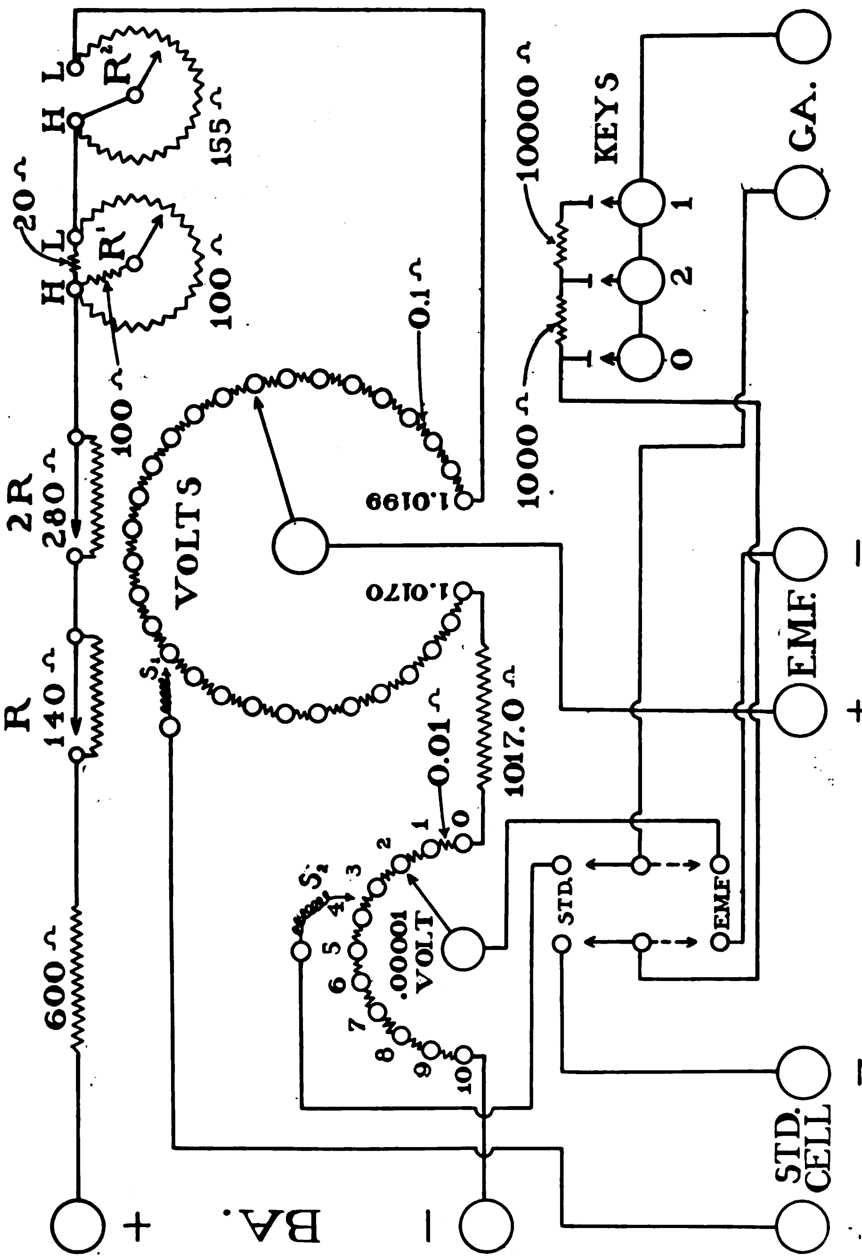

FIG. 1. Diagram of Potentiometer Connections

electromotive force of the standard, and the double-throw switch is thrown to the "Standard Cell" side, the cell is in series with the galvanometer, and is across the resistance proportional to its value. By simply throwing the switch to the E.M.F. position, the unknown is thrown in series with the galvanometer, without disturbing the standard cell connections. In this way, convenience is secured without the need of adjusting a "standard cell coil" to proportionality with the coils of the measuring circuit.

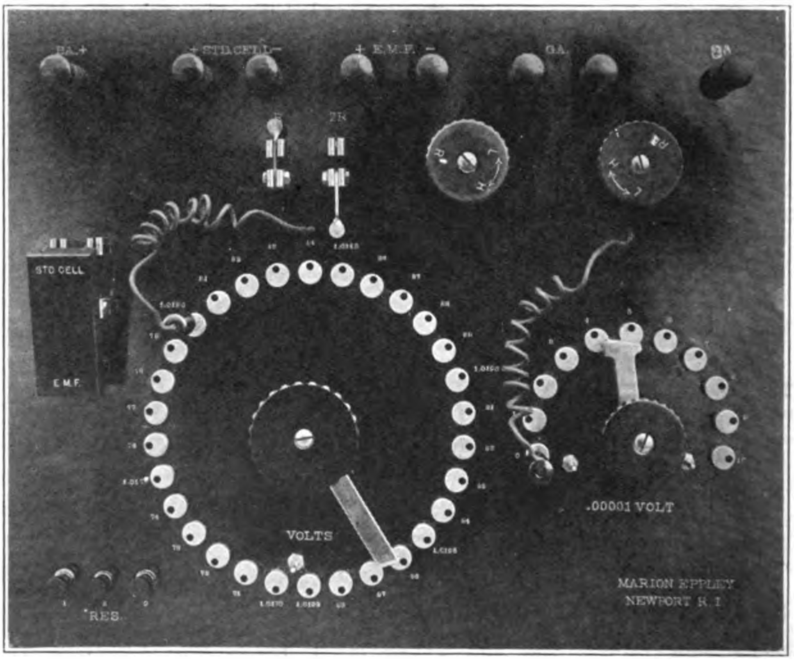

FIG. 2. Top of Potentiometer

The comparison of a cell having a value falling at the uppermost point of the range of the instrument, with a standard whose value lies at the lowest limit of the range, presents the worst condition for accuracy. An error of 0.1% in the resistance of the 1017.0 ohm coil of the potentiometer circuit would, under the above conditions, introduce an error of 3 microvolts. The same would be true of a 0.1% error in the same direction in all the resistances of the two dials, the 1017.0 ohm coil remaining constant.

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To secure uniformity of temperature, the instrument is immersed to the bottom of the ebonite plate in a stirred oil-bath.

A highly sensitive galvanometer is necessary. Most excellent results have been secured with a Leeds & Northrup Company Type HR galvanometer having the following characteristics; sensitivity 2460 megohms; 2.03 mm per microvolt; 102 ohms resistance, 12 seconds period.

The parasitic electromotive force across the standard cell binding posts, with the battery disconnected, but the galvanometer connected as usual and the key O closed, was less than five microvolts, the checking switch being closed at the "Standard Cell" position. With the checking switch at the "E. M. F." position, the parasitics across the E. M. F. binding posts were about five microvolts. Swinging the long arm over its entire range of travel twenty times, as rapidly as possible, produced an E. M. F. of 18 microvolts which in thirty-five seconds sank to 5 microvolts. Swinging the short arm twenty times in the same manner produced an E. M. F. of 10 microvolts which sank to five microvolts in thirty seconds. Measurements were made with a Leeds & Northrup Company Type K potentiometer using the "low range."

A storage cell was found to drift too rapidly for satisfactory results. A standard battery was therefore made differing from that described by Hulett[4] in the following details: A crystallizing dish 17 cm in diameter and 9 cm deep was placed in the center of another crystallizing dish 25 cm in diameter and 12 cm deep. The inner dish was partially filled with mercury and mercurous sulfate. Into the annular space molten 12½% cadmium amalgam was poured to a depth of about 2 centimeters. Crystals of cadmium sulfate were placed in both dishes and both dishes filled with a saturated, acidified solution of cadmium sulfate. A cover was provided, and suitable connectors of platinum wire in glass tubing.

[4] Hulett-Phys. Rev., *27*, 33.

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Two of these cells have proved sufficient.[5] After the lapse of two hours or more after adjustment of the measuring current, not more than a millimeter deflection of the galvanometer is customary if the current had been flowing for half an hour before the adjustment. When not in use, the circuit is kept open. Two such cells have been in use for 7 months and are still giving the same service as originally. They have received no attention.

Check-measurements on standard cells, made with this potentiometer, by the opposition method of Lindeck and Rothe,[6] and by the Bureau of Standards, usually agree to ten microvolts or better.

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[5] Note: These two cells would not operate a Leeds & Northrup Company Type K potentiometer satisfactorily.

[6] See reference 3.

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ON CONSTRUCTION OF PLATINUM THERMOMETERS AND OF RESISTANCE COILS

BY
J. R. Roebuck

While zeros of platinum thermometers are generally much steadier than are those of mercury-in-glass thermometers, nevertheless the most serious difficulty in work of high precision is the zero shift. Such observed shifts may be due to many different causes. For example; (a) to shift of lead resistance with temperature or handling, (b) to shift of insulation resistance, (c) to shift of reference coils resistance, and (d) shift in resistance of the thermometer coil itself.

Causes (a) and (b) can be located with certainty and eliminated by well known methods. In this connection it might be emphasized that lamp cord is a very unsuitable material for such leads. But (c) and (d) are very difficult, indeed, to separate. In fact, what one observes always is the difference between shifts due to these two causes. Approximate separation of the shifts may be obtained by imposing temperature excursions on the thermometer, so that the time for which it is away from the reference temperature is short. This is limited to the detection of shifts in the thermometer due to temperature change. Secular shifts are also important and here the only means of separating and measuring is by reference to a non-varying resistance. The proportional variation involved in thermometer work, say for 0.001°C at room temperature, is of the order of 3 parts in a million which is probably beyond the limit of steadiness of the best standard resistances.

The causes of resistance shift of reference coil and of thermometer coil are necessarily very much alike. They may be summarized thus: (a) gradual relief from mechanical strain due to previous handling, (b) both imposition of and relief from strain during use, (c) changes in the composition of the resistance

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material from the outside as oxidization of the surface of the resistance wire, or attack by acid in the bath liquid, or contamination of the platinum wire by silicon or metallic vapors, (d) changes in the internal character of the material due, for example, to crystal growth, or to concentration changes of solid solution and so on.

Cause (a) is usually met by annealing at a temperature well above that at which the coil is to be used. This may be made fully adequate for the Pt coil by heating electrically in air. For the resistance wires, usually manganin, this is difficult to do on account of oxidization of some components in the wire which not only changes the total resistance but also its temperature coefficient. It is undoubtedly one of the principal causes of resistance shifts in newer coils and may easily remain of very objectionable magnitude over the time for which such coils are used.

Cause (b) is one of the main reasons for shift of resistance of both coil and thermometer. Thus standard resistances are wound embedded in shellac, on a metallic cylinder. The coefficients of expansion of the wire, of the shellac, and of the cylinder are all different. The result of temperature change is to impose strains on all the parts and these may or may not remain entirely elastic. If any permanent deformation occurs anywhere, it will show itself by a shift in the resistance when the reference temperature is again attained. It is easily conceivable that the readjustment of the resistance to the reference temperature state might take time and the resistance therefore drift for hours or days.

Or consider the standard method of construction of platinum thermometers.[1] The wire is wound in notches on the edges of 4 mica strips, so that it makes a square. It is wound tight at room temperature. A rise in temperature allows the wire to become loose on its support, in annealing for instance, and on cooling it grips the points of support again. If it has shifted on the support any minute amount, the bends have to accommodate it, that is, the wire is being continually re-strained. Experimentally, it was found that a wire wound in a thread cut in a nonconductor was

[1] See Summary by T. S. Sligh, Bur. of Stand. Sci. Paper, No. 407.

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even worse in its erratic zero shifts, which is to be explained in the same way.

These conditions require that for the platinum thermometer, the wire be supported in such a way as not to strain it either from handling or from temperature change. As the wire must be of very small diameter to give the required resistance the supporting points must be close together. On the other hand, the support must not be a constraint. Some experimental work which the writer has had under way for years required the measurement of small differences of temperature at not exceeding 300°C. The difficulties encountered led to much experimental work. The following procedure for the construction of a thermometer for this range is a summary of this phase of the work.

A spool of metal for carrying four strips of mica is constructed as in Fig. 1, which shows, also, cross sections at the important

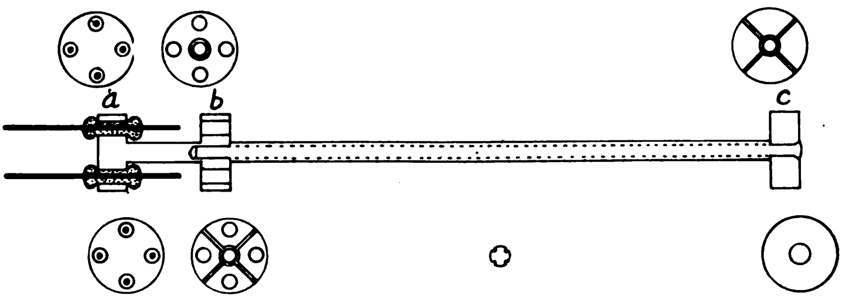

FIG. 1

places. The dimensions used are set by the space and resistance requirements. A cylinder of 3-4 cm long and 5 mm diameter can carry a meter of 0.05 mm platinum wire having about 50 ohms resistance at 0°C. The narrow and thin strips of mica are slipped into the radial slots of the spool bc, so that the outside edge of the mica is about flush with the cylinders. The strips need support over their length by the slots in the central rod. A light prick with a center punch beside the slot on the cylinders b and c will hold the mica in the slot during handling. The cylinders b and c have four holes, alternating with the slots, as indicated in the figure. Pieces of platinum wire about No. 20 and 2-3 cm long are covered with lead glass fused on to let it slip into the hole in "a" up

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to a knob at one end. A little ring of the lead glass is slipped over the wire and fused on using a small pointed flame, with the result shown at "a." By crowding the soft glass close to the brass it is possible to leave very little play. Four such leads are each silver-soldered to a copper wire long enough to reach out of the thermometer case. These are then bound together while the coil itself is constructed. The four leads thus make a very substantial support for the spool carrying the thermometer coil with very little chance for straining the fine wire.

The spool bc with the mica strips in place is cast in wax, forming a cylinder out as far as the edges of the mica. Half beeswax—half resin is a suitable wax. This cylinder is turned down in the lathe till the surface is slightly below the ends of the spool. A double thread is cut in the wax and mica with a very pointed threading tool, the number of threads being determined by the length of wire to be carried and the diameter and length of the available spindle. It is possible to use 100 threads per inch successfully both in the cutting and subsequent use. It is, of course, not necessary that the space between the threads be cut to a sharp edge, but the groove must be sufficiently wide and deep to hold the thermometer wire securely.

The compensating lead connection of the fine platinum wire is put in first. It is led through the hole in cylinder b around a semi-circumference and back to the diametrically opposite lead. It is insulated from the metal around the holes by a thin tube of glass either fused down on the wire for its length through the hole, or only at one end to prevent it slipping along the wire. Care must be taken that the wire has slack enough not to be broken by whatever looseness the leads may have. This is only important during the handling as when the coil is in the protecting tube, the leads will be unable to move materially. It is possible to proceed here to the silver soldering of the fine wire to the heavier platinum leads but it is difficult to avoid some fusion of the wax. It is safer to seal the wire in place by bringing near the wax a warm point, and proceed to wind the main coil. The two ends of the wire are insulated and held in place, as for the short wire. The spindle is mounted in the lathe and the wire stretched in a loop from the two

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fastened ends by a piece of thread tied to the loop, passed over a round bar and loaded with a weight of a few grams. The wire is annealed at a dull red heat by a suitable electric current. The two wires are then wound in the double thread with care to assure that the wires go well down into the groove and that no small shavings of wax support it irregularly. The wire near the end of the loop is caught in place by a hot point held near the wax, and the loop is then fastened at leisure by a piece of the same platinum wire passed through a fine hole bored through one of the mica strips.

All four wires are then silver soldered to their respective leads with a fine pointed flame. The operation goes more readily if the leads have been prepared with a little silver solder on the end so it is only necessary to have the fine wire in contact with the fused silver to make the joint. A little silver solder may be readily transferred from another bit of platinum wire if necessary. The making of the four joints without fusing the wax seriously required some practice and care. The wax may be protected from fusing by holding the cylinder b in a vise and slipping a narrow piece of mica under the wire being heated. An oxy-gas flame may be made very small and yet give the required heat and hence does the best work. The making of these joints may be delayed till the wax is removed, but the coil is then very easily injured. The wax is removed by a suitable solvent, hot turpentine serves well for the half-and-half and the turpentine can be removed by a volatile solvent like carbon tetrachloride.

The annealing just before winding assures that the coil will do no springing on removal of the wax. If it be now annealed by passing an electric current, the coil squirms all out of place. This may be entirely avoided by warming the coil gradually inside a hard glass tube by heating the tube with one or more gas flames. While hot a small current will make the wire incandescent and complete the annealing.

This results in a perfectly cylindrical coil with the wires as exactly spaced as the thread was cut. The wires are well supported at four points without any unusual bend at the point of support. Any adjustment due to temperature changes can result only in long radius bends which will not exceed the elastic limit. Ex-

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perience over several years with such coils shows that they have very steady zeros, the steadiest in fact of any arrangement yet tried here. Any numerical statement of the zero shift is of little value since it is complicated by the unknown variation of the reference coils and of course though probably to a smaller extent, by the variability of the ice bath temperature.

Conduction along the leads may be minimized by threading the wire through holes in short metal cylinders which fit the case and are separated from each other by nonconductors. The leads are insulated from the metal cylinders by thin glass tubes. In between the metal cylinders these are made of much thicker glass and so serve to space the metal cylinders. The result is to improve the radial conductivity without affecting the axial, so that at each point the leads will be kept near the temperature of the material surrounding the stem. The 4 platinum-copper junctions are arranged to come in the middle of the first or second metal cylinder. These four junctions are thus held together in temperature even during changing temperature. These junctions are the main location of thermals in the circuit, and this arrangement excludes them practically completely. In the set-up used by the writer for years there are no erratic motions of the light spot with a galvanometer sensibility of 10^{-9}, and almost no zero shift. It should be added that the galvanometer circuit in the bridge is entirely copper (no brass binding posts) and is never opened: the bridge is operated by manipulating the applied potential and the slide wire contact is in the battery circuit.

The introduction of a metal spindle in the thermometer and of the metal about the leads, increases the heat capacity of the thermometer and therefore of the lag when measuring a changing temperature. Where the reduction of lag is of primary importance this type of construction may not be the best. In all cases the steadiness of the zeros is of primary importance for precise work and in many cases lag is quite secondary. It is these cases where this type of construction is particularly suitable.

It is evident that a similar type of construction should improve the steadiness of the resistance coils. The conditions are somewhat different, however. The coils can be made much larger and

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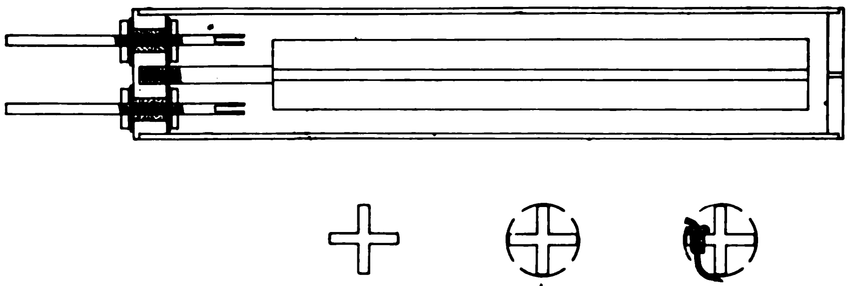
longer before the size gives rise to difficulty. The manganin wire is very stiff and a very hard wax would be needed around which to bend the wire. The difficulty of preventing surface oxidization makes it desirable to use a much coarser wire. Coils were consequently built as follows.

Four grooves are milled out of a brass cylinder ½ inch diameter and 5 inches long to give it the cross section indicated in Fig. 2a. These four grooves are filled with paraffin wax by slipping this cylinder inside a brass tube, filling with melted paraffin and after



a *b* *c*

FIG. 2

thorough cooling slipping the tube off by heating it rapidly. Four strips of mica are laid along the cylinder as indicated in Fig. 2b. The edges of the strips do not meet so that the wire when wound on does not have a complete cylindrical support but may spring in and out in each quadrant as required by the temperature conditions.

The wire is doubled for non-inductive winding and a copper wire hard soldered to the bend. This may be done with very little heating of the resistance wire by holding both wires together end on in clamps. A gas flame fed with a little oxygen enables one to heat the joint very rapidly and transfer the liquid solder from another wire with the minimum heating of the resistance wire—when done successfully less than ¼ inch of the resistance wire may show the effects of heat. The copper wire is passed through a glass sleeve inserted in a hole in a wing of the supporting cylinder as indicated in Fig. 2c. A sharp bend in the copper wire is sufficient to hold it. The ends of the resis-

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tance wire are fastened in a similar manner by hard soldering each end to two copper wires, one to serve for fastening and the other as current lead.

The mica strips are bound in place with a series of ties of string or wire. The winding of the resistance wire is begun at the doubled end and the ties removed successively as they are replaced by wire. The paraffin is then melted and washed out by suitable solvents.

There is no need for covering on the wire if it be carefully spaced in winding. Also there is nothing in the construction to prevent the coil being heated to 400-500°C. With suitable vacuum protection this should be the best treatment. The procedure so far used has been to heat the coil to about 135°C for many hours in air in a space heated by boiling xylol. The surface of the wire becomes slightly discolored at this temperature which is therefore as high a temperature as one should use in the presence of air. After this treatment the wire still remains very stiff and will spring uncoiled strongly if unfastened. This is a good reason for thinking that this annealing is a long way from adequate and that a considerable portion of unrelieved strain remains in the wire. The resistance changes show also that considerable strain has been removed.

After annealing the coil is dipped in a good grade of shellac dissolved in pure alcohol, so as to make a fairly thick varnish. It is dried by holding a few hours at near 100°C. This gives a comparatively thin coating of shellac, hardly sufficient to distort the wire materially. If the shellac penetrates between the wire and the mica it can produce scarcely any strain. Nor can the change of hygroscopic condition of the shellac produce material changes in the strain.

To minimize any effect of moisture as well as to protect the wire from mechanical injury it is best to enclose each coil in a container with acid-free dry kerosene as indicated in Fig. 2. The leads are insulated from the container by mica disks under the nuts and a glass sleeve in the holes. The whole joint is set in fused shellac which is also fused on over the outside finally. The stout copper leads have small holes bored into them axially at the coil

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ends and the copper leads from the resistance wire are soft soldered in these holes using resin dissolved in pure alcohol as flux. The brass cylinder is slipped on and soft soldered together with an iron using resin. The kerosene is put in through a small hole in the bottom which is finally closed with a lump of solder. The stout copper leads can be used for supporting the coils.[2]

Experience with such coils shows them to be materially better in steadiness than the older form. But there still remains relative variation of such a size as to be distinctly objectionable. The actual variation may easily be greater and it is these latter which come into the thermometer work.

The order of variation reached by many workers gives a fundamental interval constant to a few parts per 100 000, so that for this factor the accuracy is almost certainly better than any other associated physical measurement. But when one desires to read a small temperature difference to 0.001° or 0.0001°C the erratic or continuous variation from day to day makes such data very uncertain. This has been the writer's personal experience. Improvement in the steadiness of the system would increase one's confidence in the data very materially.

In this connection further work on manganin is very necessary. The writer has made up many coils with variations of many conditions, e. g., with and without shellac on silk insulation, with and without metallic calcium as a drying agent, oxygen and acid absorbing agent in the kerosene; on metal cylinders, or with four points of support, or without any support. Having no steady comparison resistance it has not been possible to conclude anything about the actual variation. There was always relative variation of a magnitude much greater than the limit of measurement and great enough to be very undesirable in thermometric work.

About the only experimental attack would appear to be the careful study of groups of coils made up in different ways till some means was found for reducing greatly the relative variation within a group. Differential treatment of members of the group would

[2] See this JOURNAL, 6, p. 175, 1922.

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then enable one to judge of the effect of the different conditions. For example, how quickly does a manganin coil attain a steady resistance after a change of temperature. The writer has followed the steady drift of resistance of an iron wire coil for more than 24 hours after its resistance was changed by raising the temperature about 30°C for a short time. The change was of the order of 10^{-4} and large enough to render the coil absolutely worthless for its proposed use as a thermometer.

PHYSICS LABORATORY
UNIVERSITY OF WISCONSIN
MAY, 1922

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THE DRAINAGE ERROR IN THE BINGHAM VISCOMETER[1]

BY
Winslow H. Herschel

1. Introduction

For the accurate determination of absolute viscosity it is usual to employ instruments with glass capillaries, more or less modified from the original form of Ostwald. These instruments differ in the arrangement of the bulbs in which the liquid is measured, and in the manner of filling with a definite working volume. In the instruments of Ostwald, and of Traube[2] there is a drainage error due to the variation, with the viscosity, of the volume discharged from the bulb. This error is avoided in the apparatus of Lidstone[3] by the use of mercury which does not wet the bulb, while in the Ubbelohde form of instrument the flow is into a dry bulb, and the measured volume is independent of the viscosity of the liquid tested.

The Bingham viscometer[4] shown in Fig. 1, is similar to the Ubbelohde in having two bulbs so arranged as nearly to eliminate the effect of variations in the density of the liquid, and in the use of a constant air pressure to cause the flow. The working volume is contained between the mark H and the overflow into the trap at A. When the temperature is raised, the volume is again adjusted by causing the surplus to run into the trap.

The volume of flow is contained in the bulb C, between the marks B and D. This bulb is alternately emptied and filled, if check tests are run, but it should be noted that from the method

[1] Published by Permission of the Director, U. S. Bureau of Standards.

[2] Holde-Mueller, Examination of Hydrocarbon Oils, p. *117*; 1915.

[3] F. M. Lidstone, Jour. Soc. Chem. Ind., *36*, p. 270, 317; 1917.

[4] E. C. Bingham and R. E. Jackson, B. S. Scientific Paper No. 298, p. 64; 1917. E. C. Bingham, Proc. A.S.T.M., *18*, part 2, p. 373; 1918. The word viscometer is used by Bingham and will be used throughout this paper to avoid confusion, even for instruments usually called viscosimeters.

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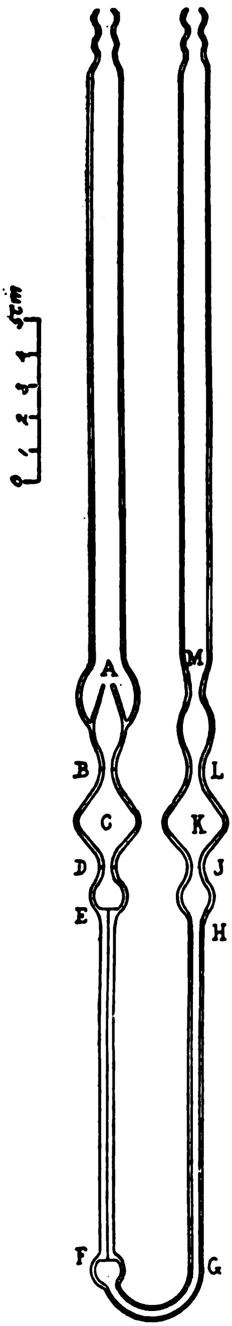

Fig. 1. The Bingham viscometer

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of adjusting the working volume, the bulb is already wet before the first run can be made, so that a comparison can not be made between the capacity of the bulb to deliver, and of a dry bulb to receive.

Since the Bingham viscometer is used only where great accuracy is required, and since it is claimed that it is "accurate to a tenth of one per cent,"[5] an error which might be of no importance in another type of instrument can not be considered as negligible. It is evident that the drainage error will increase with the viscosity, or with the amount of liquid which clings to the walls of the bulb, and the error might be considerable when the instrument is used for testing lubricating oils, with viscosities up to 20 poises and over. Thorpe and Rodger[6] who used a similar instrument, concluded that for the comparatively fluid liquids with which they had to deal, with a maximum viscosity of 0.14 poise, a cylindrical form of the bulbs was preferable, in order that the contained liquid might the more readily acquire the temperature of the bath, although with spherical bulbs, as used by Ubbelohde, less liquid adheres to the walls. In the Thorpe and Rodger instrument there is a trap on both sides and each bulb is graduated, so that the working volume may be adjusted after the first run to allow for possible evaporation.

Barr[7] used an Ostwald viscometer for testing glycerin, with a viscosity of 1.26 poise, and remarks,

The viscometer was filled by a pipette, the time of drainage of the pipette necessary for the accuracy sought being determined by experiment.

He does not appear to have considered whether the time of flow of his viscometer was sufficient or not to eliminate drainage error.

The object of this paper is to consider evidence of drainage error in the Bingham viscometer, and to estimate the magnitude of this error, or the time of flow necessary to render it negligible, with a bulb of 4 cubic centimeters capacity. While Bingham

[5] Proc. A. S. T. M., loc. cit. The effect of the uncertainty in regard to end effects, upon the demonstrable accuracy of the instrument, is considered in the discussion of this paper.

[6] T. E. Thorpe and J. W. Rodger, Phil. Trans. R. S., *185*, part 2, p. 423; 1894.

[7] Guy Barr, Reports and Memoranda, No. 755; Aeronautical Research Com., p. 4; London; 1921.

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apparently paid most attention to correctness of average pressure and of working volume, he recognized[8] the danger of imperfect drainage, for he says:

Our theoretical investigation proves therefore that the bulbs of the viscometer should be as short as convenient, but with a large radius. The constricted portions should be both narrow and short. These objects may be best achieved without sacrificing good drainage, by having each bulb like two filtering funnels placed with their ends together.

2. CALIBRATION OF THE BINGHAM VISCOMETER

In the Bingham viscometer flow is caused by air pressure applied first to one side and then to the other when a check determination is desired. In an ideal instrument the time of flow would be the same in each direction, and there would be no hydrostatic head correction, but this is rarely attained in practice.

The calibration of the instrument has been fully described in B. S. Scientific Paper No. 298, to which the reader is referred for further details than can be given here. The usual assumptions are made that there is no turbulence and that the capillary is very long in proportion to its diameter. Then the coefficient of the kinetic energy correction is equal to

$$C' = \frac{1.12\,Q}{8\,\pi\,l} \tag{1}$$

where Q is the volume of the bulb in cubic centimeters, and l is the length of capillary in centimeters.

In the equation

$$h' = \frac{\mu}{2\,C\,\gamma}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) + \frac{C'}{2\,C}\left(\frac{1}{t_1^2} - \frac{1}{t_2^2}\right) \tag{2}$$

h′ is a constant of the instrument used in correcting for hydrostatic head. t_1 and t_2 are the times of flow in the two directions, and could only be equal if the bulbs were absolutely alike. μ is the viscosity in poises, and γ is the density in grams per cubic centimeter of the calibrating liquid.

Before using equation (2) it is necessary to calculate an approximate value of C from the equation

$$C = \frac{\mu + \frac{C'\,\gamma}{t}}{pt} \tag{3}$$

[8] E. C. Bingham, H. I. Schlesinger and A. B. Coleman, Jour. Am. Chem. Soc., *38*, p. 32; 1916.

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where p is the pressure causing flow, in grams per square centimeter. For this purpose only, p may be taken equal to h = average manometer reading reduced to grams per square centimeter, or centimeters of water column, and then, after h' has been obtained, p and C may be corrected and h' recalculated if necessary.

The absolute viscosity in poises, is calculated from equation (3) in the form

$$\mu = C\,p\,t - C'\,\gamma\,/t$$

where p is the corrected manometric pressure, and is equal to

$$p = h \pm h'\gamma - K \pm L \qquad (5)$$

where h' has the plus sign when t has the lower value, and the minus sign when the flow is in the opposite direction and t is greater. K is a correction due to change in density of water with the temperature and to weight of air in the manometer, and its value depends upon the manometric head and the room temperature, while L is a minor correction due to difference in elevation of the center of the manometer and of the viscometer. Tables for K and L are given in *Scientific Paper* No. 298.[9]

For the instrument used in this work, C' was calculated to be 0.01229 by equation (1) with the approximate values, Q = 4cc and l = 14.5 cm. The diameter of capillary was approximately 0.07 cm. The two other instrumental constants, C and h', were found as shown in Table 1.

The capillary was almost too large to calibrate with water, so that it was necessary to use very low manometric pressures. In some cases the manometer reading of only 11 centimeters was not large in comparison with h' whereas Bingham[10] implies that for the most accurate work the manometer reading should be at least 30 h'. The value of .00000898 was obtained for C four times out of the fifteen tests, and as it agrees fairly well with the average, it was taken as the most probable value. In calculating Table 1, p was found and C calculated separately for each direction of flow, and the values given are the averages of the two values thus obtained.

[9] For similar tables for use with a mercury manometer, see paper by E. C. Bingham and Henry Green, Proc. Am. Soc. Test. Mat., *19*, II, p. 651; 1919.

[10] Bingham, Schlesinger and Coleman, loc. cit., p. 32.

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TABLE 1. *Calibration of Bingham viscometer*

Calibrating liquid	Temperature	C x 10^8	h'
	° C		
Distilled water......	5	886	.95
Distilled water......	5	920	.88
Distilled water......	10	889	.93
Distilled water......	10	919	.87
Distilled water......	15	898	.94
Distilled water......	20	899	.93
Distilled water......	25	898	.94
Distilled water......	25	898	.83
Distilled water......	25	909	.87
Distilled water......	25	904	.87
40% sucrose solution.	25	913	.85
60% sucrose solution.	20	868	.86
60% sucrose solution.	25	896	.87
60% sucrose solution.	30	898	.85
60% sucrose solution.	30	901	.88
Average..........................		900	.89

3. USE OF THE BINGHAM VISCOMETER

After h' has been determined, it is not necessary to refer again to equation (2), but p has to be determined for every test, by equation (5), since it varies slightly with the room temperature even if the manometer reading is not changed. The viscosity is then determined from equation (4).

A series of blends were made for use in calibrating technical viscometers, and their viscosities are shown in Table 2.

TABLE 2. *Viscosities of calibrating liquids*

Sample No.	Temperature ° C	Density g/cc	Viscosity Poises
1	20	.854	.1847
	40	.840	.0873
	55	.830	.0559
2	20	.894	.3927
	40	.881	.1573
	55	.871	.0928

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TABLE 2 (continued)

Sample No.	Temperature ° C	Density g/cc	Viscosity Poises
3	20	.884	.4702
	40	.871	.1854
	55	.861	.1072
4	20	.874	.603
	40	.860	.2315
	55	.850	.1313
5	20	.909	1.680
	35	.899	.664
	40	.896	.511
	50	.889	.320
	55	.886	.2592
6	20	.875	.872
	40	.861	.3198
	55	.851	.1787
7	10	.970	25.06*
	15	.967	15.89*
	20	.963	10.21*
	23	.961	8.00*
	25	.960	6.84
	30	.956	4.795
	35	.953	3.312
	40	.950	2.338
8	20	.875	1.232
	40	.861	.440
	55	.851	.2395
9	20	.819	.02878
	40	.805	.01860
	55	.794	.01571
10	20	.849	.1232
	40	.835	.0620
	55	.825	.04147
11	20	.859	.2307
	40	.845	.1023
	55	.835	.0638

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TABLE 2 (continued)

Sample No.	Temperature ° C	Density g/cc	Viscosity Poises
12	20	.882	.531
	40	.869	.1851
	55	.859	.1064
13	20	.876	.714
	40	.856	.2638
	55	.846	.1466
14	20	.872	1.038
	40	.858	.3691
	55	.848	.2009
	90	.832	.0690
15	20	.887	1.563
	40	.874	.4491
	55	.864	.2211
16	20	.901	1.333
	40	.888	.4175
	55	.878	.2141
17	20	.873	1.266
	40	.859	.4435
	55	.849	.2387
18	20	.911	1.686
	40	.898	.634
	55	.888	.3557
19	20	.911	2.361
	40	.898	.829
	55	.888	.445
	90	.870	.1611
20	20	.882	1.429
	40	.869	.502
	55	.859	.2701
21	20	.926	2.599
	40	.913	.935
	55	.903	.514

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TABLE 2 (continued)

Sample No.	Temperature ° C	Density g/cc	Viscosity Poises
22	20	.892	3.20
	40	.879	.951
	55	.869	.4856
23	40	.873	1.325
	55	.863	.639
24	40	.873	1.446
	55	.863	.692
25	40	.873	1.546
	55	.863	.728
	90	.842	.1982
26	40	.878	2.250
	55	.868	1.014
27	40	.881	3.221
	55	.871	1.415
	90	.848	.3360
28	20	.930	6.725
	25	.927	4.215
	30	.923	2.764
	37.8	.918	1.508
	45	.914	.933
	50	.910	.712
	54.4	.907	.554
	60	.904	.4143
	70	.897	.2714
	80	.890	.1790
	90	.884	.1236

Sample No. 5 is a French oil used in standardizing the Barbey viscometer. Sample No. 7 is castor oil, the viscosities marked with an asterisk having been estimated by comparison with another sample. No. 28 is the U oil referred to by Bingham.[11] A blown vegetable oil was used in mixing some of the more viscous samples, and the change of viscosity with the temperature is thereby decreased.

[11] E. C. Bingham, Bureau of Standards, Tech. Paper No. 204, p. 58; 1922.

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4. THE SCOTT VISCOMETER

In previous work of calibrating viscometers[12] use was made of water and ethyl alcohol solutions which are not suitable for instruments with outlet tubes of large diameter, such as are used for determining the viscosity of varnishes or fuel oils. The Scott viscometer was the first of these large tube instruments to be calibrated with liquids whose viscosity had been determined by the Bingham viscometer, and a difficulty experienced in the calibration was ascribed to the drainage error of the Bingham instrument.

The Scott viscometer[13] is not essentially different from other technical efflux instruments, such as the Saybolt, Engler, and Redwood.

Table 3 gives dimensions of the Scott viscometer, according to the designer, and also gives dimensions of two particular instruments, one as measured by the Bureau of Standards, and the other as reported by the Ordnance Department.

TABLE 3. *Dimensions of Scott viscometers*

Dimension	B. S.	Ord. Dept.	Scott	
	Cm.	cm.	inch	cm.
Diameter of oil container, at top	....	8.66	....	....
Av. diameter of oil container	8.738	8.74	3½	8.90
Diameter of outlet tube	.275	.267	.108	.274
Diam. of conical valve seat, at top	.956	.864	....	....
Length of outlet tube and lower conical flare	.628	.686	⅜	.953
Length of lower conical flare	.05	....	....	....
Length along axis of conical valve seat	.567	....	....	....
Depth of upper end of conical valve seat below surface of water with 200 cc of water in container	3.47	....	....	....
Mean diam. of widest part of flare at lower end of outlet tube	.385	....	....	....

[12] Bureau of Standards, Tech. Papers Nos. 100, 112, and 125.

[13] Proc. Am. Soc. Test. Mat., Vol. 10, p. 117–146, 1910: A. H. Gill, Oil Analysis, p. 164; 1913: Wm. M. Davis, Friction and Lubrication, p. 42; 1904: W. G. Scott, Drugs, Oils and Paints, Vol. 13, p. 50; 1897–8.

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According to Scott the outlet tube must be made by using a new No. 35 twist drill, and the time of discharge for 50 cc of water at 60°F (15°.6C) is between 10 and 11 seconds.

There is no filling mark so 200 cc of the liquid to be tested is measured in a graduated cylinder and poured into the container, the cylinder being left 10 or 15 minutes to drain if necessary. The data of table 4 were obtained with oils selected from table 2.

Fig. 2 shows a Higgins diagram[14] calculated from data of Table 4. The graph represents the equation

$$\text{kinematic viscosity} = \frac{\mu}{\gamma} = 0.0178\ t - \frac{2.74}{t} \qquad (6)$$

but since the outlet tube has a flare at both ends, it is not to be

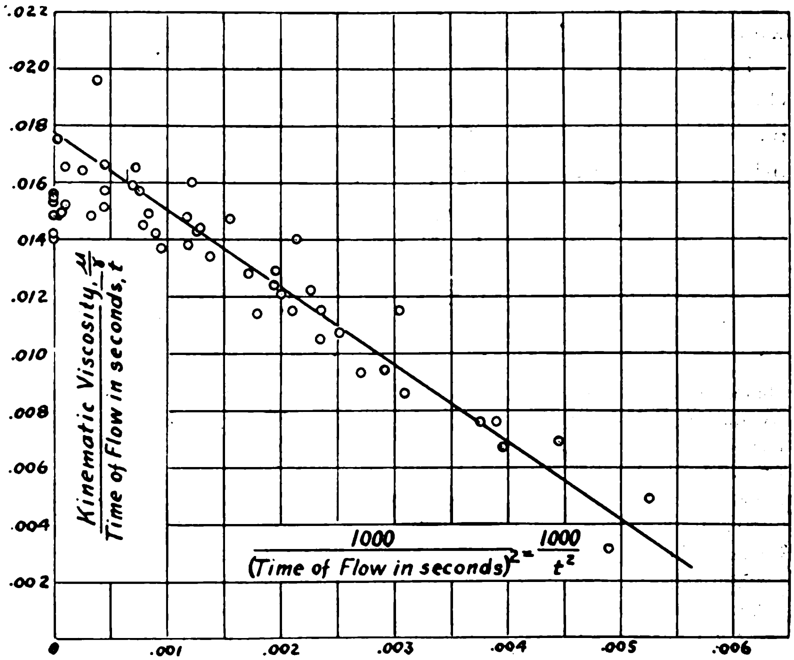

FIG. 2. Higgins diagram for Scott viscometer

expected that two instruments will have exactly the same time of flow or that equation (6) would be of general application. On account of the impossibility of accurately estimating the average

[14] W. F. Higgins, Jour. Soc. Chem. Ind., 32, p. 568, 1913.

diameter of outlet tube, no attempt was made to check equation (6) by a Herschel diagram similar to Fig. 5.

TABLE 4. *Tests of Scott viscometer, B. S. 6452*

Sample No.	Temperature	Time	Sample	Temperature	Time
	°C	Seconds	°C		Seconds
water	15.6	11.2	12	40	19.9
				55	16.3
1	20	20.6	13	40	24.1
	40	15.0		55	18.5
	55	13.8			
2	20	32.2	14	40	29.1
	40	19.2		55	20.6
	55	15.9			
5	20	129.8	15	20	118.1
	35	46.9		40	34.5
	40	36.3		55	21.0
	50	26.9			
	55	22.6			
6	20	51.0	16	40	23.1
	40	25.2		55	21.3
	55	18.4			
7	10	1865.7	17	20	95.4
	15	1139.7		40	35.5
	20	707.8		55	22.7
	23	546.3			
	25	464.8			
	30	320.9			
	35	227.5			
	40	166.3			
11	40	16.0	18	40	46.8
	55	14.3		55	28.0

It will be noted that very viscous oils, giving points at the extreme left of Fig. 2, show too low a value of the ordinate $\frac{\mu}{\gamma t}$. As there was a possibility that this was due to some peculiarity of the laws of flow through so short a tube of variable diameter,

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it seemed that this should not be accepted as an indication of drainage error in the Bingham instrument without confirmation.

5. OTHER POSSIBLE CAUSES OF THE LOW VALUES OF $\frac{\mu}{\gamma t}$

Various possible causes for the low values of the ordinates of Fig. 2, at very high viscosities, seemed worthy of consideration. The error could not be due to the contraction of the oil after leaving the outlet tube of the Scott viscometer, because the error is most marked when tests are run at temperatures lower than room temperature. A drainage error in the Scott viscometer would lower the average head, increase t and thus decrease the ordinate, but it seemed unlikely that the error would be appreciable in an instrument having such a large, shallow container. The tests for drainage error were therefore made on a Saybolt Universal viscometer tube.

After making a run in the usual way, vacuum was quickly applied to the bottom of the outlet tube to draw out all the oil except that clinging to the walls. The maximum film thickness, calculated from the difference in weight of the dry and wet container, was 0.011 centimeter, with an oil of 5000 seconds viscosity. Further tests were made by filling the container to the level reached at the end of a normal run, 8.8 cm below the overflow rim, and applying a vacuum as before. The results showed that only 8 per cent of the oil which adhered to the walls in the first series of tests, was above the 8.8 centimeter level, so that the average film thickness on the walls exposed during a normal run would be only 0.0009 centimeter. Although this value does not strictly apply to the Scott viscometer, it seems safe to regard the film thickness as inappreciable in comparison with the 9 centimeter diameter of this instrument.

6. INDICATIONS OF DRAINAGE ERROR OF BINGHAM INSTRUMENT BY COMPARISON WITH SAYBOLT AND REDWOOD VISCOMETERS

Figs. 3 and 4 show calibration diagrams for Saybolt Universal viscometer No. 109. Fig. 3, like Fig. 2, shows too low an ordinate at high viscosities, while in Fig. 4 values of the abscissas near the

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bottom of the graph are too large. Figs. 5 to 7 show Higgins diagrams for the Redwood No. 1, Redwood Admiralty, and Saybolt Furol viscometers, respectively. It will be noted that they all show the same signs of drainage error in the Bingham instrument, or some cause producing the same effect, so that it may be concluded that the evidence noted in Fig. 2 is not due to any peculiarity of the Scott viscometer.

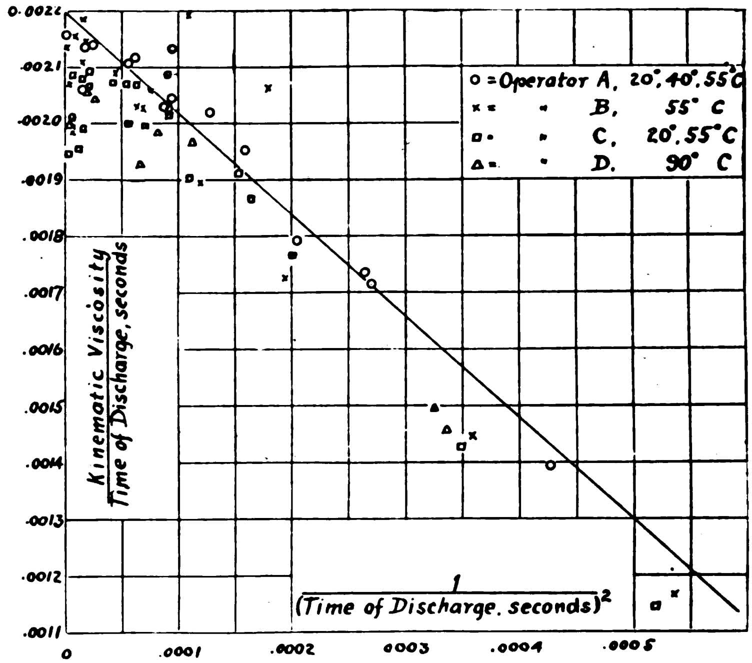

FIG. 3. Higgins diagram for Saybolt Universal viscometer No. 109

Fig. 8 shows another method of indicating the drainage error and was suggested by a diagram of Higgins.[15] The upper graph, from the work of Higgins, shows the relation between the time of flow of the Redwood viscometer and the kinematic viscosities obtained with a viscometer similar to that used by Thorpe and Rodger. The lower graph shows results obtained in calibrating

[15] W. F. Higgins, Collected Researches, National Physical Laboratory, *11*, p. 12; 1914.

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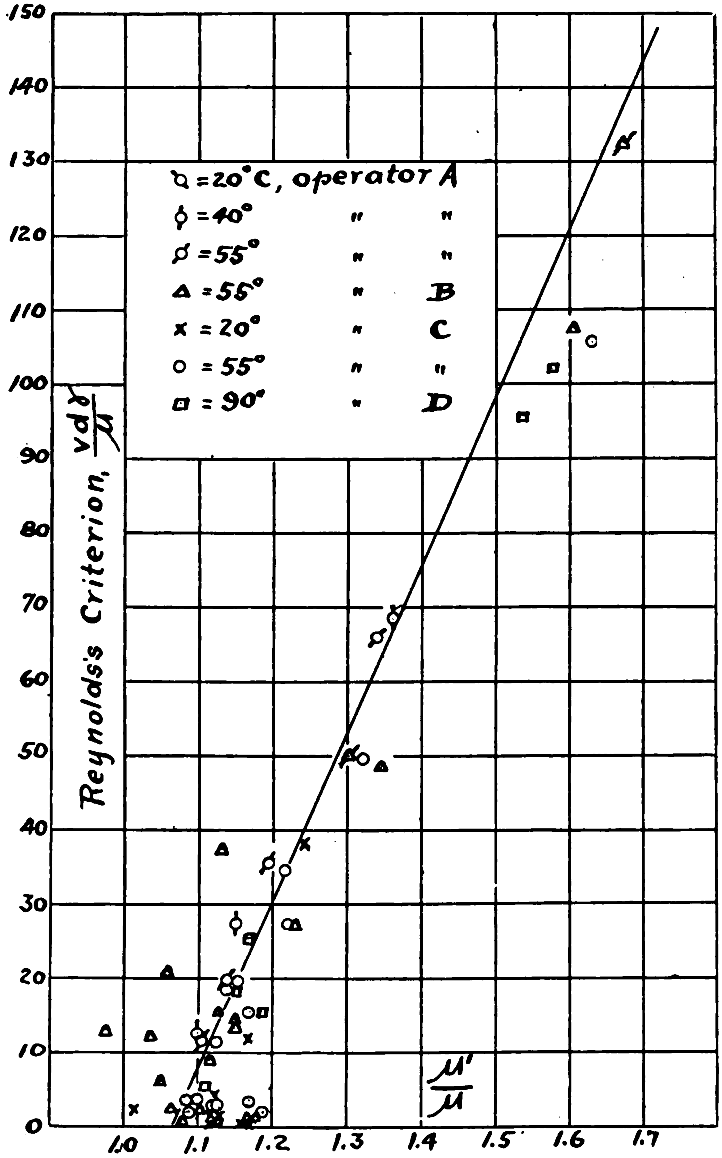

FIG. 4. Herschel diagram for Saybolt Universal viscometer No. 109

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the Redwood viscometer at the Bureau of Standards,[16] the kinematic viscosities being obtained by an instrument of the Bingham type. The curve at the lower end of the graph is due to kinetic energy, but the rest of the graph should be straight, since the equa-

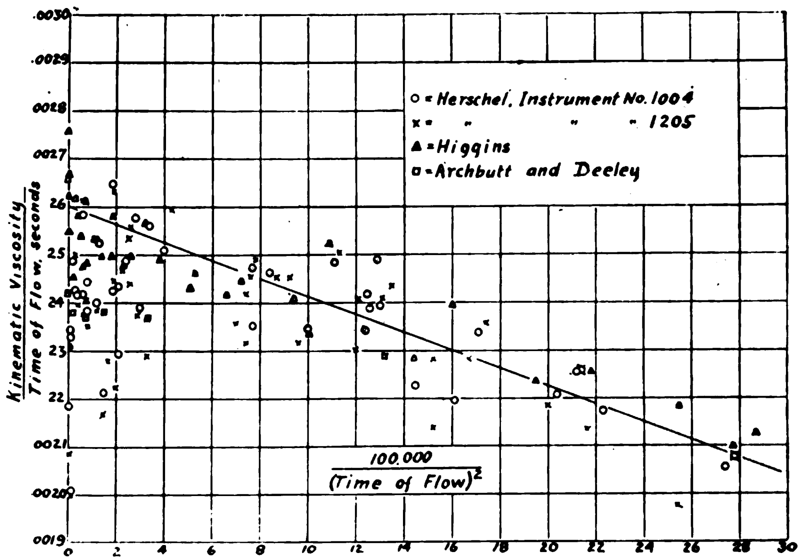

FIG. 5. Higgins diagram for Redwood viscometer

tion for the Redwood viscometer is of the same form as equation (6). The upper part of the line from Higgins tests is straight, but the lower line bends downward at the upper end. This difference in the two lines shows that the drainage error of Higgins's instrument with cylindrical bulbs, is less than the drainage error due to the specially shaped bulb in the Bingham instrument.

7. CALIBRATION OF BULB OF BINGHAM INSTRUMENT BY OILS OF VARIOUS VISCOSITIES

In order to get a more direct and accurate measurement of the drainage error, a special jacketed pipette was built with a bulb of the same size and shape as in the Bingham viscometer used in the work above recorded. There was no capillary and the rate of dis-

[16] Winslow H. Herschel, B. S. Technologic Paper No. 210; 1922.

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charge could be regulated by a stop cock. The volume discharged could be readily determined from the density of the oil and the weight discharged.

Since the viscosity, by equation (4), is proportional to C, or inversely proportional to the volume discharged, the true viscosity will be to the apparent viscosity (as calculated from a value of C

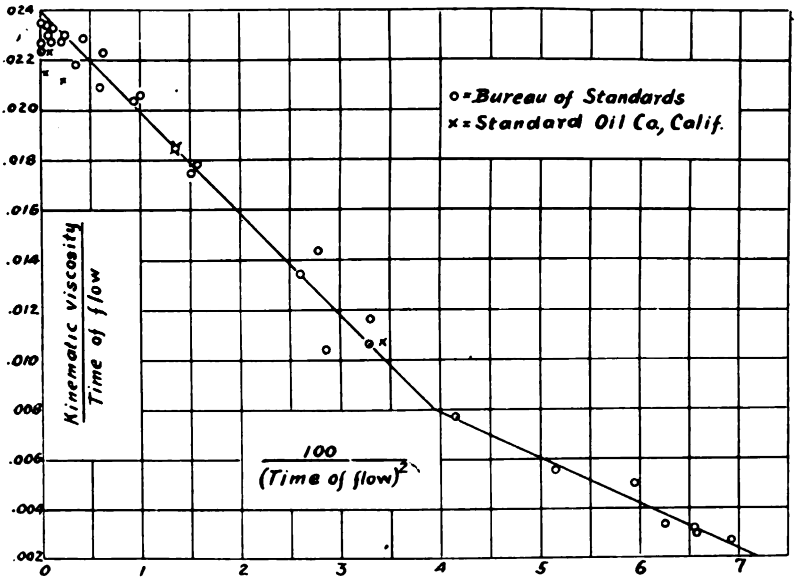

FIG. 6. Higgins diagram for Redwood viscometer, Admiralty type

determined by calibration with water) as the volume of water discharged from the bulb is to the volume of oil. The capacity of the bulb when calibrated with water was 4.232 cubic centimeters.

Table 5 gives results of calibration of the bulb with oils of various viscosities, the correction factor being the ratio of capacity of the bulb to discharge water and oil.

Fig. 9, from data of table 5, may be used in estimating the drainage error in a given test, or in selecting a time of flow so that the error will be negligible. The numbers on points indicate the correction factors, and lines of equal factors have been drawn. Data has been added from the work of Bingham and Young[17]

[17] E. C. Bingham and H. L. Young, paper presented before the Birmingham, Ala., meeting, A. C. S.; April, 1922.

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TABLE 5. *Factors to correct for drainage error of Bingham viscometer*

Corrected viscosity	Time	Factor	Time	Factor	Time	Factor	Time	Factor	Time	Factor
Poises	Sec.		Sec.		Sec.		Sec.		Sec.	
.1278	463	1.000	......	......	......	......	......	......	......	......
.242	58	1.003	255	1.000	......	......	......	......	......	......
.430	80	1.005	312	1.001	900	1.000	......	......	......	......
.722	87	1.007	400	1.000	......	......	......	......	......	......
1.247	103	1.012	240	1.007	420	1.000	......	......	......	......
2.140	150	1.015	212	1.010	585	1.007	1524	1.003	......	......
3.642	296	1.012	600	1.011	1280	1.007	1070	1.009	2332	1.005
6.473	300	1.013	420	1.012	1181	1.007	......	......	......	......
9.560	343	1.018	846	1.010	1491	1.008	......	......	......	......
12.160	744	1.015	1168	1.011	1680	1.010	2862	1.006	......	......
17.960	728	1.019	1351	1.012	2700	1.010	......	......	......	......

showing a somewhat smaller drainage error, due it is believed to the use of a larger bulb.

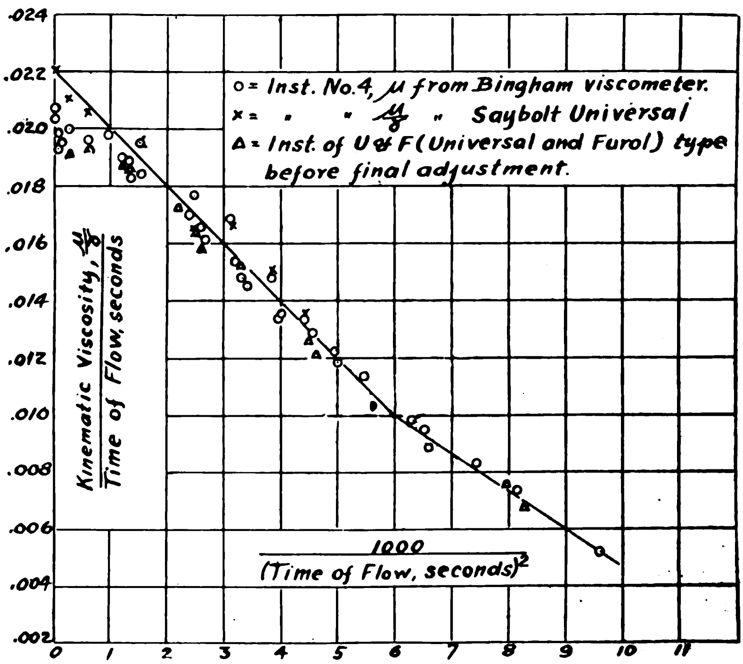

FIG. 7. Higgins diagram for Saybolt Furol viscometer

8. METHODS OF AVOIDING DRAINAGE ERROR

Drainage error may be avoided by using torsional viscometers, but these instruments are outside the scope of this paper. In the Lidstone viscometer, previously referred to, the liquid to be tested rests upon mercury, and the separation of the two liquids would

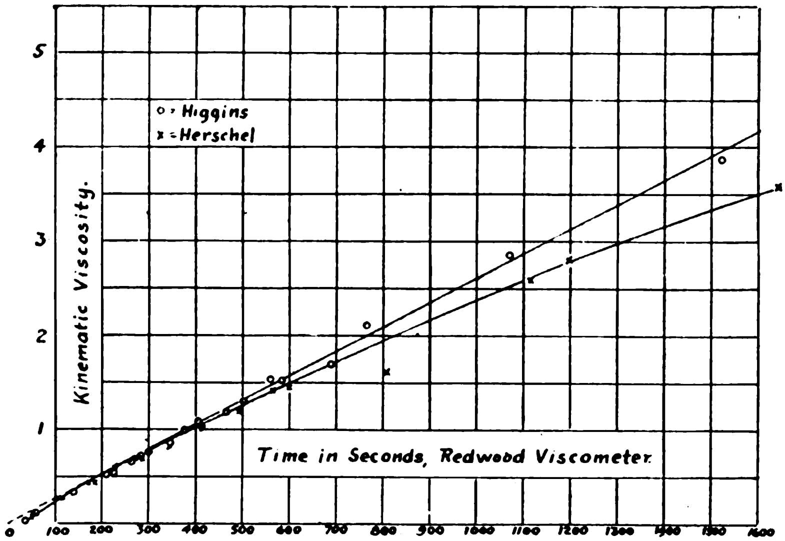

FIG. 8. Relation of kinematic viscosity to time of flow of Redwood viscometer

apparently offer considerable difficulty. In technical instruments for oil, of which the Saybolt Universal and Furol may be taken as examples, there is no drainage error, but there is another error, due to the contraction of the oil after it leaves the outlet tube, which may be called the cooling error. Barbey avoids both the drainage and the cooling errors by using a constant hydrostatic head, and running a test for the constant time of ten minutes. The receiving flask is heated in the bath to the nominal temperature of test, before reading of the volume discharged is taken, and results are expressed as "fluidity" or the rate of discharge in cubic centimeters per hour.

Bingham[18] suggests that the drainage error may be avoided in his instrument if the flow begins in the neighborhood of the trap

[18] E. C. Bingham, Fluidity and Plasticity, p. 66; 1922.

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opening, for then a certain amount of liquid may flow into the measuring bulb after the record of time has begun, and this will tend to offset the effect of any liquid left in the bulb at the end of the time of flow.

The Ubbelohde form of Ostwald instrument has, as ordinarily used, the disadvantage that it is necessary to refill it before running a check test, but there is no drainage error. This advantage and that of ease in running a retest may both be retained by the

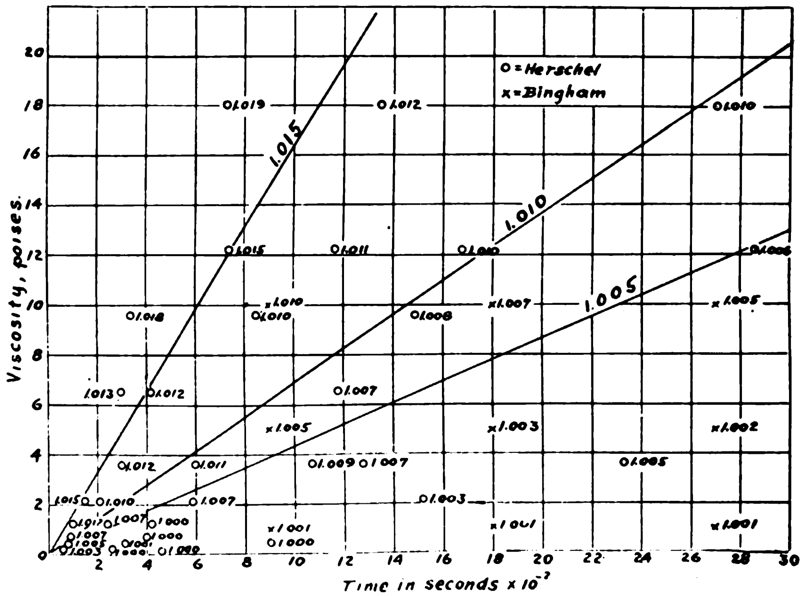

FIG. 9. Correction factors for Bingham viscometer

following slight changes. Referring to Fig. 1, graduate bulb K with marks similar to B and D. Fill from A to H, keeping the bulb K dry, and empty the trap. Apply air pressure on the left limb and make the first run into the dry bulb. Apply air pressure on the right limb and make the second run into bulb C which will be wet, and the third run will be into bulb K which will also be wet. The difference in the time for the first and third runs will be due to drainage error.

It should be noted that there is normally a difference in time between a run from left to right and one from right to left, so that

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the difference in time between the first and second runs would not usually be due entirely to the drainage error. The drainage error could however be calculated as the per cent difference between the viscosities as calculated from the first and second runs, the instrumental constant h' allowing for the difference in time of flow in the two directions when the volume is measured both times in a wet bulb.

9. CONCLUSIONS

1. The drainage error is negligible only for comparatively light oils when a sufficiently long time is taken for the discharge. A diagram is given to show the relation between viscosity, time of flow and per cent drainage error.

2. The drainage error may be avoided by having the trap on the opposite limb from the bulb which measures the volume discharged.

APPENDIX. COMPARISON OF VISCOSITIES AS OBTAINED BY THE SCOTT VISCOMETER, AND BY THE PLUMMET METHOD OF BASSECHES[19]

The investigation to be described was undertaken in the hope of explaining the difficulty in calibrating the Scott viscometer, afterwards shown to be due to drainage error of the Bingham instrument used in determining the viscosity of the calibrating liquids. While the results are somewhat discordant, this may be attributed largely to the rapid change of viscosity with the temperature in the case of such highly viscous substances. Even with lubricating oils, an error of one per cent in viscosity, will be caused by an error of 0.2 to 0.1 °C, depending upon the viscosity and upon the source of the crude oil.

While the tests were inconclusive as far as proving or disproving that there was a serious drainage error in the Bingham instrument, it is believed that a description of the Basseches method will be of interest since this method seems to be unsurpassed as a very rapid though approximate method of measuring the consistency of very viscous or plastic substances.

Basseches method is as follows. The liquid or soft plastic substance to be tested is placed in a shallow dish on a stand under a

[19] Based upon unpublished manuscript of J. L. Basseches, formerly of the Bureau of Standards.

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plummet hung from the arm of a chemical balance the beam of which can be lifted. The plummet is of cylindrical shape with a conical base, the juncture of cylinder and cone forming the line to which the plummet is immersed by raising the stand. The beam is then lifted and if necessary the stand is readjusted so that the pointer comes to the first division on the right end of the scale. A weight is placed on the pan of the balance, causing the plummet to be pulled out of the material under test, and the time required for the pointer to traverse five scale divisions is taken by means of a stop watch. The pan also contains the counter-poise for the plummet. As will appear later, this operation must be repeated with a different weight on the pan, in order to find both of the numerical values determined by the test.

It will be assumed that two properties or groups of properties are to be measured, one of which, viscosity, causes a resistance in proportion to the velocity, or inversely as the time, while the second group of properties causes a resistance which is independent of the velocity. Let W_v be that portion of the weight required to overcome viscosity, and W_s the portion required to overcome the constant resistance. Then

$$W = W_v + W_s \tag{7}$$

Basseches regarded W_s as a measure of surface tension only, believing that the "yield value" of plastic materials is only a form of surface tension manifested in the adsorbed liquid films on the surfaces of the solid particles. Without attempting to decide this matter, W_s will be considered here as a measure of all other resistances than viscosity, which may include surface tension, yield value and friction of the balance. Part of the weight must also be expended in lifting the film which adheres to the plummet.

If all the weight were exerted in overcoming viscosity, t would be inversely proportional to W, or for two trials,

$$W_1 t_1 = W_2 t_2 = \text{constant} = V \tag{8}$$

where W_1 and W_2 are the weights used in the two trials, and t_1 and t_2 are the corresponding times, the units employed being grams and seconds. But since surface tension, or some similar resistance must also be overcome, the net weight expended in overcoming

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viscosity, and which must be inserted in equation (8) is $W-W_s$ so that the equation becomes

$$(W_1 - W_s)\,t_1 = (W_2 - W_s)\,t_2 = V \qquad (9)$$

from which

$$W_s = \frac{W_2 t_2 - W_1 t_1}{t_2 - t_1} \qquad (10)$$

After W_s has been found from equation (10), V may be found from equation (9).

Two terms which are used to indicate the consistency of printing inks are "tack" or "tackiness" and "length" and Basseches proposed that VW_s should be used as a measure of the former, and $\frac{V}{W_s}$ as a measure of the latter.

Since V is proportional to the viscosity in poises, if the liquid is truly viscous, this method may be used to determine viscosity if the plummet is calibrated with a liquid of known viscosity. If μ is the viscosity of the calibrating liquid in poises, then

$$\mu = KV \qquad (11)$$

where K is an instrumental constant.

For example a sample of sodium silicate gave the following results at 22°C (71.°6F) using the 3 centimeter plummet.

W in grams	t in seconds
1.874	6.0
1.904	5.0

Then $W_s = \frac{11.244 - 9.520}{1.0} = 1.724$ and by equation (9), $V = 0.90$. Since under the same conditions castor oil gave $W_s = 0.88$ and $V = 0.78$, and since the viscosity of castor oil at 22°C is about 8.34 poises, from equation (11), $K = \frac{8.34}{0.78} = 10.7$

In a comparison of the plummet method using another sized plummet, with the Scott viscometer, and with the falling ball method,[20] the results were obtained as shown in Table 6 and Fig. 10.

[20] S. E. Sheppard, Jour. Ind. and Eng. Chem., 9, p. 523; 1917; W. H. Gibson and L. M. Jacobs, Jour. Chem. Soc. London, p. 473; May, 1920.

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The viscosity by plummet was calculated from equation (11) using a value of 65.7 for K as obtained with blown rape seed oil at 30°C.

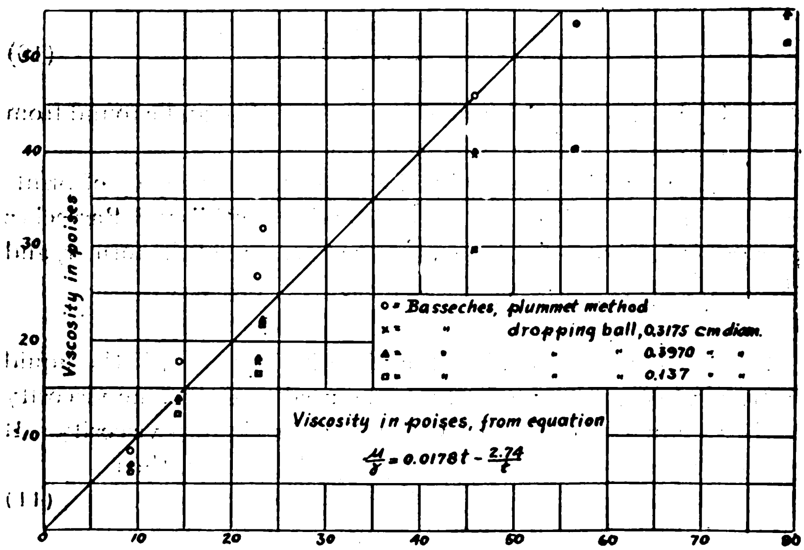

FIG. 10. Comparison of different methods of measuring viscosity

TABLE 6. *Comparison of different methods of measuring viscosity*

Liquid	Temp.	V	Basseches: Absolute viscosity, poises. Plummet	Basseches: Dropping ball of diameter in cm. .3175	.3970	.137	Herschel: Viscosity in poises by equation (6)
	°C	Sec/g					
Castor oil	23	.128	8.4	6.7	6.9	6.3	9.3
Burnt litho No. 1	30	.270	17.7	13.5	13.8	12.3	14.4
Burnt litho No. 1	23	.408	26.8	17.9	18.1	16.5	22.7
Rosin oil	30	.487	31.9	22.2	22.3	21.8	23.3
Blown rape	30	.698	45.8	39.6	39.9	39.6	45.8
Rosin oil	23	.817	53.6	38.4	37.9	40.3	56.6
Blown rape	23	1.15	75.5	54.2	54.6	51.4	78.9
Half tone varnish	23	5.00	328.0	......	......	......	....

BUREAU OF STANDARDS, WASHINGTON, D. C.

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SPECIFICATIONS FOR BOURDON TUBE PRESSURE GAGES FOR AIR, STEAM, AND WATER PRESSURES, AND FOR USE AS REFERENCE STANDARDS

BY
FREDERICK J. SCHLINK

This constitutes the second and concluding installment of specifications for recording thermometers and pressure gages begun in the September number of this JOURNAL. The specifications now presented cover pressure gages for ordinary service, and those for reference or test purposes. It is found desirable to differentiate the construction and accuracy requirements of these two types of gages in a number of respects, though common commercial practice has been to make the difference between them mainly a matter of richer appearance and of more detailed individual marking of graduations in the case of reference or test gages.

The other more important differences from common practice in these specifications are itemized in the introductory note preceding the first installment of the specifications, in the September number of this JOURNAL, to which reference should be made also for information as to the practicability and the use hitherto made of these specifications, the rationale of the statement of tolerance adopted, and the rather surprising precision that may be expected of good Bourdon tube gages under controlled conditions of use.

SPECIFICATIONS FOR BOURDON SINGLE TUBE PRESSURE GAGES FOR STATIONARY SERVICE: AIR, STEAM, WATER

1. GENERAL

a. *Size.* The gage diameter as given in the order will be construed to refer to the available or effective diameter, that is, in the usual case, the smallest diameter of the bezel ring.

b. *Capacity.* The limit of graduation or capacity referred to in the order is the actual maximum working limit of operation of the gages, and the construction shall be of such rigidity and the parts so stressed, as to carry pressures approximating but

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not exceeding the capacity pressure named, for an indefinite period if necessary, without permanent set beyond the limits of the tolerance hereinafter provided.

c. *Minimum Graduation.* The minimum graduation as called for in the order will require that at all parts of the graduated scale, the pressure equivalent of a single graduation interval shall be the same, and will be construed to prohibit the anomalous number and arrangement of graduations commonly and erroneously provided on pressure gages between the zero graduation and the first major graduation; provided, however, that the omission of all intermediate graduations between the zero and the first major graduation will be permitted; for example, in the case of a gage marked from zero to 80 lbs. by 1 lb., intermediate graduations between the zero and the 5 lb. graduation may be omitted at the option of the supplier.

d. *Marking.* Each gage shall bear the name of the actual manufacturer and a distinctive serial number, the latter to be legibly stamped, engraved or etched either upon the dial or the case flange, preferably the former.

e. *Items not Supplied.* The maker is not to supply gage cocks, nipples, or siphons.

f. *Delayed Rejection.* In case it is found expedient not to remove the dials and pointers of new gages at the time of the acceptance test, the purchaser reserves the right to return such gages at any later time for correction in case internal defects of such character are found as to make it certain that defective construction or workmanship existed in the gages at the time of delivery.

g. *Transportation Costs on Rejected Gages.* All costs of transportation on gages rejected for non-compliance with these specifications shall be borne by the supplier.

2. Mechanical Construction

a. *General.* These gages shall be of the bottom-connected type, of single-tube construction, with accurately cut and finished segmental gear and pinion. The linkwork shall be so arranged as to permit of general adjustment to correct multiplication and a secondary adjustment to correct for angularity of the linkwork connections.

b. *Mounting.* The mounting of the Bourdon tube and pointer mechanism is to be a single unit rigidly secured to the case. On gages of 500 lb. per sq. in. capacity or over, solder shall be employed only to secure tightness of joint.

c. *Permanence.* The materials and construction of these gages shall be such that a reasonable permanence of the initial adjustment will be maintained in service; gages which show any considerable secular change of the zero upon being placed in normal service or when given a reasonable oscillatory-pressure test within their rated capacity, will be returned for replacement. The materials are to be such that no corrosion interfering with the appearance or accuracy of the gage will occur in ordinary service.

d. *Linkwork—General.* Connections in the linkwork shall be either of the fork-and-pin type or shall consist of shouldered screws seated solidly in the members to which they are affixed. The length of the segmental gear and the properties of other elements of the movement, including the length of the hair-spring, shall be such that the pointer, in case of accidental overload, may move to the extent of 100% of the total graduated arc above the upper limit of graduation, and to the extent of 50% of the total graduated arc below the zero graduation, without in either case causing disengagement of the segmental gear and the pointer pinion.

e. *Links.* The connecting links used in the movement shall in no case be less than .05 in. in thickness, and bending of these links flatwise in order to obtain adjustment of the gage shall not be permitted.

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f. *Gearing.* The segmental gear and pinion shall have working faces not less than .09 in. in width.

g. *Fit and Finish.* All spindles and bearings are to be neatly and accurately fitted, and excessive side or end shake will be deemed sufficient cause for rejection. The surface of the pointer spindle upon which the pointer is applied shall be smoothly finished and true in order that in adjusting the gage the pointer may be readily set in the exact position desired.

h. *Hair-Spring.* These gages shall be equipped with the usual hair-spring to minimize the backlash error, this hair-spring to be of a non-rusting alloy. It shall be so tensioned that at any reading of the pointer on the dial, it will be effective in eliminating backlash by turning the pointer pinion into contact with the mating gear teeth. The coils of the hair-spring must not at any position of the pointer within the interval of graduation, be in contact with other coils or with the parts of the gage itself except at the points where the spring is secured to the movement frame and to the pointer spindle respectively.

i. *Holes for Mounting.* The mounting flange shall have three uniformly spaced holes for attaching the gage to its support, one hole to be located at the top of the gage diametrically opposite to the pipe connection.

j. *Glass and Bezel.* The cover glasses shall be clear and free from visible blemishes; the means of securing glass in bezel and bezel to case shall be such that the case will be substantially dust tight.

k. *Type of Case.* These gages shall be supplied in flanged iron case with nickel-plated close bezel ring without ogee flare[1] (similar to figure —, page —, — Co's. Bulletin.—)

3. Pointer, Dial, and Graduations

a. *Range of Graduation.* The total range of graduation shall cover an angular interval of not less than 270 degrees.

b. *Graduations—General.* The dials shall be of nickeled or silvered metal, with permanent black graduations, deeply incised; numerals to be neat, simple in outline, disposed horizontally and entirely within the circle of graduations. The minor graduations shall not be less than .01 inch nor more than .02 in. in width. The major graduations shall not be less than two nor more than three times as wide as the minor graduations. The major graduations shall be suitably distinguished also by an increase in length over the minor graduations.

c. *Length of Graduations.* The length of the minor graduation lines, or the difference between the radii of the outside and inside graduation circles, shall not exceed the mean graduation interval; that is to say, the radial length of the minor graduation lines shall not exceed the mean center to center interval between these lines.

d. *Diameter of Outside Circle.* The diameter of the outside graduation circle of the dial shall not be less than 90% of the smallest diameter of the bezel ring, referred to in specification a, Section 1.

e. *Attachment of Dial.* The dial screws shall fit the holes in the dial accurately, with a diametral clearance not exceeding .005 in. and they shall also be snugly fitted as to their threads so that they may not be readily jarred loose by vibration. The

[1] This requirement relating to appearance only, is subject to modification to meet individual taste or the need of uniformity with other equipment.

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manner of attaching the dial to the gage is to be such that it will be securely and permanently affixed.

f. *Pointer Clearance.* The pointer shall be so pivoted as to travel parallel to the dial, and the clearance between pointer and dial shall be not less than .03 in. and not greater than .09 in. at any point of the graduated scale.

g. *Shape and Length of Pointer.* The pointer shall be symmetrical about its longitudinal axis, shall be of neat appearance, and slender and sharp as to its index end. Its length shall be such that the index end will not entirely cross both circles limiting the graduation lines but will stop somewhat short of the outer circle, in order that, when desired, fractional intervals can be estimated.

h. *Stop Pins Omitted.* The gages furnished on this order are to be without zero stop pins.

4. ACCURACY

a. *Test Cycle—Tolerance.* These gages shall be calibrated and adjusted at a temperature of 80 deg. F.[2] approximately. The tolerance to be allowed in excess or deficiency shall not exceed one-half of one of the minimum graduations, as specified in the order, at any point within the range of graduation. The foregoing allowance of error is to include that due to friction and imperfect elasticity, the test being carried out, at the purchaser's option, without tapping or jarring the gage. The test to be performed will consist of a cyclic calibration through a range from zero to the upper limit of graduation and back to zero. Before the acceptance calibration is begun, the gage may be subjected to a pressure 25% in excess of its rated capacity, for a period not to exceed one hour, followed by a rest period not to exceed one-half hour.

SPECIFICATIONS FOR BOURDON TUBE PRESSURE GAGES TO BE USED AS REFERENCE STANDARDS FOR TEST PURPOSES

1. GENERAL

a. *Size.* The gage diameter as given in the order will be construed to refer to the available or effective diameter, that is, in the usual case, the smallest diameter of the bezel ring.

b. *Capacity.* The limit of graduation or capacity referred to in the order is the actual maximum working limit of operation of the gages and the construction shall be of such rigidity and the parts so stressed, as to carry pressures approximating but not exceeding the capacity pressure named, for an indefinite period if necessary, without permanent set beyond the limit set by the tolerance hereinafter provided.

c. *Marking.* Each gage shall bear the name of the actual manufacturer and a distinctive serial number, the latter to be legibly stamped, engraved, or etched upon the dial. The units in which and the temperature at which the gage is calibrated shall be clearly marked upon the dial in the following form: lb. per sq. in. at 80° F.

d. *Items not supplied.* The maker is not to supply gage cocks, nipples, or siphons.

e. *Transportation costs on rejected gages.* All costs of transportation on gages rejected for non-compliance with these specifications shall be borne by the supplier.

2. MECHANICAL CONSTRUCTION

a. *General.* These gages shall be of the bottom-connected type, of single-tube construction, with accurately cut and finished segmental gear and pinion. The link-

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work shall be so arranged as to permit of general adjustment to correct multiplication and a secondary adjustment to correct for angularity of the linkwork connections. In the gages as delivered, there shall remain not less than one fourth of the adjustment interval on each of the two adjustments referred to in this specification, to provide for carrying out readily such subsequent adjustments as may be required during the life of the instrument.

b. *Mounting*. The mounting of the Bourdon tube and pointer mechanism is to be a single unit rigidly secured to the case. On gages of 500 lb. per sq. in. capacity or over, solder shall be employed only to secure tightness of joint.

c. *Permanence*. The materials and construction of these gages shall be such that a reasonable permanence of the initial adjustment will be maintained in service; gages which show any considerable secular change of the zero upon being put into use or when given a reasonable oscillatory-pressure test within their rated capacity, will be returned for replacement.

d. *Linkwork-General*. Connections in the linkwork shall be either of the fork-and-pin typě or shall consist of shouldered screws seated solidly in the members to which they are affixed. Smaller diameters and bearing lengths of spindles than those employed in the usual service gages are desired in these test gages since the conditions of service favor a much lower rate of depreciation, due to the less severe usage and handling than in the case of ordinary pressure gages. The segmental gear and pinion shall have a minimum working face consistent with reasonable durability, not less than .04 in. and not more than 0.10 in. The length of the segmental gear and the properties of other elements of the movement, including the length of the hair-spring, shall be such that the pointer, in case of accidental overload, may move to the extent of 50% of the total graduated arc above the upper limit of graduation, and to the extent of 25% of the total graduated arc below the zero graduation, without in either case causing disengagement of the segmental gear and the pointer pinion.

e. *Links*. The connecting links used in the movement shall in no case be less than .05 in. in thickness, and bending of these links flatwise in order to obtain adjustment of the gage shall not be permitted.

f. *Fit and Finish*. All spindles and bearings are to be neatly and accurately fitted, and excessive side or end shake will be deemed sufficient cause for rejection. The surface of the pointer spindle upon which the pointer is applied shall be smoothly finished and true, in order that in adjusting the gage the pointer may be readily set in the exact position desired.

g. *Hair-Spring*. These gages shall be equipped with the usual hair-spring to minimize the backlash error, this hair-spring to be of a non-rusting alloy. It shall be so tensioned that at any reading of the pointer on the dial, it will be effective in eliminating backlash by turning the pointer pinion into contact with the mating gear teeth. The coils of the hair-spring shall lie substantially in a plane and must not at any position of the pointer within the interval of graduation, be in contact with other coils or with the parts of the gage itself except at the points where the spring is secured to the movement frame and to the pointer spindle respectively.

h. *Glass and Bezel*. The cover glasses shall be clear and free from visible blemishes; the means of securing glass in bezel and bezel to case shall be such that the case will be substantially dust tight.

i. *Type of Case*. No special details of case design are required, but the screw bezel construction will be given preference. Either brass or iron cases will be accept-

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able, but in any event the finish is to be of the best quality, with smooth surfaces, well japanned if of iron, and of neat appearance throughout. A narrow bezel ring without flare is preferred, and the glass should lie as close to the pointer as practicable.

3. Pointer, Dial, and Graduations

a. *Arc of Graduation.* The total range of graduation shall cover an angular interval of not less than 270°.

b. *Graduations—General.* The dials shall be of nickeled or silvered metal, with permanent black graduations, cleanly incised; numerals to be neat, simple in outline, disposed horizontally and entirely within the circle of graduations. The surface finish of the dial shall be such that clear marks and graduations can be made upon it with an ordinary lead pencil and that such marks can be easily erased if necessary. The graduations are not to exceed .01 in. in width and the major graduations shall be distinguished from the minor graduations not by an increase in width of line but by moderate extension of their length alone. The emphasizing and numbering of the graduations shall be upon a decimal basis and the fifth and tenth graduations rather than the second or fourth shall be emphasized.

c. *Length of Graduations.* The length of the minor graduation lines, that is, the difference between the radii of the outside and inside graduation circles, shall not exceed the mean graduation interval; that is to say, the radial length of the minor graduation lines shall not exceed the mean center to center interval between these lines.

d. *Diameter of Outside Circle.* The diameter of the outside graduation circle of the dial shall not be less than 90% of the smallest diameter of the bezel ring, referred to in specification a, Section 1.

e. *Attachment of Dial.* The dial shall be attached to the case or movement in such manner as to be securely and definitely fixed in position, independently of the friction under the heads of the holding-down screws, its accurate and permanent centering being assured either by accuracy of fit of the holding-down screws or by dowel pins.

f. *Pointer Clearance.* The pointer shall be so pivoted as to travel parallel to the dial, and the clearance between pointer and dial shall be not less than .03 in. and not greater than .05 in. at any point of the graduated scale.

g. *Shape and Length of Pointer.* The pointer shall be symmetrical about its longitudinal axis, shall be of simple design and neat appearance, and slender and sharp as to its index end. Its length shall be such that the index end will not entirely cross both circles limiting the graduation lines but will stop somewhat short of the outer circle, in order that when desired, fractional intervals can be estimated.

h. *Stop Pins Omitted.* The gages furnished on this order are to be without zero stop pins either on the dial or within the case, except that a special locking device capable of being disengaged at will may be provided in order to secure the gage against damage due to the shocks incident to shipment and handling.

4. Accuracy

Test Cycle—Tolerance. These gages shall be calibrated and adjusted at a temperature of 80° F.[2] approximately, the naming of this temperature being necessary in

[2] Subject to variation to suit special requirements.

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order to give definiteness to the meaning of the tolerance of error below. The tolerance to be allowed in excess or deficiency on the gages as delivered, shall not exceed one-fourth of one of the minimum graduations as specified in the order,—at any point within the range of graduation. The foregoing allowance of error is to include that due to friction and imperfect elasticity, the test being carried out, at the purchaser's option, without tapping or jarring the gage. The test to be performed will consist of a cyclic calibration through a range from zero to the upper limit of graduation and back to zero. Before the acceptance calibration is begun, the gage may be subjected to a pressure 15% in excess of its rated capacity, for a period not to exceed one hour, followed by a rest period not to exceed one-half hour.

AMERICAN ENGINEERING STANDARDS COMMITTEE,
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A METHOD OF MAINTAINING SMALL OBJECTS AT ANY TEMPERATURE BETWEEN −180°C AND +20°C

BY
P. P. Cioffi and L. S. Taylor

In the determination of the crystal structure of solid mercury by X-Ray analysis,[1] difficulty was encountered in the formation of ice, condensed from the atmosphere around the tube containing the mercury. Since the presence of the atmosphere was not otherwise objectionable, a method was devised which may readily be adapted to any small object within the temperature range between −180°C and +20°C, by which the temperature may be maintained constant for many hours to within about 4°. The method consists in surrounding the object to be cooled by a stream of cold dry air, while keeping any surfaces exposed to the free atmosphere above 0°C.

The cold dry air is obtained by evaporating liquid air in a Dewar flask (Fig. 1) by an electric heater immersed therein. The cold air is forced by the pressure thus created past the substance under examination through a vacuum heat-insulated delivery tube. The design of the delivery tube may be varied widely depending upon the nature of the problem. If visual observation is not necessary, the whole delivery tube may be of silvered glass, and may extend beyond the point where the measurements are taken, thus minimizing the expenditure of liquid air. Where visual observation is necessary or where, as in the case of X-Ray analysis no crystalline material other than the substance being analyzed can be permitted to be in the X-Ray beam, the part of the delivery tube surrounding the substance may be of clear glass. To reduce X-Ray absorption, the walls of the tube in the path of the beam should be pyrex or borosilicate glass drawn very thin. Absorption may be reduced still further if the sample is small and

[1] L. W. McKeehan and P. P. Cioffi. Phys. Rev. (2), *19*, 444–446; 1922.

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of streamline shape, by placing it just beyond the open end of the delivery tube. It is necessary that the cold air be conveyed across the substance as an unbroken stream, without mixing with the surrounding air, which is relatively warm and moist. The necessary conditions can be maintained for several diameters of the delivery tube beyond its open end by placing at this distance

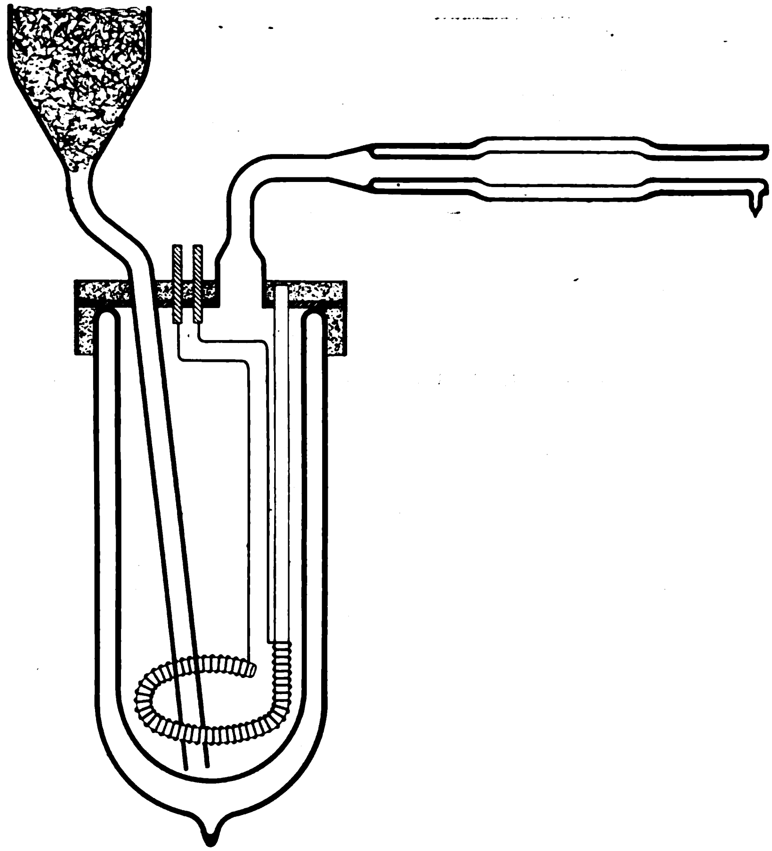

FIG. 1. Section of Complete Apparatus

the mouth of a tube connected to a rough vacuum pump of sufficient capacity (Fig. 2). This tube should have a diameter about twice that of the inner delivery tube, and a length about twice that of the gap. The connection to the vacuum pump must be warmed to prevent clogging by ice condensed

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from entrained air. Flaring the mouth of the delivery tube preserves the integrity of the stream, and prevents fringes of frost from building across the gap. The vacuum heat insulated delivery tube may be vertical but where a horizontal section is required, the construction of the right-angled double walled tube (Fig. 3) is a matter of some nicety. If tubes over

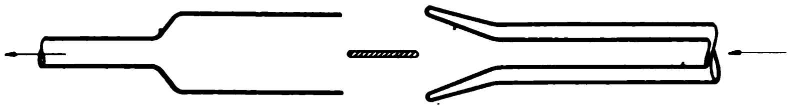

FIG. 2. Arrangement for Cooling in Open Air

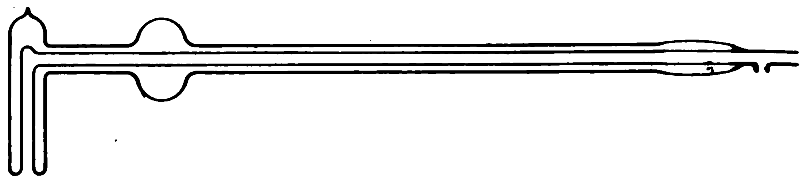

FIG. 3. Section of Long Horizontal Delivery Tube

25 cm long are required, flexibility of construction is necessary owing to the difference in expansion between the outer and inner walls. This may be obtained by including a flattened bulb section in the outer tube to serve as an expansion joint. The use of pyrex glass has here the additional advantage of reducing the difference in expansion between the two tubes. The heater is a coil of resistance wire with a low temperature coefficient of resistance. It is wound around a horizontal glass ring supported from the cover of the flask so as to hang near the bottom of the flask, and has its leads brought thru the cover. The cover makes airtight sealing-wax joints with the neck of the flask and with its connections, is conveniently made from a fibre disk suitably perforated, and is flanged on both sides by the addition of a short length of tubing. The Dewar flask requires frequent replenishment and occasional cleaning, both of which processes are facilitated by leaving it unsilvered. The flask is charged with liquid air by means of a funnel passing through the cover and reaching nearly to the bottom. This funnel is provided with a plug of glass wool for filtering out ice and solid carbon dioxide. In spite of this precaution particles of frozen impurities gradually accumulate and

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finally make it difficult to observe the liquid air level, so that it becomes necessary to interrupt the experiment and clean out the flask. This is most easily done by maintaining a small current thru the heater, while passing air thru the system.

Temperature regulation is effected by changing the rate of evaporation of the liquid air, i. e., by changing the current thru the heater, and the mean temperature may be varied over a range from just below room temperature to a minimum depending upon the dimensions and heat insulation of the delivery tube. The regulation being electric can easily be made automatic if constancy of temperature is essential. Where considerable work is to be done covering a wide range in temperature, it is advantageous to make a temperature calibration of the apparatus in terms of the heater current, replacing the sample with a resistance thermometer of about the same dimensions and thermal capacity. In special cases the sample itself may be used as a thermometer, or the temperatures may be measured during the experiments by a thermometer adjacent to the sample. In a particular case, with a tube about 50 cm long and 1 cm inside diameter, a temperature of —150°C was maintained at the outlet with an expenditure of 94 watts in the heater, and this required a supply of about 2 liters of liquid air per hour. The maintenance of a constant temperature required the recharging of the flask when the level of the liquid air reached to within 3 cm of the heater. At this rate of boiling the system required cleaning about once in six hours which was therefore the limit for continuous operation.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY
AND THE WESTERN ELECTRIC COMPANY, INCORPORATED
JULY 21, 1922.

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A TUNGSTEN FURNACE FOR EXPERIMENTS ON DISSOCIATION AND IONIZATION

BY
K. T. COMPTON

Requests have been received from several sources for a description of the tungsten furnaces in use in this laboratory for the investigation of radiating and ionizing potentials and for the excitation of spectra of dissociated diatomic gases. The accompanying sketch gives the details of construction of the furnace used by Dr. Duffendack in his work on low voltage arcs in hydrogen, nitrogen and iodine. The figure is drawn to scale.

The furnace is made of sheet tungsten, which may be obtained from the Elkon Works of the General Electric Company at Weehawken, New Jersey, in thicknesses down to 0.002 inch or less. A piece, cut to the right dimensions, is bent into the form of a cylindrical tube and clamped by the end pieces. Each of these consists of a small steel "napkin ring" fitting in the split rectangular steel block. The tungsten sheet is held between the "napkin ring" and the block. These blocks are mounted on water-cooled brass tubes, which serve as leads for the heating current. These leads pass into the surrounding glass or metal vessel through brass tubes which pass through the water-cooled brass end pieces (not shown). Short glass tubes surround the water-cooled leads for purposes of insulation. The tungsten tube furnace is encircled, at two or three points between the end blocks, by loops of fine tungsten wire drawn tight enough to prevent bulging of the tungsten sheet along the line of the cut. The central electrode is a straight length of 20 mil tungsten wire, welded to heavier molybdenum leads. By adjusting the length of this wire, it is possible to secure equal potential drops along the furnace and the filament, so that they act essentially as equipotential electrodes.

Although we have used steel clamps, molybdenum clamps would have been preferable, since the steel at the ring of contact melts

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and alloys with the tungsten before the highest temperatures possible with the furnace have been reached. Molybdenum blocks can be machined to required dimensions and can be secured from the Elkon Works of the General Electric Company at a very reasonable price. We first tried seamless brass tubing for the water-cooled leads, but these invariably spring leaks after continued use, causing the burning out of the tungsten electrodes. This occurred even when the tubing was thoroughly "tinned" with

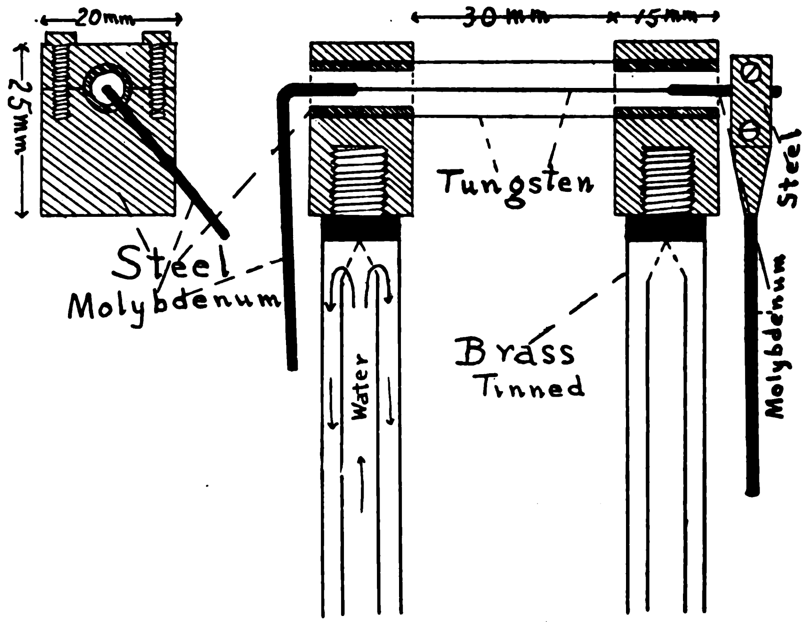

FIG. 1. Detail of Tungsten Furnace

solder, inside and out. With the outer tube bored out of solid brass rod and thoroughly "tinned," no appreciable leakage developed. The molybdenum leads to the filament were held in clamps and bent slightly so as to keep the filament under sufficient tension to prevent its sagging at high temperature. The temperatures of the furnace were estimated from its resistance, after making allowance for the resistance of the leads. The degree of dissociation of any gas in the furnace may be calculated by

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means of Nernst's equation of the "reaction isobar."[1] if the heat of dissociation is known and if the chemical constant is known[2] or can be calculated.[3]

The furnace of the dimensions shown above required about 200 amperes, with a potential drop of 6 volts, to reach its melting point. It reached a good white heat with 100 amperes. We used storage batteries, but it should be possible to use a transformer, being careful that the phases of the heating currents in the furnace and the filament were equal, so as to insure their equivalence to equipotential surfaces.

In addition to its use in studying low voltage arcs, it has been used to investigate conditions of spectral excitation, observations being made through the open end of the furnace. By mounting parallel plates, alternately, positively and negatively charged, just beyond the open end of the furnace, ions may be prevented from passing from the furnace into the region beyond these plates. Another plate placed in this ion-free region so as to receive radiation excited within the furnace acts as a photoelectric detector of this radiation. By this means Dr. Olmstead and the writer have proved the excitation of the first four and the convergence members of the Lyman series in atomic hydrogen at the successively higher voltages 10.1, 12.0, 12.6, 13.0, 13.6.[4]

Palmer Physical Laboratory,
Princeton, New Jersey.

[1] W. Nernst, "Theoretische Chemie."
[2] Nernst, loc. cit.
[3] Schames, Phys. Zeit. *21*, p. 41; 1920.
[4] Not yet published.

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NOTE ON THE ENERGY EXCHANGES IN THE FORMATION OF THE LATENT IMAGE OF A PHOTOGRAPHIC EMULSION[1]

By S. E. Sheppard and E. P. Wightman

A number of determinations have been made of the energy of light of different wave-lengths necessary to produce, on development, a blackening of unit density of various photographic plates; and also the energy necessary to produce the least perceptible visible blackening has been found for many commercial plates.

Not so many calculations have been made, however, of the energies involved in the formation of the latent image itself. Mees,[2] in 1915, gave certain calculations of Nutting as to the amount of energy necessary, after full development, to produce a deposit of unit density and from this deduced that "the energy incident on a grain during exposure may be sufficient to affect only one molecule in the grain, and the latent image," he said, "may be composed of grains in each of which, on the average, only one molecule has lost an electron by the action of light."

Lately, Volmer[3] made some calculations, based on the measurements of Leimbach,[4] of the ratio of altered to unaltered moles of

[1] Communication No. 151 from the Research Laboratory of the Eastman Kodak Company.

[2] C. E. K. Mees, J. Franklin Inst., *179*, p. 164; 1915. A recent recalculation of Nutting's value shows that it is probably much too small.

[3] M. Volmer, Phot. Korr. *58*, p. 226; 1921.

[4] G. Leimbach, Z. wiss. Phot. *7*, 181; 1909. A preliminary determination in this laboratory of a similar value gives an amount of energy of the same order of magnitude.

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silver bromide in a grain of a certain size. He used for this purpose, however, the energy of blue light, λ = about 450$\mu\mu$, which would produce a density of unity, and failed to take into account that only about ½ the total surface of the grain would be exposed. He assumed the grain to be spherical, whereas in most emulsions, grains of the size which he used for his calculations are tabular with an average thickness from ½ to 1/12 the mean width.

Instead of the energy used by Volmer for his calculations let us take Leimbach's value for the "Schwellenwert," that is, the visible threshold value after development for the Schleussner plate—upon which the former made his calculations.

For $\lambda = 450\mu\mu$ this is given as

$$\epsilon_1 = 10.5 \times 10^{-11} \frac{\text{watt}}{\text{cm}^2} \text{ sec.}$$

which in ergs is

$$= 10.5 \times 10^{-3} \text{ ergs/cm}^2$$

Assume, for convenience, that the grain has the shape of a square tablet with length of side $= 10^{-4}$ cm, and thickness $= 10^{-5}$ cm (this is much nearer the average size grain in photographic emulsions than that assumed by Volmer). The energy per quantum is

$$\epsilon_2 = h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-27} \times 3 \times 10^{-10}}{4.5 \times 10^{-5}} = 4.37 \times 10^{-12} \text{ ergs.}$$

Then from Einstein's photochemical law, the number of molecules affected by the light will be

$$n = \frac{\epsilon_1}{h\nu} = \frac{\epsilon_1}{\epsilon_2} = \frac{10.5 \times 10^{-11}}{4.37 \times 10^{-12}} = 24.0 \text{ molecules per grain.}$$

The total number of molecules per grain may be calculated in either of two ways, from its weight or from the lattice structure. Both give the same result. Take the first case and assume the above values for width and thickness and the density of silver bromide crystals to be 6.47. The volume is then

$$v = 10^{-4} \times 10^{-4} \times 10^{-5} = 10^{-13} \text{ cc.}$$

and its weight is,

$$M = 6.47 \times 10^{-13} \text{ g}$$

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Dividing by the weight of one molecule of AgBr, $m = 1.66 \times 10^{-24} \times 188 = 3.12 \times 10^{-22}$, we get for the number of molecules of AgBr in the grain

$$N = 2.07 \times 10^{9}$$

On the other hand if we calculate from the lattice structure and use Wilsey's value[5] of the lattice constant, 2.89 A. U. $= 2.89 \times 10^{-8}$ cm, then

$$N = \frac{10^{-4}}{2 \times 2.89 \times 10^{-8}} \cdot \frac{10^{-4}}{2.89 \times 10^{-8}} \cdot \frac{10^{-5}}{2.89 \times 10^{-8}} = 2.06 \times 10^{9}$$ molecules per grain

The number of molecules in the surface on which the light falls is of course obtained by omitting the last factor, the thickness, in the preceding equation, and is

$$N_s = 6 \times 10^{6} \text{ molecules per surface exposed}$$

The ratio of altered to unaltered molecules is therefore

$$n : N = 1 : 8.6 \times 10^{8}$$

and

$$n : N_s = 1 : 2.5 \times 10^{5}$$

This ratio is different, of course, for every different sized grain but for this particular grain it is still too large,[6] that is, if we consider, not the visible threshold value, but the actual threshold value, which could only be detected by the high power microscope.

As a matter of fact, work in the last few years on crystal structure in general, and on the crystal structure of AgBr in particular, by means of X-ray analysis; and on the applications of the quantum theory of light, have changed our views of the mechanism of the formation of the latent image.

It is certain that we cannot regard the grain of AgBr any longer as a heterogeneous clumping of molecules, and hence in speaking of "molecules per grain" all we mean is the number of pairs of (Ag^+, Br^-) that occur in the grain.

[5] R. B. Wilsey, Phil. Mag., *13*, p. 262; 1921.

[6] If just 1 molecule is affected by 1 quantum of energy then $n = \frac{\epsilon_1}{\epsilon_2} = 1$, and the ratio for the exposed surface becomes $n : N_s = 1 : 6 \times 10^{6}$. It might be of interest in this connection to note that R. Gans (Ann. d. Physik. *52*, p. 291; 1917) has detected quantities of light smaller than the quantum, but it is not known if these would be sufficient to affect a photographic plate.

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The mechanism itself of latent image formation, we shall not discuss here but would refer to two papers[7] by one of us recently published.

Let us consider the possible energy changes which can occur when light falls on the grain:

1. *Energy of separation of Ag^+ from Br^- in the lattice structure:*
$$(Ag^+, Br^-) = Ag^+ + Br^- - \Delta_1 \text{ cal.} \tag{1}$$
2. *Energy of the loss of an electron by the Br^- ion:*
$$Br^- - \Theta = Br \text{ (liquid)} + \Delta_2 \text{ cal.} \tag{2}$$
3. *Energy of the addition of an electron to the Ag^+ion:*
$$Ag^+ + \Theta = Ag \text{ (atom, solid state)} - \Delta_3 \text{ cal.} \tag{3}$$
4. *Energy of formation of Br_2 molecule:*
$$2\,Br = Br_2\text{(gas)} + \Delta_4 \text{ cal.} \tag{4}$$
5. *Energy of formation of Br addition product with gelatin:*
$$\text{Gelatin} + Br = (\text{Gelatin. } Br) + \Delta_5 \text{ cal.} \tag{5}$$
6. *Energy of formation of Br substitution product with gelatin:*
$$\text{Gelatin} + 2\,Br = (R\ .\ Br) + HBr - \Delta_6 \text{ cal.} \tag{6}$$

NOTE: The gelatin$-$Br substitution product would probably be an endothermic compound but the formation of HBr is exothermic to a greater extent, hence Δ_6 cal. is positive.

7. *Energy of loss of an electron by the silver molecule or atom:*
$$Ag - \Theta = Ag^+ + \Delta_7 \text{ cal.} \tag{7}$$

Of these seven possibilities the first five appear to be the most probable; the seventh may play an important catalytic part, however, in that silver metallic silver atoms may supply the electron for reversing equations (3) and (2), and assist the local surface concentration of the latent image. Under such conditions, therefore, the previous energy changes become of less relative importance. We shall return to the evaluation of these energy changes in a later paper.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.,
JULY 31, 1922.

[7] S. E. Sheppard, Silver nucleus theory of development, Phot. Korr, Jan.-April, 1922, p. 76. The action of soluble iodides and cyanides on the photographic emulsion, Phot. J. *60*, p. 88; 1922.

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THE OPTICAL CONSTANTS OF ISOLATED TELLURIUM CRYSTALS

By George Dewey Van Dyke

This paper is a brief report on one phase of an extended research which is being conducted by Dr. L. P. Sieg of the State University of Iowa, on the optical properties of small metallic crystals. The information obtained from such investigations should reveal much in regard to the crystalline structure of the pure metal and possibly throw some light on the atomic structure of metals in general.

Skinner[1] has published results on the optical constants of a selenium crystal, finding the crystal doubly refracting, the index when the principal axis of the crystal is parallel to the plane of incidence being higher than that of any other known element. Skinner employed the ordinary polarimetric method of determining the elliptic constants. Weld[2] has made a fairly extensive study of the elliptic constants of light reflected from a small selenium crystal using the "Crystelliptometer," a special apparatus designed by himself. The same apparatus was used by the writer in obtaining the optical constants of tellurium crystals. The crystals were made by sublimation at the State University of Iowa, by Dr. A. R. Fortsch.

Drude[3] has developed the theory of metallic reflection for crystalline bodies.

Let Φ_1, Δ_1, Ψ_1, be respectively, the angle of incidence, phase difference, and azimuth when the principal crystal axis is parallel to the plane of incidence;

Φ_2, Δ_2, Ψ_2 be, respectively, the angle of incidence, phase difference, and azimuth when the principal axis of the crystal is perpendicular to the plane of incidence, and

n_1, n_2, k_1, k_2, ρ_1, ρ_2, be the indices of refraction, absorption coefficients, and coefficients of reflection, respectively, for the

[1] Skinner, Phys. Rev., N. S., *9*, p. 148; 1917.

[2] Weld, *Journal of the Optical Society of America*, *6*, p. 67; 1922.

[3] Drude, Ann. d. Physik, *34*, p. 529; *32*, p. 616, 1887; *35*, p. 518; 1888.

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two principal positions of the crystal axis, n_1, referring, for example, to the index of refraction when the electric vector agrees with the principal axis.

Then

$$\frac{\sqrt{a}}{\cos\phi_1}-\sqrt{\gamma}\cos\phi_1=\frac{\cos 2\Psi_1+i\sin 2\Psi_1\sin\Delta_1}{1\ -\ \sin 2\Psi_1\cos\Delta_1}$$

$$\frac{\sqrt{\gamma}}{\cos\phi_2}-\sqrt{a}\cos\phi_2=\frac{\cos 2\Psi_2+i\sin 2\Psi_2\sin\Delta_2}{1\ -\ \sin 2\Psi_2\cos\Delta_2}$$

where a and λ are auxiliary complex constants employed in determining the final optical constants. The letter i represents $\sqrt{-1}$.

Let the values of a and γ as given by the above equations be

$$a=a_{11}+i\,a_{12} \qquad\qquad \gamma=a_{31}+i\,a_{32}$$

Then if we define X by $\tan X=\frac{a_{12}}{a_{11}}$,

and E by $\tan E=\frac{a_{32}}{a_{31}}$, we have

$$k_1=\tan\frac{X}{2}, \qquad\qquad k_2=\tan\frac{E}{2},$$

$$n_1^2=\frac{2\sin\frac{X}{2}\cos^3\frac{X}{2}}{a_{12}}, \qquad\qquad n_2^2=\frac{2\sin\frac{E}{2}\cos^3\frac{E}{2}}{a_{32}}$$

$$\rho_1=\frac{n_1^2(1+k_1^2)+1-2n_1}{n_1^2(1+k_1^2)+1+2n_1},\ \text{and} \qquad\qquad \rho_2=\frac{n_2^2(1+k_2^2)+1-2n_2}{n_2^2(1+k_2^2)+1+2n_2}$$

It should be noted that the values of a and γ do not depend upon the angles of incidence, hence for each wave-length we are able to determine a and γ for several angles of incidence and average the results.

The optical constants for five different wave-lengths in the visible spectrum were obtained. The results of the investigation (c. f. Figs. 1, 2, and 3) show that a crystal of tellurium is doubly refracting, the index of refraction being higher in the horizontal than in the vertical position. The reflecting power of the crystal

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Table of Elliptic Constants

λ ($\mu\mu$)	Φ	Δ_1	$2\Psi_1$	Δ_2	$2\Psi_2$
437	60°	141°30′	74°	136°	66°48′
437	65°	126°6′	73°48′	119°24′	64°12′
437	70°	108°54′	72°12′	103°54′	60°36′
437	75°	89°42′	69°48′	78°54′	61°
450	60°	127°30′	69°12′	120°24′	63°24′
450	65°	117°12′	68°48′	113°36′	61°12′
450	70°	99°30′	66°24′	86°30′	62°48′
508	60°	136°30′	73°36′	133°24′	61°12′
508	65°	122°6′	67°24′	119°48′	58°24′
508	70°	108°36′	62°24′	105°	56°
508	75°	84°12′	61°48′	76°36′	55°12′
590	60°	137°36′	71°	133°	67°
590	65°	132°12′	65°12′	117°18′	60°24′
590	70°	108°6′	64°48′	100°36′	57°12′
590	75°	89°18′	59°24′	80°6′	54°24′
650	60°	127°54′	68°48′	115°12′	57°24′
650	65°	115°30′	65°	102°24′	55°48′
650	70°	102°24′	62°	92°42′	55°36′

Averages of Crystal Constants

λ	α	γ
437	$.0528+.0503i$	$.0513+.0811i$
450	$.0751+.0918i$	$.0800+.1232i$
508	$.0493+.0738i$	$.0391+.0967i$
590	$.0416+.0688i$	$.0426+.0896i$
650	$.0659+.1034i$	$.0641+.1532i$

Table of Optical Constants

Crystal Axis Horizontal				Crystal Axis Vertical		
λ	n_1	κ_1	ρ_1	n_2	κ_2	ρ_2
437	3.44	.399	.33	2.52	.551	.30
450	2.62	.466	.28	2.29	.543	.26
508	2.96	.535	.31	2.57	.672	.30
590	3.07	.563	.34	2.68	.632	.30
650	2.50	.548	.29	2.05	.666	.27

varies very little for the two positions and for the various wave-lengths used, a fact verified by Sieg[4] by direct measurement of the reflecting power. The curve obtained from plotting the index of refraction against the wave-length shows a distinct

[4] Unpublished results.

minimum and maximum over the range investigated. This characteristic is also shown in Skinner's[5] results for selenium,

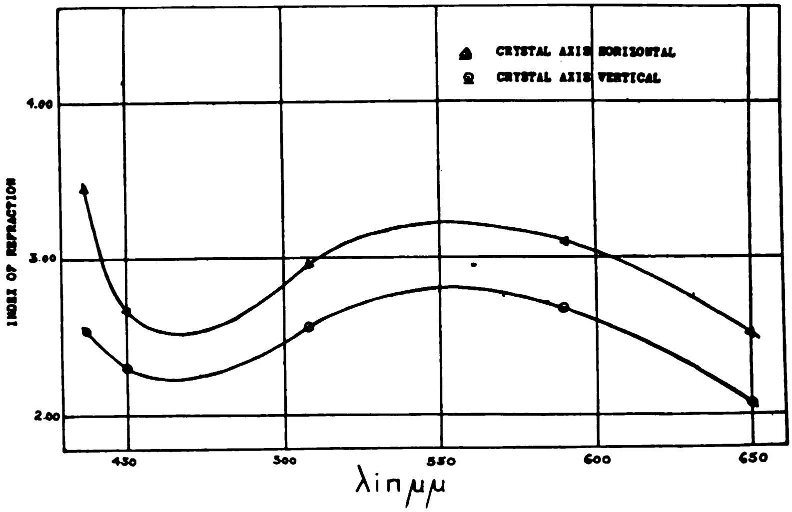

FIG. 1. *Principal indices of refraction of an isolated tellurium crystal for various wave-lengths.*

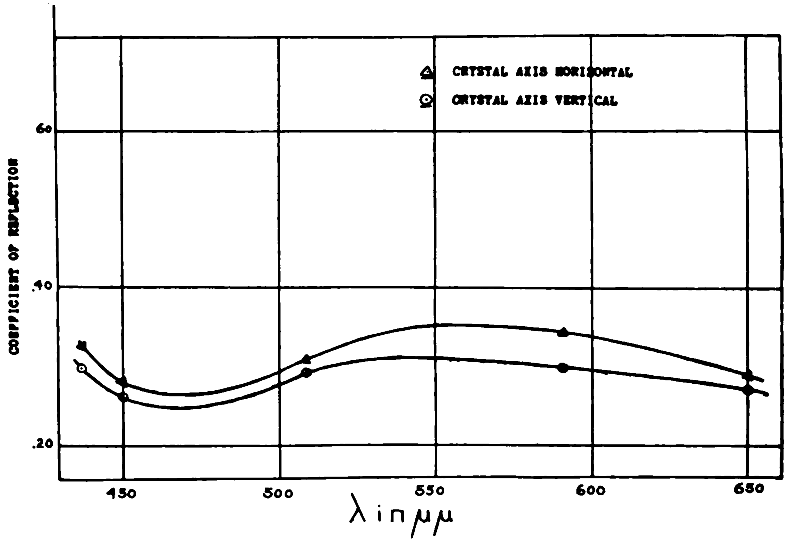

FIG. 2. *Principal reflecting powers of an isolated tellurium crystal for various wave-lengths.*

[5] Loc. cit.

although to a far less degree. The peculiarity found in the index in the horizontal position was not evident in tellurium, the two indices following almost similar curves.

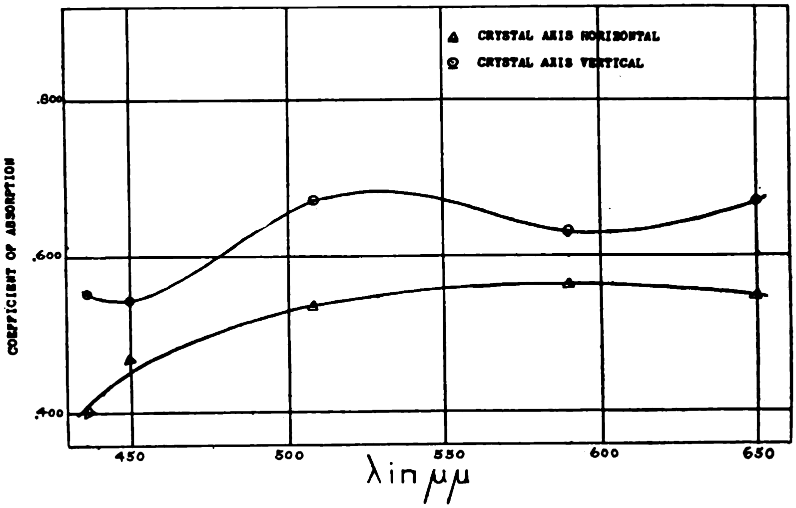

FIG. 3. *Variation in the coefficients of absorption of a crystal of tellurium with a variation in the wave-length.*

The above problem was suggested to me by Professor L. P. Sieg, to whom I wish to express my appreciation.

STATE UNIVERSITY OF IOWA,
AUGUST, 1922.

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THE STUDY OF VISUAL PROCESSES BY MEANS OF MOMENTARY RETINAL SHADOWS

By Frederick W. Ellis

In 1901 I made an extensive series of experiments in the study of certain visual phenomena,[1] and I have recently supplemented and extended these researches with others in which new methods have been employed. It is the object of this paper to call attention to what seems to be a new method of studying several visual phenomena.

In 1891 Jastrow and Moorehouse[2] published an article describing an interesting phenomenon which had been brought to Jastrow's attention by Münsterberg some time before. When a slender horizontal rod is moved vertically in front of a rapidly revolving disc having a white or colored sector of greater luminosity than that of the rest of the disc, a number of horizontal bands are seen. Jastrow and Moorehouse studied this phenomenon under varying conditions, but gave no explanation of it, and spoke of it as an optical illusion. The word illusion does not apply to it, for it is due to well known optical and physiological laws. It demonstrates in a rather novel way the persistence and independence of visual perceptions, and affords a method of measuring the duration of the perceptions. The bands are due to the perception, and the persistence of the perception of the retinal shadow of the rod which is formed at each passage of the sector behind it. The word shadow is used here to denote an area of diminished luminosity irrespective of color.

In order to obtain a clear understanding of the mode of production of the bands it is best to use a white sector on a black ground. The sector should be comparatively narrow; one of twenty degrees is sufficiently broad. When the disc revolves rapidly enough to eliminate flicker it assumes a uniform dark gray shade. If a

[1] Studies in the Physiology and Psychology of Visual Sensations and Perceptions, American Journal of Physiology, Vol. 5.

[2] American Journal of Psychology, Vol. 4.

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slender black rod or strip, held horizontally, is now moved up and down at the rate of about once in a second, a number of very distinct black bands are seen on the dark gray background. If the motor to which the disc is attached is stopped, and then slowly turned by hand while the rod is held in its horizontal position, it is easy to study the way in which the retinal shadows are formed. It is only necessary to recall that what we see is the projection of the retinal image. As the white sector sweeps behind the rod a variable portion of it is covered by the rod at successive instants, and the retinal image or shadow of the rod is formed in successive sections. When the disc revolves rapidly the continuously varying parts of the shadow are perceived as a whole, owing to the persistence of vision. At each revolution of the disc the sector encounters the moving rod in another position in the field, and, consequently, another shadow is formed. As the persistence of vision is a considerable part of a second, several bands are seen at the same time. The number of the bands perceived depends upon the rate of revolution of the disc, and the distance between them is governed by the rate at which the rod is moved across the disc.

Figs. 1 to 9 illustrate the mode of formation of the bands. The oblique lines of the first four figures represent the fusion color or shade of the rapidly revolving disc; the vertical shading of the remaining figures indicates that the disc is at rest, or moving so slowly that it can be readily followed with the eye. The heavy horizontal line, extending beyond the disc on either side, corresponds to the horizontal rod, and the other broad lines, which are parallel to this line and terminate at the border of the disc, represent the shadows that we are studying. In Fig. 1 the rod is not moving; in Fig. 2 the rod is moving downward, and in Fig. 3 upward. Fig. 4 illustrates the appearance when the rod moves downward more rapidly than in Fig. 2. No sectors are seen in these four figures, for, in accordance with Talbot's law, the color of the sector is lost in the uniform fusion color of sector and ground. It is evident that none of these four figures represents a real retinal image. We may regard the four as psychic images due to the composite blending in the brain of

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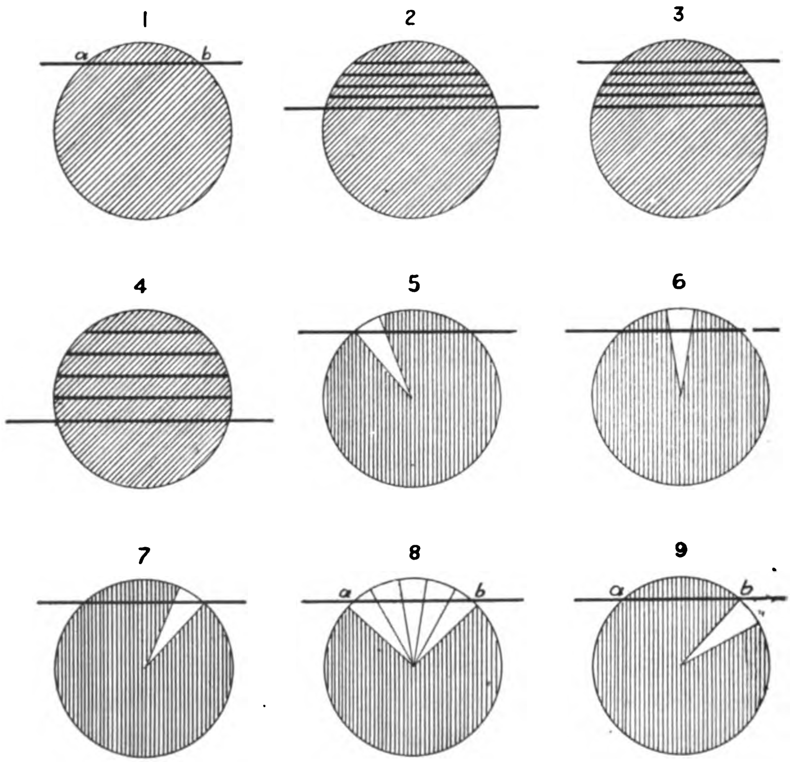
successive retinal images. The formation of these images is easily explained by referring to the remaining figures.

If the disc revolves clockwise, Fig. 5 will be an instantaneous view of it when the sector has just passed behind the rod. That part of the image of the rod which crosses the sector obliquely divides the sector into two parts separated by a retinal shadow,



or an area of less luminosity. Fig. 6 is a retinal image at a little later period, and the next figure one still later at the instant when the sector begins to emerge from behind the rod. In these instantaneous views it will be seen that the retinal shadows occupy varying positions with respect to the radial axis of the sector. Fig. 8 represents a composite of five different images in which it will be seen that the retinal shadows lie on the same straight line. Owing to the combination and persistence of the

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changing impressions a shadow seems to extend from *a* to *b*. In Fig. 1 the projected shadow coincides with the stationary rod. If the rod is moved vertically downward while the disc completes its revolution the shadow will be seen behind the rod. When the disc makes the next revolution another subjective shadow is formed, and is seen directly behind the rod, while the first shadow still persists, and forms the second band. If the first shadow persists during five revolutions, the rod and bands will have the appearance shown in Fig. 2, and the band farthest from the rod will be the one first formed. It is evident from Fig. 9 that no retinal shadow will be formed while the sector continues its revolution from *b* back to *a*, and, if the rod moves during this period, it will give rise to another band when the sector again passes behind it.

The dark bands can also be shown very distinctly when a part of the revolving disc is a colored sector of considerable luminosity, and the ground is black. The distinctness of the bands varies with the brightness of the colored sector. When the colored disc is placed on a gray disc of the same luminosity the bands almost entirely disappear. Based upon this fact is a method of color photometry first employed by Rivers[3] in measuring the luminosity of colored papers by comparing them with a series of gray papers. He found that the bands nearly or completely disappeared when the gray had the same luminosity as the sector, and the results that he obtained in this way were nearly the same as those in a parallel series of experiments with the flicker method.

A complementary experiment with a slit in a sheet of black cardboard demonstrates in a striking way the mode of formation of the bands which we have been studying, and confirms my previous statements regarding their origin. The slit should be 4 or 5 mm wide, and of any convenient length. When the slit is held horizontally, in front of the revolving disc, and the screen is moved up and down about once in a second, a series of light bands corresponding to the dark ones of the previous experi-

[3] The Journal of Physiology, *22*, 1897.

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ments, and having a color resembling that of the sector, is seen. The colored sector as it passes behind the slit illuminates its different parts at successive instants, and these partial views are blended into an impression of the entire slit by the persistence of vision. The point to emphasize in this connection is that these images of the slit correspond to the dark bands of the rod experiment, and prove that the explanation of the origin of these bands given above is the correct one, and that they may be illustrated by the same figures.

It is possible to demonstrate another set of subjective shadows of different origin from that of those which we have described, if the revolving disc is placed in a darkened room, and is illuminated by a single lamp placed at a little distance in front of it. When the white sector is behind the rod, under these conditions, a real shadow of part of the rod is cast on the sector at each instant while it passes behind the rod. This comparatively short objective shadow sweeps across the disc in a nearly straight line, and gives rise to the subjective impression of a dark band extending across the disc, when the disc is regarded from one side. When the position of the observer is favorable two sets of bands may be seen; those due to retinal shadows, and those caused by the projection of a real shadow on the sector. It is evident that both kinds of bands may be similarly explained by the figures already given. The bands due to the retinal shadows of the rod are the more important, as they may be obtained under a greater variety of conditions, and are the ones best adapted for utilization in experimental work.

It is evident that the experiments which have been described afford the basis for two methods of measuring the duration of visual perceptions. In this instance, as in a former paper, I am impelled to emphasize the difference between visual perceptions and sensations. This is especially necessary in studying the subject of visual persistence. Vision is always accompanied by a perception, and it is the duration of the perception that ordinarily interests us. The duration of a visual sensation apart from a perception cannot be measured, for it has no existence. It seems preferable, therefore, to speak of the duration of visual perceptions

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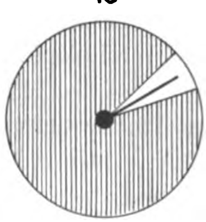
rather than that of sensations. An added reason for this distinction is that I have shown, in the paper which I published in 1901, that the duration of visual perceptions may be decidedly affected by psychological conditions, but comparatively little by the intensity of the luminous stimulus. This would appear to indicate that the persistence of vision is largely due to the activity of the higher visual centers. It is evident that, if we could count the number of the bands in the rod experiment, knowing the rate of revolution of the disc, we could estimate their duration. This cannot be done satisfactorily, but it is quite possible to measure the duration of the perceptions by employing another method. When one end of the rod or strip which casts the shadow is held so that the end coincides with the prolongation of the axis of the disc, and the other end is made to describe a circle slowly, the bands are radial. If the rate of revolution of the rod is properly adjusted the bands can be made to cover the entire disc. The slowest rate of revolution of the rod which enables us to complete the circle of bands is the measure of the duration of the perceptions, and I have found this to be about four-tenths of a second. The essential part of the mechanism employed for this purpose is illustrated by Fig. 10.

10



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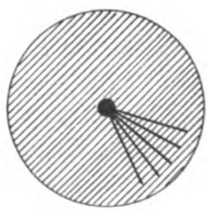

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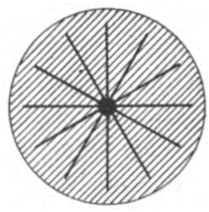

The small black circle in the center of Fig. 10 represents a section of a cylinder to which one end of a slender rod is attached, which is seen projecting radially over the middle of the sector. The axis of the cylinder is the prolongation of that of the disc. The cylinder with the rod may be revolved independently of the disc about the common axis, and in either direction. The rate of revolution of the rod may be varied continuously within the required limits. If the rod revolves slowly, clockwise, while

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the disc revolves with sufficient rapidity, the subjective bands will be radial, as in Fig. 11. If the disc revolves clockwise, the first radial line in this figure represents the rod, and the remaining four radii the dark bands which we are studying. When the rod revolves more rapidly, the bands may be made to extend completely around the circle, as is shown in the last figure. In Fig. 11 the band farthest from the rod is the one formed first, and is about to disappear. As the rate of revolution of the rod increases the oldest band recedes from the rod until it is diametrically opposite. When the bands cover more than a half circle, and the rod revolves still faster, the disappearing band approaches the rod on the other side of the disc until it finally coincides with it, as in Fig. 12; the rate of revolution of the rod is then the measure of the duration of the band. When the rate of revolution of the rod remains constant, the number of bands seen will depend on the rate of revolution of the disc. If the disc revolves at constant speed the radial bands will become more widely separated when the rate of revolution of the rod increases, but it will also be noted that a greater number of bands will be seen when they extend entirely around the disc. This depends upon the fact that the mind can note and retain a larger number of the visual impressions which give rise to the bands when they are widely separated in space. As I have noted before, the duration of visual perceptions is chiefly affected by psychological factors.

The successful use of the apparatus just described requires some practice and concentration of attention, and, in using it, it is especially important to resist the tendency to follow the revolutions of the rod with the eye. As the perception becomes less vivid before it entirely disappears, an allowance has to be made for this fact in determining when the shadows are distributed entirely around the disc. It requires but little experience with the apparatus to demonstrate that with 150 revolutions of the rod per minute, and probably with a rate somewhat less, the perception of the shadow persists during an entire revolution. This indicates a duration of at least four-tenths of a second. This result is in accordance with those obtained with other

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methods, and recorded by me in a former paper. It has been amply proven that this duration is much longer than has been generally supposed, and that it depends comparatively little on the intensity of the stimulus. The factors that abridge the duration are especially those that interfere with attention. By the use in a dark room of the apparatus to which I have given the name of Prism Stroboscope, which was described in the paper which I published in 1901, it has been shown that luminous impressions may last three-fourths of a second. In a dark room all distracting influences are at a minimum, and there is the greatest possible contrast between the image and the field. In the shadow experiments just described the whole field of vision is receiving simultaneously many impressions which cannot be ignored, and which lessen contrast.

The attentive study of the results of the band experiment sheds light on a number of important visual processes. Visual inhibition is most clearly demonstrated in the bands. In the case of a white sector on a black ground the fusion color is a dark gray, but the radial bands are nearly or completely black. By reducing the angular extent of the sector it is possible to lessen the duration of the retinal shadow to a thousandth of a second, or even to a period much less than that. When this is done the resulting gray is very dark, but the bands are distinct and much darker. Unless the illumination is very intense there will be complete fusion of the black and white without flicker, nevertheless, the withdrawal from the light of the sector of the shaded portion of the retina for a thousandth of a second is sufficient to inhibit the previous white constituent of the gray, which persists in the unshaded region with nearly undiminished intensity. In other words the previous gray sensation is inhibited, and a perception of a black band corresponding to the retinal shadow remains. This inhibition of a sensation can be shown even more strikingly by using a colored sector of considerable brightness on a colored ground of feeble luminosity. The brighter color will then be inhibited, and the bands will have the color of the ground, or one closely approaching it.

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Another condition, known in physiology as the refractory state, plays a part in the production of the bands. It should be remembered that when the retinal shadow lasts only the thousandth of a second, the perception of the shadow continues for four-tenths of a second, or possibly a little longer. I have even succeeded in obtaining with a very narrow white sector distinct bands due to shadows that lasted less than three-thousandths of a second, and the duration of the bands was as given above. It is, therefore, easy to prove that under appropriate conditions the perception may outlast the retinal shadow more than a thousand times. During the period of four-tenths of a second the part of the retina upon which the shadow fell was stimulated at least twelve times by the repeated passages of the sector without evoking any sensation in the corresponding part of the visual field. The perception of the shadow made that part of the visual field refractory to stimulation by the reflected light of the sector. This indicates that in the complicated processes of vision sensations which interfere with the clearness of perception may be suppressed or prevented. Visual perceptions are due to the activity of the higher visual centers, and it is in these centers that it is most probable that inhibition and the refractory state occur. The evidence of the band experiments is in accord with that furnished by my early experiments, and authorizes the same conclusions.

If we replace the revolving rod with a black disc having a narrow radial slit, and view the other disc as it revolves rapidly, through the slit, gray or colored bands will be seen if the object disc is well illuminated. These bands correspond to the dark ones due to the revolving rod, and their duration may be estimated in the way that has been described. This duration seems to be slightly longer than with the rod, or about one-half of a second. A white or colored sector may be used on a black or colored ground. The distinctness of the bands depends on the size of the slit and of the sector, the difference in the luminosity of the sector and ground, and the illumination of the object-disc. In order to insure sufficient illumination, the object-disc, revolving at the rate of about 1,800 revolutions a minute, should be placed

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20 or 30 cm behind the slit, and near a window, or a lamp in a darkened room if artificial illumination is desired. It is desirable that the slit should be adjustable, and it should be viewed at a distance of one or several meters. This makes a convenient arrangement for measuring the persistence of vision, and for proving that this persistence is very constant under widely varying conditions.

FROM THE AUTHOR'S LABORATORY
MONSON, MASSACHUSETTS.

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A COMPARISON OF THE FECHNER AND MUNSELL SCALES OF LUMINOUS SENSATION VALUE

By Elliot Q. Adams

A critical study of the Munsell scale of (luminous sensation) value has been made by Priest, Gibson, and McNicholas.[1] Their results "verify in a remarkable manner the consistency of the Munsell values for different hues . . . considering the uncertainties of heterochromatic photometry which were necessarily involved in Munsell's work." They establish that "the squares of the Munsell value numbers are directly proportional to the reflection of sunlight"—as might be expected from the construction of the Munsell photometer, which employs an Aubert diaphragm,[2]—and object that "the implication that values, read directly as the diagonal of the shutter, are proportional to sensation in the sense of Fechner's law is quite wrong." Since it is well known[3] that Fechner's law is only an approximation to the law relating brightness and sensation, and an equation which constitutes a closer approximation has recently been published,[4] it will be well to examine, in the light of this newly-found relation, both the Fechner and the Munsell (or Stefanini)[5] scales.

Before proceeding to a quantitative comparison, it will be well to point out anew[6] the difference between the range of valid-

[1] I. G. Priest, K. S. Gibson, and H. J. McNicholas, Technological Paper No. 167, Bur. Stds. (Sept. 1920) "An Examination of the Munsell Color System. I. Spectral and Total Reflection and the Munsell Scale of Value."

[2] H. Aubert, Grundzüge der physiologischen Optik, pp. 489, 547. Leipzig, W. Englemann, 1876.

[3] And is explicitly conceded by the authors on page 29 of the reference in footnote 1.

[4] E. Q. Adams and P. W. Cobb, J. Exp. Psych., *5*, pp. 39-45; 1922.

[5] A. Stefanini, Nuov. Cim. (3) *22*, p. 97; 1887 (for sound). Atti della R. Acc. Lucc. di Sc. Lett. ed Arti., *25*, pp. 383-400 (for light and weight). The Stefanini formula is the special case of Plateau's formula $E=kR^{\epsilon}$, in which $\epsilon=0.5$.

[6] See for example, A. Elsas, Wundt's Philos. Stud. *4*, 162-79 (1888), also E. B. Titchener, Experimental Psychology, Instructor's Manual, Quantitative, pp. 210-32. §29. New York, Macmillan, 1905. The method of equal sense distances: historical and critical.

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ity of Weber's law, and the range over which Fechner's sensation law holds. This comparison can be made more concrete by the analogy of the measurement of current by a tangent galvanometer[7] provided with an assortment of shunts. Such an instrument gives greatest percentage precision when the scale reading is in the neighborhood of 45°, and the percentage precision at that (or any other constant) angular deflection is the same whenever the shunt is selected so that the current measured gives that deflection. Hence with a sufficiently varied supply of shunts the percentage precision may be made constant over the range covered by the shunts, that is:

$$\Delta I = \frac{\Delta s}{b} I = cI \tag{1}$$

where ΔI is the least detectable increase in current, I; b and c constants, and Δs the least perceptible change in scale reading, s. At the same time *with any one shunt the* scale reading, s, is related to the current by the equation:

$$I = I_m \tan s \tag{2}$$

where I_m is the current which for that shunt gives a scale reading of 45° and hence maximum precision. Yet the equation obtained by integrating (1), in the form $Ids = bdI$;

$$s = b \ln I \tag{3}$$

does not hold at all, for over the range of validity of (1), the scale reading is always near to 45°. Now if I represent light intensity and s sensation, (1) and (3) become Weber's and Fechner's Laws respectively.

The analogy is, of course, not perfect, for while equation (1) in the form

$$\Delta B = cB \tag{4}$$

holds over a considerable range of brightness, the equation[8]

[7] This instrument is chosen because its deflections follow a simple mathematical law and remain finite as the current is indefinitely increased. For the sake of continuous variation of the shunt resistance a slide wire might be used.

[8] Equation (2) of the article referred to in footnote (4), based on the assumption that visual impressions are transmitted along each fiber of the optic nerve by a series of impulses whose effect depends only on their frequency,—the All-or-None hypothesis of Keith Lucas, "The Conduction of the Nervous Impulse," p. 9, London; 1917. Cf. also L. T. Troland, J. Opt. Soc. Am., *4*, p. 160; 1920.

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which has been found to relate sensation to brightness ***at constant adaptation***, (the analog of galvanometer readings with a given shunt) is not of the form of equation (2), but is

$$s=\frac{B}{B+k} \tag{5}$$

where k is a constant dependent on the state of adaptation, being equal to the brightness at which photometric precision is a maximum (for the given state of adaptation).

s expresses sensation as a fraction of the maximum possible range of sensation; if it is desired to express it in other units, a coefficient, a, must be inserted in equation (5). Similarly if sensation is to be measured from any other point of reference than the sensation corresponding to (physically) complete blackness, a term, s_o, for that sensation, must be introduced into equation (5) which thus becomes

$$s=s_0+a\,\frac{B}{B+k} \tag{6}$$

The relation between sensation and brightness thus assumes an infinite number of forms according to the value of k. Since in equation (6) s becomes independent of B at both limits, $k \doteq 0$ and $k \doteq \infty$, the law of variation in these limiting cases may be found as follows. For $k \doteq 0$, i.e., for dark adaptation

$$s=s_o+a\,\frac{B}{B+k}=s_o+a\,\frac{(\;1\;)}{1+k/B}\doteq s_o+a(1-\frac{k}{B})=(s_o+a)-\frac{ak}{B} \tag{7}$$

while for adaptation to infinite brightness, $k \doteq \infty$,

$$s=s_o+a\,\frac{B}{B+k}\doteq s_o+\left(\frac{a}{k}\right)B \tag{8}$$

that is, the sensation approaches in the first case a linear function of the reciprocal of brightness, in the second a linear function of brightness itself.

Many of the other formulas which have been found empirically are special cases of the Plateau equation

$$s=k'B^{\epsilon} \tag{9}$$

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where ϵ is an exponent lying between 0 and 1, and k' a constant.

If ϵ be given the value $\frac{1}{2}$, equation (9) becomes the Stefanini[5] equation, on which the Munsell scale is based:

$$s = k'\sqrt{B} \tag{10}$$

If sensation be not measured from the sensation produced by the physical absence of light, a term, s_o, must be added in equation (9) as in the case of equations (5) and (6), giving

$$s = s_o + k'B^{\epsilon} \tag{11}$$

If ϵ be made equal to unity, this becomes the Merkel[9] proportionality law:

$$s = s_o + k'B \tag{12}$$

which is identical with (8).

When ϵ approaches O, (11) becomes

$$s = s_o + k'e^{\epsilon lnB} \doteq s_o + k' \, (1 + \epsilon lnB) = (s_o + k') + k'\epsilon lnB \tag{13}$$

that is, Fechner's law, which is, therefore, the other limit of the Plateau equation.

It will be noted that equations (5) and (9),—and hence also the equations derived from them,—retain their form if B be measured in other units, provided the appropriate changes are made in the constants of the equations. Since for any constant illumination the relation between sensation and test object *albedo*[10] (the brightness relative to that of a perfect diffusely reflecting surface similarly illuminated) will depend upon the albedo of the surroundings but will be independent of the illumination,—within the range of brightnesses for which Weber's law holds,—equations (4) to (13) may be made the same for the relation between sensation, s, and test object *albedo*, as for the relation between sensation and brightness. In what follows the symbol, B, and the term "albedo" will both signify *test object albedo*.

Priest, Gibson, and McNicholas give in their Fig. 16, (on p. 31), a comparison of the Merkel, Munsell, and Fechner scales, made to coincide for Nos. 1 and 9 of the Munsell scale.

[9] Julius Merkel, Wundt's Philos. Stud., *4*, pp. 117-60, 251-91, 541-94, 1888; *5*, pp. 245-91, 499-557, 1889; *10*, pp. 140-59, 203-48, 369-92, 507-22 especially p. 517; 1894.

[10] This term is used habitually by astronomers in stating the reflecting powers of the planets.

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TABLE 1. *Relation Between Sensation and Test Object Albedo, According to Various Scales and Theories*

Sensation	Scale No.	Albedo according to:								
		Munsell Scale		Merkel $\epsilon=1$	Fechner $\epsilon=0$	Plateau $\epsilon=2.3$	Adams and Cobb			
Arbitrary Scale		meas. by Priest *et al*	Theoretical (Stefanini)				k=0	k=0.01	k=0.09	k=0.50
0.0	0	....	.00	......	.0058	....	.0089	.0078	0	......
0.1	1	.018	.01	≡.01	≡.01	≡.01	≡.01	≡.01	≡.01	≡.01
0.2	2	.041	.04	.11	.0173	.0578	.0114	.0128	.0225	.0522
0.3	3	.090	.09	.21	.0300	.126	.0133	.0165	.0386	.102
0.4	4	.161	.16	.31	.0520	.211	.0159	.0215	.0600	.162
0.5	5	.234	.25	.41	.0900	.310	.0198	.0290	≡.09	.234
0.6	6	.343	.36	.51	.156	.420	.0261	.0412	.135	.325
0.7	7	.465	.49	.61	.270	.542	.0386	.0656	.210	.441
0.8	8	.602	.64	.71	.468	.670	.0736	.1267	.360	.595
0.9	9	.772	.81	≡.81	≡.81	≡.81	≡.81	≡.81	≡.81	≡.81
1.0	10	....	1.00	.91	1.403	.958	.....	.....		1.130
Equation No. ...			10	8,12	13	11	7	6	6	6
s_0 ...			0	.09	.938[b]	.0549	.91[b]	−.72	0	.0738
Coefficient[a]			1	1	2.147[a]	1.972	−.0081	1.64	1	1.336

[a] The coefficient, in the appropriate equation, of B, log B or 1/B respectively. For Fechner's Scale the coefficient is that for *common* logarithms.

[b] These values are that for s_1, the sensation for unit *albedo*, in the case of the Fechner scale, and that for S_∞, the sensation for infinite relative brightness, for the limiting case $k=O$, since in both s_0 is infinite.

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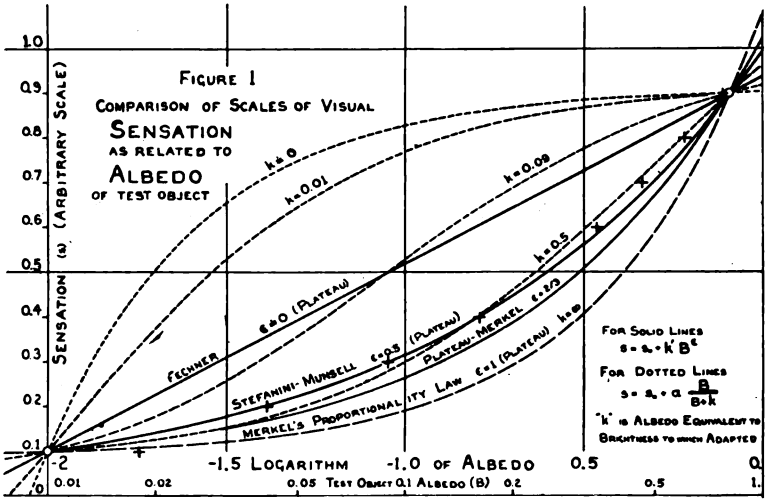
A similar comparison is given in Table 1 and Fig. 1 of this paper. The figure differs from that of Priest *et al*,—besides giving curves for several other formulas than the three named,—in two respects; sensation (or value) has been plotted against log albedo, instead of log albedo against sensation, and the theoretical curves have been made to agree at numbers 1 and 9 of the *theoretical* Munsell scale, (equation 10) while the *albedo* of the Munsell papers, as measured by Priest *et al*, is indicated by crosses. Solid lines indicate formulas derived from that of Plateau (equations 9 to 13) for the indicated values of the exponent ϵ, dotted lines show the relations given by the equation of Adams and Cobb



(equations 4 to 8) for various values of k, the brightness to which the eye is adapted, (expressed in *albedo* units). The Merkel proportionality law, being a limiting case of both the foregoing, is indicated by a dashed line.

From the figure it can be seen that all the other curves lie within those representing the limiting cases of constant adaptation to zero and infinite brightness, respectively; hence, by an appropriate constancy or variation in the state of adaptation during the measurements, any of the relations shown could be obtained experimentally. Again, it will be noted that with adaptation to the

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geometric mean ($k = 0.09$) of scale numbers 1 and 9 (of the theoretical Munsell scale) the relation between sensation and albedo approximates the scale of Fechner, whereas with adaptation to the arithmetic mean[11] of the same brightnesses ($k = 0.41$), it agrees well with the Stefanini equation on which the Munsell scale is based. It is noteworthy that the actual Munsell scale agrees fully as well with the equation of Adams and Cobb (for $k = 0.5$), as with the Stefanini equation.

In view of the criticisms by Priest, Gibson, and McNicholas it may be well to point out the *physical basis* of the Munsell scale. Its numbers represent the *amplitude* of the light waves from a diffusely reflecting surface relative to that of the waves from a *perfect* diffuse reflector, similarly illuminated.

In view of the marked dependence of the subjective scale of (luminous sensation) value on the state of adaptation, it is doubtful if the axiom of Priest, Gibson and McNicholas (p. 29): "It will probably be agreed by all who are interested in the subject and consider it carefully, that the steps in the value scale should be apparently equal; that is, the visual contrast between the cards of any two adjacent numbers should equal that between any other adjacent two,"—can be applied to the grading of the series of grays. It may well be preferable to use the actual albedo of the surfaces, since this scale is one of the *limits* of the subjective scale, namely that approached as the adaptation brightness is increased.

SUMMARY

1. Only if the state of adaptation of the eye is maintained constant, is it proper to speak of luminous sensation as a function of brightness.

2. With constant adaptation, k, the functional relation between sensation, s, and brightness, B, is well represented by the equation of Adams and Cobb.

$$s = \frac{B}{B+k} \tag{5}$$

[11] The curve for $k = 0.41$ in the equation of Adams and Cobb has not been represented on Fig. 1, but it can easily be seen that it would lie only slightly above that for $k = 0.50$.

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3. All the equations connecting sensation and brightness are of such a form that,—within the range of validity of Weber's law,—the relation between sensation and test object *albedo* may be made independent of the absolute level of brightness (for any constant illumination of test object and surroundings) but will depend on the albedo of the surroundings to which the eye is adapted.

4. Between numbers 1 and 9 of the Munsell scale of (luminous sensation) value, the sensations of an eye adapted to a brightness corresponding to the arithmetic mean[11] of the albedo of those scale numbers (i.e., 0.41) approximate the values of the Munsell scale.

5. Within the same limits, the sensations of an eye adapted to a brightness corresponding to an albedo of 0.09,—the *geometric* mean of the albedos corresponding to Munsell scale numbers 1 and 9,—approximate the values of the Fechner scale.

6. In view of the marked dependence of subjective value on the state of adaptation of the eye, grays should be rated according to their *albedo*, which is a physically determinate property.

NELA RESEARCH LABORATORIES
CLEVELAND, OHIO
AUGUST, 1922.

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INSTRUMENT SECTION

THE MEASUREMENT AND SPECIFICATION OF OPTICAL CHARACTERISTICS IN PROJECTOR PERFORMANCE

By G. W. Moffitt

In order to facilitate further improvement in the art of optical projection it has become desirable to definitely recognize those optical characteristics of projector performance which should be the basis for judging the merits of any projector of whatever construction. The measurement and specification of these characteristics should also be considered and a system adopted which will not only make possible a critical study and comparison of the various types of projector but will also make the results of such study and comparison generally available instead of leaving this information entirely with a few.

Optically the performance of projectors may vary in definition, in screen illumination, and in quality. In addition to these there are certain other phenomena—such as shutter effects and vibration—that affect the optical performance but which are due to causes entirely mechanical. For the presence of these defects the optical system should not be blamed, but nevertheless account must be taken of them in the specification of the optical characteristics of projector performance. Moreover, it is obvious that a consideration of the fine points in the optical performance of projectors may be profitably taken up only after the objectionable mechanico-optical effects have been reduced to an inoffensive degree.

Definition

The first demand on the optical system of a projector is that it produce a well-defined screen image. Usually the projection lens is held entirely accountable for the definition, but unless it is properly supported by the machine good results may not be obtained even with the best of lenses. If the film is not maintained in a proper fixed position relative to the optical system, or if it is not allowed to remain stationary during the projection

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intervals, then the performance will be unsatisfactory no matter how excellent the lens may be. Moreover, there are certain optical relations between condensing system and projection lens that may affect the definition to some extent. This matter will be considered in a subsequent paragraph.

Of recent years the increasing use of the motion picture in the home and in the school at rather short viewing distances and with small screens has resulted in the production of apparatus using miniature films. On the basis of conservative study of this matter the conclusion is justifiable that there is a proper place for films of this kind. In some cases there has been a tendency to crowd on the magnification and make these miniature outfits project a picture much larger than was necessary for satisfactory results. This practice requires a film magnification as high—sometimes higher—than that required for standard projection of very large pictures where the viewing distance is great. It means that the miniature film demands a degree of correction in the projection lens hitherto not called for in the standard sizes. If these small films come into general use a very superior type of projection lens will be required.

Formerly the specifications for good projection lenses did not tax the resources of the designers. But now the increasing demands for higher efficiency have made refinement necessary. Relative apertures have been increased until—as seems to be the opinion of many—further advance in this direction is limited by the decreasing depth of focus in its relation to the mechanical limits in maintaining the film position and contour. More and more we find designers turning their attention to improvement in definition, in transmission, and in quality of image. Along with these advances in projection lens design the need has arisen for a workable system of measurement and specification of projection lens performance and of the optical characteristics of projector performance in general.

No doubt there are those who would advocate the use of precision lens bench tests as the basis for the specification of definition in projector performance. But while it is true that the lens bench is a very useful aid in the critical study of the

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performance of lenses intended for use as visual or as photographic objectives it does not necessarily follow that it is universally applicable, except in the hands of an experienced worker. Because of the power of the instrument to show up the defects of a lens system there is sometimes a tendency to judge too critically when examining a lens in this manner, and the necessity for making allowances in the interpretation of the results gives an opportunity for bias and temperament to affect the judgment. Of course this ability to reveal image defects makes the instrument indispensable to the progressive designer who wants to know *in what way* the lens is failing. Lens bench tests are made with a uniform distribution of light over the aperture of the objective whereas in actual projection this might be, but usually is not, the case. In fact the intensity distribution over the area of a projection lens aperture is usually very far from uniform. This lack of uniformity is such that the performance of a projection lens will often be better than a lens bench test would lead one to expect. Obviously then, the projection lens should be tested in position and under actual working conditions. Illuminating systems differ considerably in the matter of uniformity of intensity at the lens. Therefore a lens possessing considerable zonal aberration may show up better on some projectors than on others. As higher and higher optical efficiency is attained the opening of the projection lens will become more and more uniformly and completely filled. Here again, it may be noted, the conditions for improved efficiency demand better correction of the aberrations of the projection lens.

The definition tests should be made when the machine is completely adjusted for showing a picture of normal size at normal screen distance. If the sum total of optical performance as affected by the mechanical defects is desired the machine should be running at full speed while readings are being made. If it is suspected that the mechanical effects are of considerable magnitude and that therefore the test may not be a fair one for the optical system the readings should also be made with the mechanism at rest. Thus whether the tests be made with the machine in motion or stationary depends on whether one is

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testing *the individual complete machine* or *the optical system of the machine.*

A suitable standardized plate, such as is shown in Fig. 1, of sufficient rigidity to prevent warping may be placed in the film-gate and the focus set for the best delineation of the central hole. It is then a simple matter to find the position, with respect to the screen plane, of the primary, the secondary, and the best image, and also the distortion for any or all of the field points. This may easily be done with a white card and a meter scale.

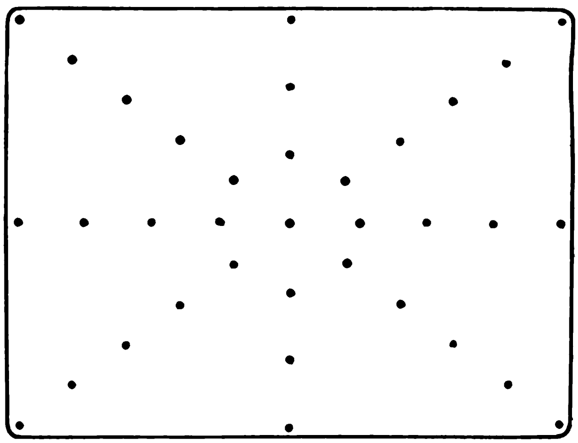

FIG. 1. *Showing the arrangement of pin-holes in a proposed film-gate stencil plate suitable for the study and measurement of the aberrations of a projection lens.*

If any of the desired images fall behind the screen one may place the focus for the central spot at a suitable small distance in front of the screen and all of the images which it is desired to measure may thereby be made accessible. Thus all that is necessary to map the field is the standardized plate, a meter stick, and a white card to serve as a search screen. Coma, as usual, refuses to submit to numerical specification and we must be content with a more or less qualitative statement regarding it.

In case the quality of the axial definition is in question one may resort to a consideration of the best image size—after making proper allowance for the geometrical magnification—or to resolving power tests carried out in the usual way although

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resolving power tests of this kind made under conditions of uniform flux density at the lens should be accepted here with some reserve for the reasons already pointed out. The design of a satisfactory resolving power test slide to fit into the film-gate is a matter that might well receive considerable attention.

Screen Illumination

The measurement of screen illumination is comparatively easy but deductions from such measurements should be drawn cautiously for the contrast rendering ability is sometimes affected by gain in illumination. Illumination gained at the expense of contrast rendering ability may be no net gain at all. In order that the illumination measurements may be complete it is necessary to determine the intensity, the uniformity, the screen size and distance. Measurements should always be taken with the machine running for there are certain types of shutter that have a vignetting action. Since the screen brightness always depends on the characteristics of the screen itself it is obvious that *screen illumination* and not *screen brightness* should always be measured when the performance of the projector alone is under consideration. Machines with freak shutters having translucent blades, or blades otherwise diffusive, offer some difficulty in the determination of effective screen illumination and unless care be exercised the conclusion may be reached that the effective illumination is higher than is actually the case, the apparent gain being only diffuse light that can do nothing but fog the screen and degrade the quality of the projected picture. On the other hand perforated flicker blades, or flicker blades regularly transmitting a portion of the light, may actually increase the effective illumination to some extent.

Uniformity of screen illumination is, perhaps, best expressed by stating the ratio of the illumination at the various screen points in question to that at the center of the screen. It would be advisable to select and standardize a certain set of screen points to be used in this connection. A stencil to fit the film-gate could be made that would indicate on the screen the points at which to make illumination readings. Fig. 2 shows a proposed stencil

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of this kind that is easily made of fine wires stretched on a frame. The illumination is to be read in the small squares, these areas being so chosen that a good idea of the distribution is obtained. Whatever the arrangement of such a set of selected points may be they should be so chosen that any lack of symmetry either along horizontal or vertical lines will be apparent. There are a number of factors entering into the question of the best distribution of screen illumination which would seem to indicate that a uniform distribution, or even a symmetrically vignetted distribu-

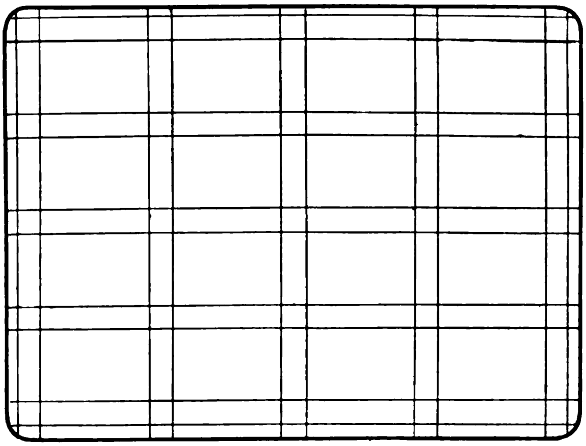

FIG. 2. *Stencil of fine wires for use in the measurement of screen illumination. Readings are made on the centers of the small squares.*

tion, is not generally the best, and that there is no one type of distribution that is best for all installations. Therefore a rather complete statement of the distribution is desirable, in order that one may estimate correctly the performance of the projector in any given auditorium.

The results of a screen illumination test should be accompanied by a statement of screen size and distance, for both the distribution of the light and the intensity may be changed in a manner other than the size of the screen would lead one to expect. The change in focus for different distances may necessitate slight readjustments of the illuminating system and so change slightly the effective diaphragming. And, of course, the performance of a given machine for a given picture size with a projection

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lens of one focal length should not be taken to apply to the performance of the same machine with a projection lens of the same make but of another focal length the distance being changed so that the picture size is maintained. It is well known that the illumination falls off for a given picture size as the focal length of the projection lens is increased. But it does not seem to be so well known that there is no fundamental theoretical reason for this and that present-day machines have this characteristic simply because manufacturers have gotten into the habit of making them that way.

Qualitative statements as to color effects, local patchiness, and striations not revealed by the values of illumination at the standardized points should also be included in the complete statement of screen illumination, although no projector can be considered good if it is not possible to eliminate such defects.

Quality

Quality in projection is determined by those characteristics of a projector which affect its ability to render faithfully the contrasts of the film. It deserves more attention than it has received in the past. Under this head we find the defects generally known as "flare-spot" and "flare." Scattered light which does not come through the lens but reaches the screen by other paths need not be considered here for such light can always be controlled.

Fortunately nearly all good projection lenses are of a type especially free from inherent flare-spot and flare, that is, the design is such that reflection images are few in number and advantageously located. But with the newer trend toward higher relative aperture and higher film magnification a tendency may be noted in some quarters to depart from the time-tried types and exploit others neither so well suited to give high transmission of image-forming light nor so fortunate in the small number and desirable position of their reflection images.

Flare-spot.—This defect is noticeable whenever an image of a strongly radiating point is formed by reflection between some of the lens surfaces, the image so formed falling near the screen. The result is that irregular areas of fog and more or less concen-

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trated spots of light may play about on the screen, their position, intensity, and motion depending on the character of the film being shown. They may be more conspicuous with some condensing systems than with others.

To detect the presence of flare-spot in a projector place the definition test slide in the film-gate precisely as in the definition test. Any other opaque slide having a number of small distributed holes will serve. If splotches of light appear among the screen images of the holes the system is afflicted with flare-spot. Obviously nothing more than a qualitative statement can be made as to the extent to which this defect is present.

Flare.—Flare is usually due to out-of-focus flare-spot. That is, the light which in some cases goes to make a flare spot is so much out of focus that it is quite uniformly distributed over the screen. Another source of degraded contrast in projection is found in poorly polished surfaces and surfaces that are not clean. Obviously all these causes of flare are more marked in their effects the greater the number of air-glass surfaces.

If the machine is equipped with a shutter having any of its blades translucent, or otherwise diffusing, the tests for flare should be made with the machine running. Solid blades with edges blackened would not be expected to have any noticeable influence on the quality of the picture, but shutters with some of the blades of diffusing material add to the total illumination only by the amount of scattered light they throw onto the screen with the result that the picture is degraded thereby.

The specification of flare may best be made by stating the maximum contrast which the projector is capable of rendering in the most unfavorable case, that is, when a small opaque object is depicted on a fully illuminated screen. The necessary measurements can be made by placing a small opaque and well-blackened disk or strip in the film-gate as shown in Fig. 3, and then determining the ratio of full illumination on the screen adjacent to the image of the disk or strip to that in the image itself. Care must be taken to see that the measuring instrument is capable of measuring the contrast obtained with the best lenses. Furthermore it must be used in such a way that there is no question of

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diffusely reflected light getting into the dark half of the photometer field. A statement of the full illumination and of the picture size should also be included.

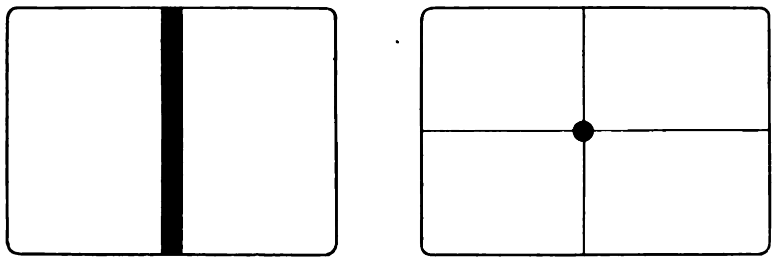

Fig. 3. *Proposed stencils for flare determination.*

Conclusion

By the use of some system of measurement and specification, such as that here outlined, the optics of projection may be placed on a definite basis quite as readily as has already been done for the photographic objective. The improvement of the performance by the improvement of one or more of the component parts is, of course, a matter for the designer and its consideration is beyond the scope of this paper.

Frankford Arsenal,
Philadelphia, Pa.
July, 1922.

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AN ELECTRON TUBE TUNING FORK DRIVE*

BY E. A. ECKHARDT, J. C. KARCHER AND M. KEISER

In view of the increasing use of tuning forks, both in research and in industry, the maintenance of tuning fork vibrations has become a matter of some practical importance. Applications of the tuning fork (1) as a sound source; (2) as a small scale time standard; (3) as a frequency or pitch standard; (4) as a current interrupter, and (5) as a synchronizer or speed-controlling device, are suggestive of the general wide field of usefulness of the tuning fork as a research and engineering instrument.

The invention of the electrically-maintained tuning fork is probably due to Lissajous,[1] although frequently ascribed to Helmholtz.[2] The method used by both involves the periodic opening and closing of the electrical circuit by the vibrating forks. Many modifications of detail involving this general principle, however, are to be found described in the literature. The difficulties encountered in the microphonic behavior of the interrupting contact were made the basis of the microphonic method of maintenance by Appleyard.[3] Until 1919 the interrupted contact and the microphonic methods for maintaining tuning fork vibrations electrically were practically the only ones available, the former being almost universally used.

During 1919 papers appeared by Abraham and Bloch[4] and Eccles and Jordan[5] describing methods for maintaining tuning fork vibrations electrically without circuit interruptions, by means of electron tube circuits. Contemporary work by the present authors[6] led to similar results. The purpose of this paper is to describe apparatus and circuit arrangements which we have found advantageous and to point out advantages of this method.

It is well known that the maintenance of tuning fork vibrations by means of circuit interrupters is more difficult the higher the

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frequency of the fork. Beyond 300 cycles the difficulties are very great.[7] Helmholtz[8] vibrated a series of eight tuning forks, whose respective frequencies were 120 and integral multiples thereof, by means of an interrupter fork of frequency 120 cycles per second. This procedure of driving a high frequency fork by means of a sub-harmonic interrupter fork has recently been used by Curtis and Duncan[9] for maintaining the vibrations of a 500-cycle fork in a tuning fork chronograph. The driving and driven fork frequencies must be adjusted to have a precise multiple ratio. This adjustment is quite critical and is not easily maintained, particularly if the frequency ratio is high. We have applied the electron tube drive to a large number of forks in the frequency range from 50 to 2,000 cycles. There has been no occasion to attempt higher frequencies, but the experience with the 2,000-cycle fork indicates that it is entirely feasible to go higher in the frequency scale.

The Principles of the Electron Tube Circuit Tuning Fork Drive

The electron tube circuit in general consists of a plate or power circuit and a grid or control circuit. Both are completed through the electron tube and have common circuit elements therein. These circuits may be coupled externally in such a manner that the total action becomes regenerative. This is the case when the coupling is such that the current rise in a plate circuit is followed by a potential rise of the grid which, operating through the tube, will serve to augment the current rise in the plate circuit which started the action. The current thus generated is oscillatory because of the nature of the circuit and soon reaches a maximum which is determined by the closeness of coupling and the characteristics of the vacuum tube.[10]

A characteristic form of regenerative circuit is shown in Fig. 1. The external coupling between the grid and plate circuits is here inductive. If the circuit constants are suitably related[11] such a circuit will act as a generator of electrical oscillations the frequency of which may be controlled by an adjustment of the circuit constants.

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Forced vibrations of a telephone diaphragm or a tuning fork are obtained if the windings of the telephone receiver or the driving electromagnet of a tuning fork are included in the plate circuit of such an oscillation generator or coupled to it through a transformer. By adjustment of the oscillator frequency to resonance relatively large vibration amplitudes may be obtained. The

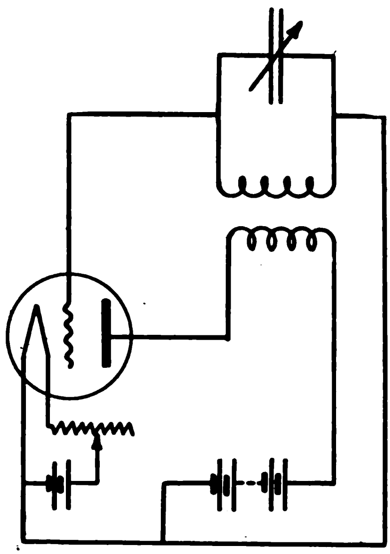

FIG. 1. *Regenerative electron tube circuit.*

maintenance of tuning fork vibrations in this manner is not satisfactory, however, because it is more or less difficult to maintain the frequency of an oscillator circuit sufficiently constant while yielding sufficient power to assure an adequate amplitude of vibration.

As pointed out by Eccles and Jordan the coupling in a regenerative circuit may be wholly provided by a vibrating mechanical system, the interaction between the regenerating circuit elements being due to the mechanical motion, or the coupling may be partly due to the mechanical system, and partly electrical. In the apparatus developed by the authors the mechanical coupling predominates and the inductive coupling between the grid and plate circuits is adjustable. The arrangement is shown

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schematically in Fig. 2. Moving the laminated iron yoke which carries the grid coil up to or away from the driving electromagnet changes the electrical coupling within suitable limits. The variability of the coupling thus provided is of importance in adjusting the circuit for any one fork or in making the same circuit available for driving forks of different frequencies. The complete

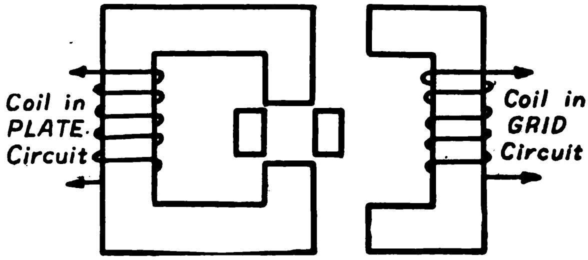

FIG. 2. *Schematic diagram of tuning fork drive.*

circuit in idealized form is shown in Fig. 3, and Fig. 4 shows an equipment with a 500-cycle tuning fork mounted for driving.

The functioning of the circuit is in brief as follows: The closing of the plate circuit (with filament bright) starts the plate current through the driving electromagnet. The fork-prongs are pulled together and thereupon vibrate feebly. Inspection of Fig. 2 will

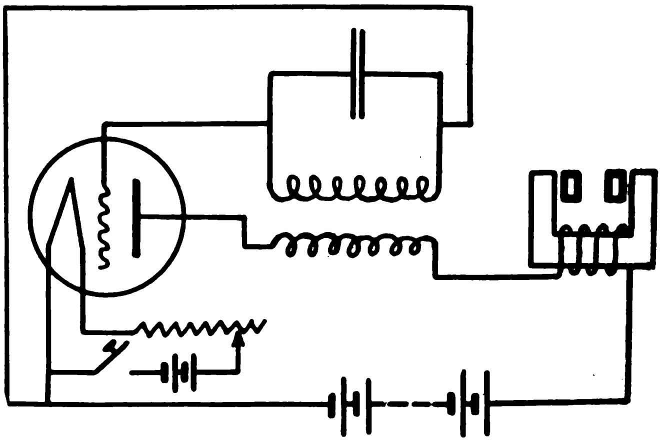

FIG. 3. *Idealized tuning fork drive circuit.*

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indicate that this feeble motion of the fork has a relatively large effect on the reluctance of the magnetic circuit. The reluctance

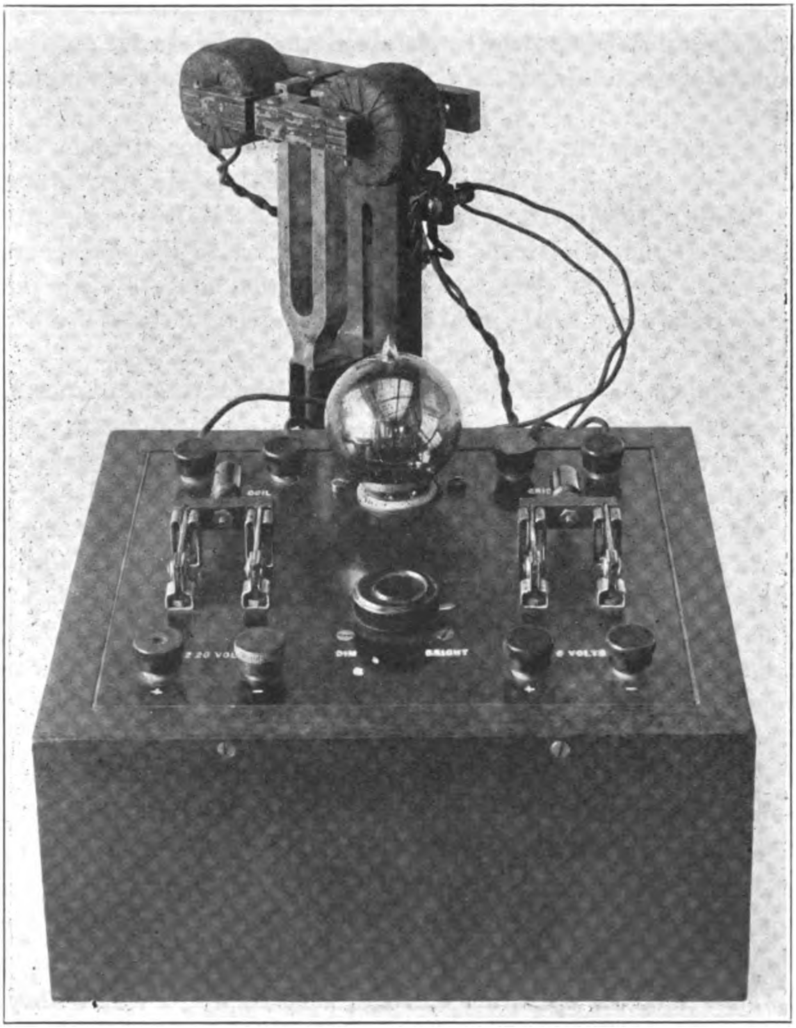

FIG. 4. *500 cycle tuning fork unit.*

changes are determined by and are in synchronism with the free period of the fork. Corresponding variations in the plate

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current result, which by virtue of the coupling existing between the plate and grid circuits are regeneratively amplified. The alternating component of the plate current, as its growth progresses, causes the fork to vibrate with progressively greater amplitudes, in this way acelerating the regenerative action. This process continues until limited by the properties of the tube.

In adjusting the filament current to the proper value the singing of the circuit is a valuable guide. As the free period of the fork is approached beats between the two frequencies are heard. If the adjustment is sufficiently close so that the beats are slow, the beat period will automatically increase until the beats disappear, when the circuit is completely controlled by the fork.

A condenser is connected across the grid coil for tuning purposes. Mica condensers of the type generally used in radio circuits are far superior to paper condensers for this purpose. In general the driving arrangement illustrated in Fig. 4 is sufficiently flexible so that commercially available condensers may be used. For a 500-cycle fork the plate coil has 1,200 turns of No. 28 B. & S. gage silk-enamel wire and the grid coil 3,000 turns of No. 30. The tuning condenser has a capacity of 0.25 microfarad. Any tube of 5-watt rating, providing a steady plate current of approximately 50 milliamperes with 220 volts on the plate is suitable for use in the circuit.

Fig. 5 shows a circuit arrangement in which all the necessary power for operation is derived from a 220-volt power circuit. The diagram is practically self-explanatory. The 3,300-ohm rheostat permits the adjustment of the grid to positive mean potentials, which is sometimes advantageous. The ammeters shown in the filament- and plate-circuits are very convenient because once the operating currents have been established they facilitate adjustment when the power line voltages are not uniform.

The drive discussed for tuning forks is adaptable for maintaining the vibrations of a diaphragm at its natural frequency.

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Fig. 6 shows the method of drive applied to a Webster phone. Fig. 7 shows the phone with the cylindrical resonator case removed. A small piece of iron is mounted on the diaphragm. Its position is adjusted to correspond to that of the right hand prong of the tuning fork in Fig. 2. The position of the driving magnet M is adjustable by sliding along the guide rods R_1 and R_2.

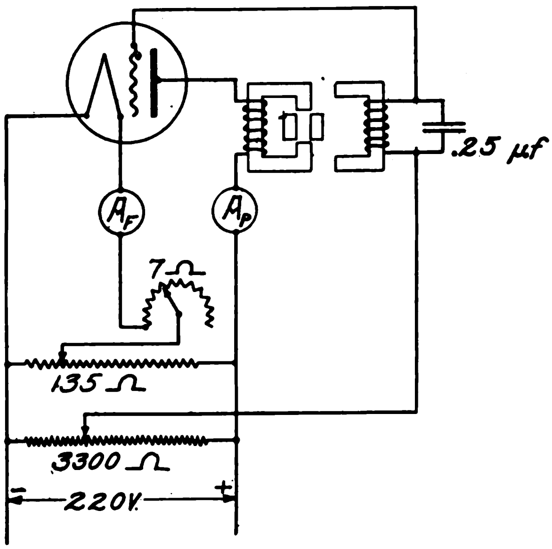

FIG. 5. *Circuit for operation from 220 volt power supply.*

The clamp screws C_1 and C_2 hold it firmly in the chosen position. The grid magnet G is mounted behind the driving magnet to slide on the same guide rods, the relative position being adjustable during operation by means of the adjusting screw S.

This type of tuning fork drive is especially available for use with tuning fork chronographs where precision of frequency is a matter of great importance.[12]

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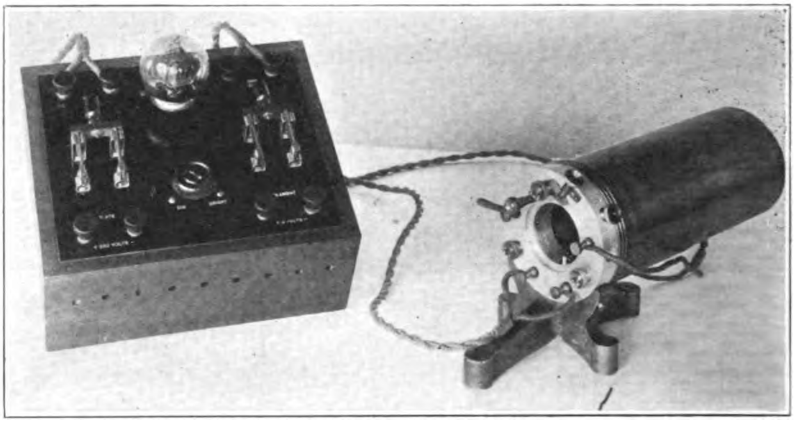
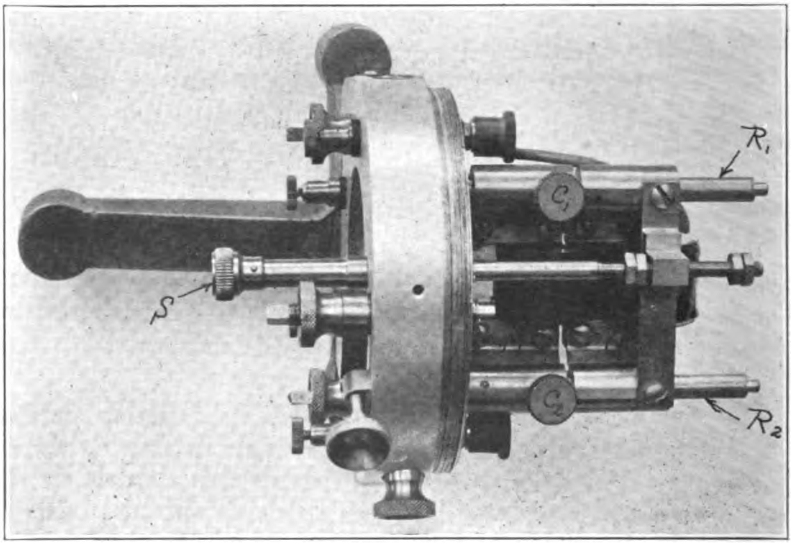
4. Abraham and Bloch, C. R. pp. 1197, 1198, June 16, 1919; Jour. d. Phys., Ser. V, *9*, pp. 225-233; 1919.





FIGS. 6-7. *Webster Phone with electron tube circuit drive of diaphragm.*

5. Eccles and Jordan, The Electrician, *82*, p. 704, June 20, 1919.

6. Eckhardt, Karcher, Keiser, Phys. Rev., Ser. 2, *17*, pp. 535-536; 1921.

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7. Rayleigh's Sound, *1*, (2d ed.), p. 456.

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9. H. L. Curtis and R. C. Duncan, Jour. Wash. Acad. Sc., *9*, p. 642; 1919; Phys. Rev. 2nd Ser., *15*, pp. 511-512; 1920.

10. See Bureau of Standards Circular No. 74, Radio instruments and measurements, secs. 56-58.

11. L. W. Turner, The oscillatory valve relay, The Electrician, *83*, pp. 4-5; 1919.

12. E. A. Eckhardt, A precision high speed oscillograph camera, Jour. Frank. Inst., *194*, pp. 49-67; 1922.

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A LABORATORY HYPSOMETER*

By E. F. Mueller and T. S. Sligh, Jr.

A very simple form of hypsometer, consisting of a nearly closed space into which steam from a boiler can be admitted, will serve to maintain, in the steam space, a temperature which differs, at most, by a few hundredths of a degree from that corresponding to saturated steam at atmospheric pressure. The well known Rudberg or Regnault hypsometer is a very simple piece of apparatus and is capable of serving the requirements of all but the most precise thermometric measurements.

Of the various more elaborate forms of hypsometers which have been devised for precise measurements two were designed at the International Bureau, while most of the remainder originated at the Reichsanstalt. In the Chappuis hypsometer[1] which was an improved and simplified form of a very elaborate instrument devised by Pernet,[2] the design was determined almost entirely by the requirement for facility in changing the position of the thermometer from vertical to horizontal, while in the steam. In this hypsometer, the steam is lead from the boiler through piping to a tube in which the thermometer is placed, thence into an annular space surrounding the tube, and thence to the condenser. A water manometer is used to measure the difference between the pressure in the steam space and that of the atmosphere. A copy of this instrument has been in use for many years at the Bureau of Standards, and the only feature of it to which objection might properly be made, is the water manometer.

Thiesen, Scheel and Sell[3] describe a very elaborate hypsometer, which was a modification of an earlier instrument described by Pernet, Jaeger and Gumlich.[4] The hypsometer as modified con-

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[1] Described by Guillaume, Trav. et mem. du Bur. Int., *5*, p. 39; 1886.

[2] Trav. et Mem. du Bur. Int., *1*, p. B-15; 1881.

[3] Wiss. Abh. der Phys. Tech. Reichsanstalt *2*, p. 138; 1895.

[4] Wiss. Abh. der Phys. Tech. Reichsanstalt *1*, p. 87; 1894.

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sisted of a large gas heated fire tube steam boiler, from which the steam was piped through a pressure regulator, to the space in which the temperature was to be measured. In the bottom of this space was a water seal, through which the entering steam was made to pass in order to ensure saturation. The steam escaped from the steam space through a second pressure regulator. An important feature was the manometer for measuring the excess pressure in the steam space. The one limb of this manometer which was in contact with the steam, was surrounded by water which was kept boiling violently by means of steam supplied by a separate boiler. The other limb of the manometer included a water surface of large area.

In the description of this apparatus, the errors due to possible superheating, and to incorrect measurement of the pressure in the steam space, are emphasized, and it is evident that elaborate measures were considered necessary to eliminate such errors. It is reported that the apparatus functioned satisfactorily.

A later apparatus described by Guitzmacher[5] has the general appearance of the Regnault hypsometer, but differs from it in provision of special means to eliminate superheating. The steam from the boiler flows through a number of short tubes, through a water seal forming the bottom of the thermometer space, and thence past the thermometers. A number of water manometers were connected for indicating the excess pressure of the steam, but evidently it was not considered necessary to boil them as in the apparatus previously described.

It does not appear necessary to refer to all of the various other forms which may be found described in the literature. It will be sufficient to refer to the hypsometer used by Henning & Heuse[6] in their recent determination of the expansion of gases. Steam was generated in a small boiler, heated by an electric heating coil immersed in the water. The steam was piped to the top of the annular space surrounding the thermometer space, flowed downward in the annular space and up past the thermometer

[5] Wiss. Abh. der Phys. Tech. Reichsanstalt, *3*, p. 259; 1900.
[6] Zs. für Physik *5*, p. 295; 1921.

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and thence to the condenser. The excess pressure was read on a water manometer. All parts were thoroughly insulated to avoid fluctuations of temperature due to drafts.

The last two forms of apparatus are noteworthy as indicating a tendency to depart from the somewhat monumental form attained in earlier instruments. It is also noteworthy that the last apparatus while omitting the very elaborate precautions observed in the design of earlier forms, was used with platinum resistance thermometers, which would have made it possible to detect errors and irregularities so small as to escape detection entirely in work with mercurial thermometers.

The above brief review will indicate that in the design of hypsometers, in addition to the essential precaution of steam jacketing for the space in which the thermometer is placed, which is the feature that makes the distinction between the Rudberg or Regnault hypsometer and an ordinary tin can, the refinements which have been emphasized are (1) avoidance of superheated steam in the space around the thermometer, (2) accurate measurement of the pressure in this space, (3) provision for securing constancy of pressure and temperature in this space. To these the authors would add purity of material as an essential feature, to be attained by rapid and thorough removal of air from the steam.

In designing a new hypsometer[7] for general laboratory use, it appeared that an improvement on the instruments already described could be obtained by introducing the steam into the thermometer space and the surrounding annular space at the top, allowing the steam to flow downward in parallel in the two spaces. This arrangement secures steam jacketing of the thermometer space, avoidance of superheat in the steam, since the boiler is located at a distance and ample cooling surface can be provided between boiler and thermometer space, and eliminates entirely the necessity for a water manometer to measure the excess pressure of the steam. The density of steam being less than that of air, a column of steam flowing downward in a pipe

[7] Briefly described in Jour. Wash. Acad. *11*, p. 167; 1921.

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open to the air at the bottom, is stable, while if the steam is flowing upward in a pipe, stability can only be attained by restricting the escape of steam sufficiently to cause the pressure in the pipe to exceed that of the atmosphere. The downward flow of the steam also greatly facilitates the removal of air from these spaces.

The details of construction will be evident upon reference to the photograph, Fig. 1 and the schematic line drawing, Fig. 2. Electric heating was chosen for convenience. The boiler is made of a brass tube 5 cm in diameter and 20 cm high, the outside reservoir serving to maintain a nearly constant water level. The steam pipe from the boiler is a 13 mm brass tube, which enters the steam space around the thermometer tangentially. The small boiler makes it possible to heat up rapidly, and its small cross section and the small steam pipe provide for relatively rapid steam flow in these portions thus facilitating rapid removal of air. The upper 5 cm of the thermometer space are uninsulated to provide surface for condensation, while the remainder of the space is provided with an air jacket which is apparently sufficient to prevent fluctuations due to drafts. The steam after passing through the thermometer space, escapes into the air or may be condensed and returned to the boiler.

All parts in contact with steam or water were tinned. The heating coil consists of two sections which may be connected by means of the switch, either in series or parallel. With the coils in series on a 110-volt circuit the input is about 125 watts which is sufficient to maintain just a trace of steam escaping into the air. Increasing the input up to 640 watts produced no determinable change in the indications of a resistance thermometer in the steam, thus indicating the absence, both of superheating and of excess pressure.

A series of steam point determinations with a resistance thermometer, using both the new hypsometer and the Chappuis hypsometer, indicated no systematic difference between the two, although the precision was slightly in favor of the new instrument. Fortuitous errors of about 0.005° persisted and are apparently due

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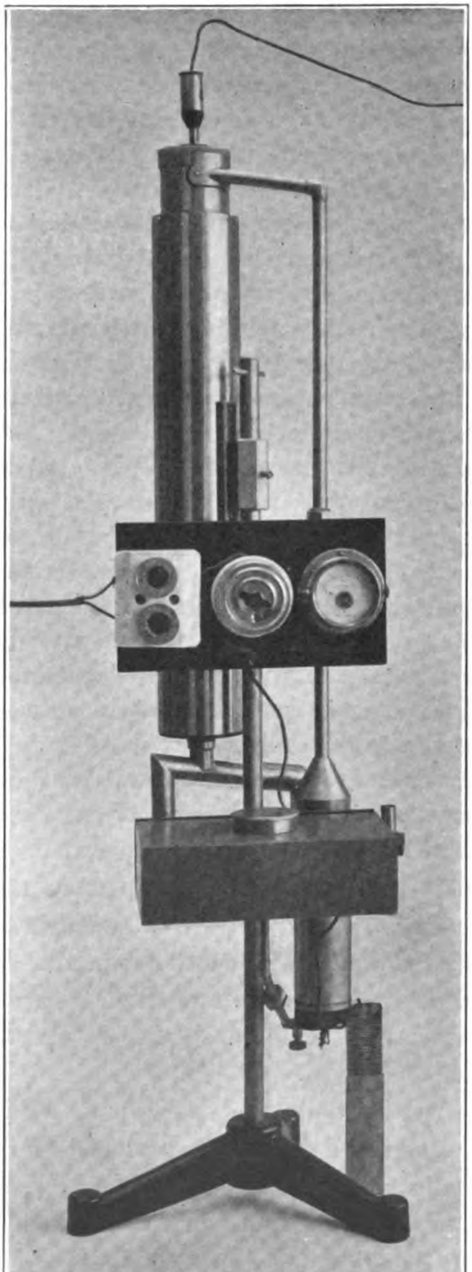

FIG. 1. *Laboratory Hypsometer*

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to irregular fluctuations in atmospheric pressure or to errors in measurement of barometric pressure indicating that improve-

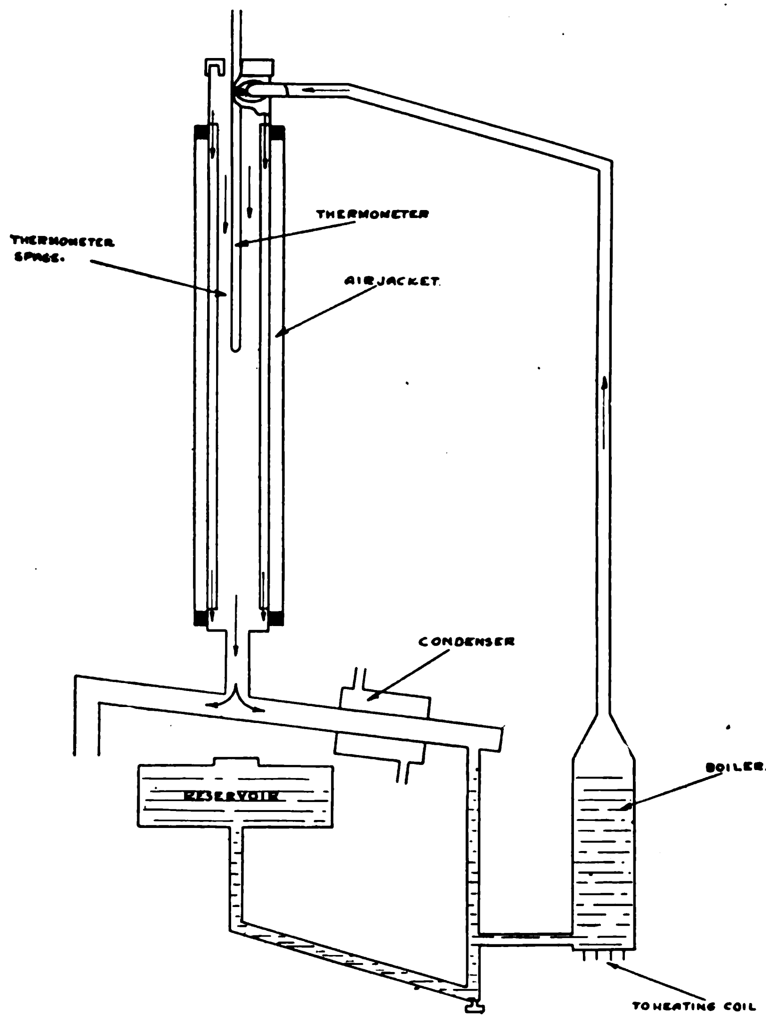

FIG. 2. *Schematic Diagram of Hypsometer*

ments in this respect will require the use of a closed system and better temperature control of the barometer.

While there is no indication that results hitherto obtained with other hypsometers are in error, it is believed that the principles

applied in the design of the present instrument can be used to advantage in the construction of new equipment.

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A HIGH TEMPERATURE REGULATOR FOR USE WITH ALTERNATING CURRENT

By Howard S. Roberts

For many processes, in the factory as well as in the laboratory, elevated temperatures must be held within narrow limits for considerable periods. Where this can be accomplished automatically a part, at least, of the personal element is eliminated and more satisfactory results are secured. Sometimes, where electrical heating is used, the voltage of the power available fluctuates to such an extent that hand regulation is out of the question, and without automatic regulation no processes that involve close temperature control can be employed.

In a recent publication[1] the author described a furnace temperature regulator for direct current resistance furnaces, depending on the temperature coefficient of resistance of the heating element of the furnace. The heating element formed one arm of a Wheatstone bridge and its change of resistance with temperature caused the heating current to be increased and decreased alternately by a system of relays replacing the galvanometer of the bridge. Adams and White[2] had previously described a regulator operating on this Wheatstone bridge principle; while Haagn[3] has described one in which the ratio of current to voltage-drop thru the furnace is maintained constant by a differential relay.

The present paper has to do with the adaptation of the author's direct current regulator for use with alternating current, thus meeting the limitation that direct current is not everywhere available. A possible advantage of using alternating, rather than direct current is that the effect of leakage on the readings of thermoelements at high temperatures is considerably reduced.

[1] J. Wash. Acad. Sci. *11*, pp. 401-409; 1921.

[2] Phys. Rev., *14*, pp. 44-48; 1919.

[3] Elektrotech. Zs. *40*, pp. 670-672; 1919. Zs. Instrumentenk., *41*, pp. 92-93; 1921.

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OPERATION OF THE REGULATOR

Since the fundamental principles underlying the operation of regulators of this type have been discussed elsewhere,[4] the present paper will include only those matters peculiar to the alternating current apparatus.

The various parts are essentially the same as were used in the direct current apparatus, except that a more satisfactory relay has been developed and a simple rectifier placed in the galvanometer circuit, thus making possible the use of the original direct current galvanometer.

The functions of the various parts of the regulator will be sufficiently clear from the diagram, Fig. 1, to admit of postponing their detailed description until the operation of the regulator has been described.

We shall ignore, for the moment, the effect of self induction in the heating element and in the other resistors that together form the bridge, and assume that at a given instant the temperature of the heating element, and therefore its resistance, is such that the bridge is balanced. There is now no current flowing from *a* to *b* thru the galvanometer circuit. If the temperature of the heating element rises, its resistance increases and an alternating emf is set up between *a* and *b* which may be considered as in phase with the current flowing thru the bridge; if the temperature falls from this higher value, the emf *ab* passes thru zero and becomes 180° different in phase; i.e., it reverses just as would be the case if the bridge were supplied with direct current.

The current for the galvanometer passes thru a rectifier which, as will be shown later, causes a direct current to flow thru the galvanometer in one direction when the emf *ab* is in phase with, and in the opposite direction when the emf *ab* is 180° from the bridge current. Thus, the current thru the galvanometer passes thru zero and reverses as the temperature of the heating element passes a particular value, determined by the positions at which the contact arms *a* and *b* have been placed. The boom of the galvanometer (in the figure) tends to swing toward the right with rising temperature and toward the left with falling temperature.

[4] Roberts, op. cit., pp. 405-407.

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The actual control of the temperature is brought about by cutting in and out a fixed series resistance, the "regulating resistance" in Fig. 1, as the temperature falls and rises, respec-

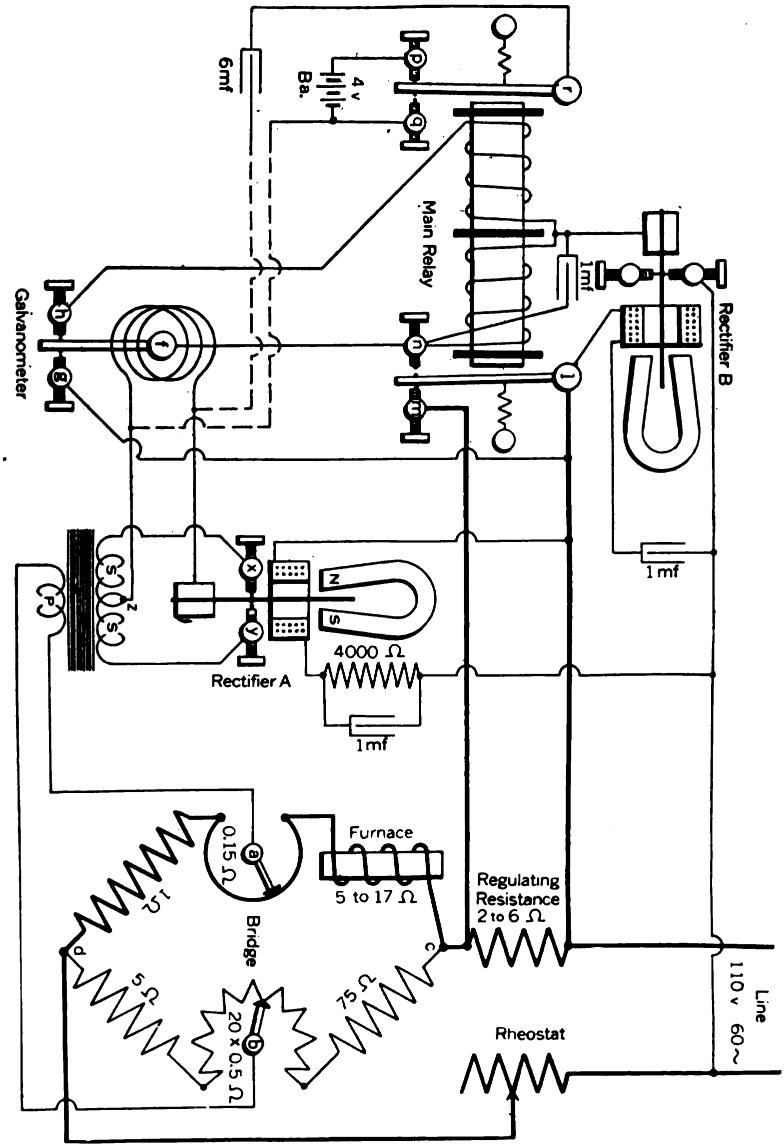

Fig. 1

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ively. The outside rheostat must therefore be set to such at value that the temperature will fall when this resistance is in, and rise when it is out.

With the main relay in the position shown in Fig. 1 the "regulating resistance" is short circuited, the temperature is rising and the boom of the galvanometer is moving toward the right. When the boom reaches the right hand contact *g* a circuit is completed thru the right hand coil of the main relay, causing the relay to close. This does two things (we are not at present considering the contacts shown at the left hand end of the relay): it removes the short circuit thru *m* around the regulating resistance; and provides an alterantive path, at *n*, thru which the energizing current of the relay may flow after the galvanometer contact *g* has opened.

The removal of the short circuit around the regulating resistance causes the heating element to cool, and the boom of the galvanometer begins to move toward the left. As has been pointed out, the current continues to flow thru the main relay because of the alternative path thru *n* and is not interrupted when the boom leaves contact *g*. When the boom reaches contact *h* a circuit is completed thru the second coil of the relay; the two coils oppose each other and the relay opens. By this action the whole supply of current to the relay coils is interrupted at *n*, and the short circuit around the regulating resistance restored at *m*. The latter causes the temperature of the heating element to rise, completing one cycle of the operation of the regulator.

Thus the temperature of the heating element oscillates between two fixed limits, a very few degrees apart. The thermal lag between it and the body whose temperature is to be held constant is usually sufficient practically to prevent oscillations in the temperature of the latter.

Where the bridge is supplied with direct current, it is quite obvious that for any position of the sliders *a* and *b*, there is a critical value of the resistance of the heating element for which the difference of potential between *a* and *b* is zero, and that this difference changes sign as the resistance of the heating element varies from a value slightly less to one slightly greater

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than the critical value. In the case of alternating current, this potential difference can only be zero in the particular cases where the reactances as well as the resistances of the four arms of the bridge are in proportion, or where the resistances are in proportion and the reactances are zero. As is customary in dealing with alternating currents, the potential *ab* may therefore be represented by a vector whose magnitude and direction will, in general, both vary with the temperature of the heating element. If, however, we resolve this vector into a component in phase with, and one at 90° from the potential impressed on the bridge (from the potential *cd*), there will, as before, be some critical value of the resistance of the heating element for which the former (in-phase) component of *ab* is zero. Further, this component must change sign as the resistance of the heating element varies from a lower to a higher value than the critical value. On separating this component selectively from the 90° component and rectifying it, we obtain a pulsating direct current whose sign changes as the temperature of the heating element passes the particular value for which the sliders *a* and *b* are set. The selection and rectification may be accomplished by means of the synchronous vibrator described herewith or, for example, by means of a commutator driven in synchronism with the line voltage. The same result might also be obtained without rectification by making use of the electro-dynamometer principle (alternating current galvanometer).[5] In any case the apparatus must be adjusted so that the component of *ab* selected by it is in phase with the voltage impressed on the bridge, in order to eliminate the effect of reactance in the bridge resistors.

So long as the reactances of the four arms of the bridge remain unchanged, the magnitude and direction of the vector potential *ab* is definitely fixed by the resistance of the heating element; but if they vary, because of a change in frequency, for instance, *ab* must also vary. It has already been shown that the galvanometer is affected only by changes in the power component of *ab* (the component in phase with *cd*). The effect of small changes in

[5] As suggested in the author's previous paper, op. cit., p. 407.

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reactance on the power component is small compared to the effect on *ab* as a whole; and where, as will usually be the case in practice, the ratio of reactance to resistance is itself low, the absolute change in *ab* as a whole is so small that the effect, of changes in reactance, on its power component can be neglected.[6] We should therefore expect the temperature of the furnace or bath to be practically unaffected by any accidental variation in the frequency of the alternating current supply.

This expectation was verified, in the case of a resistance furnace, by comparing the temperature at which the regulator held it when supplied with alternating current, and with direct current. The temperature in the first case was 1081.1° and in the second, 1081.6°, a difference of 0.5°. This corresponds to the maximum possible frequency change of 100 per cent.

DESCRIPTION OF APPARATUS

The details of the bridge, indeed of the apparatus as a whole, will depend on the particular conditions under which it is to be used, but it may be worth while to take up certain general considerations, as well as to describe one form of the regulator.

The heating element should be made of some material having a rather high temperature coefficient of resistance, and should not oxidize readily at the temperature at which it is used. Copper and iron are satisfactory at low temperatures; nickel up to about 500°. The alloy "alumel" is said to have a high temperature coefficient and might prove satisfactory up to 800° or 900°. At still higher temperatures platinum is probably best.

Although the various resistance coils need not be wound non-inductively, the use of iron in their construction had better be avoided; this is particularly true of the bridge coils. The fine adjustment *a*, Fig. 1, may be a piece of heavy resistance wire bent into an arc of a circle, and the contact a short piece of the same wire set radially on a pivoted arm. In the case of the coarse adjustment *b*, a dial switch whose points are connected to the resistances is to be preferred to a rheostat whose slider makes contact directly with the turns of wire on a resistance coil: in

[6] All of this can be shown rather simply by means of a vector diagram.

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the latter case the contact arm is very likely to shunt one or more turns of the wire and, as the contact resistance changes, the setting of the bridge is disturbed.

As the apparatus is capable of responding to changes of 1 part in 20,000 in the resistance of the heater, the apparatus should be so constructed that the change in bridge setting from all other causes is considerably less than this. This means that the bridge resistances must be made from one or another of the low temperature coefficient alloys such as manganin, therlo, constantan, ideal, advance, etc.; the temperature coefficients of nichrome and chromel are a little too high. The connections within the bridge circuit should be of one of these alloys or of heavy copper and the joints should be soldered wherever possible. In order to prevent electrical or magnetic leakage from affecting the galvanometer, the connections to it should be thoroly insulated and the transformer must not be located too close to any of the resistance coils.

In the apparatus used by the author the values of the various resistances and capacities are those given in Fig. 1. These are chosen for use with any suitable heating device whose resistance, when in use, lies between 5 and 17 ohms. Power is supplied at 110 volts, 60 cycles, and the main relay has a capacity of about 10 amperes.

The galvanometer is the same instrument used in the direct current temperature regulator described elsewhere.[7] It is a Weston "Model 30, five binding post galvanometer," having a resistance of 50 ohms and a period of about 2/3 second. It is essentially a millivoltmeter in which the pointer is replaced by an arm carrying a contact button of "iridium alloy" which makes contact with one or the other of two contact screws, depending on the direction of the current in the coil. A potential of 1 millivolt across the coil terminals causes the contact button to move about ½ mm.

When the apparatus is running continuously, each of the galvanometer contacts may be called upon to operate as often as

[7] Roberts, op. cit., p. 404.

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40,000 times per day, and, unless the current flowing thru them is very small they quickly become pitted and stick. With a current of about 10 milliamperes, however, the contacts seldom stick for more than a few seconds before breaking loose, and will usually run for several days without attention. Contacts of Acheson graphite have been found very satisfactory; they need to be cleaned much oftener than the alloy contacts but have the advantage that they may be cleaned off with a nail file without interrupting the regulator.

Sticking of the galvanometer contacts when it does occur causes temperature fluctuations, so that for very precise control of the temperature some means must be adopted to prevent it. The inductive device as used in the direct current apparatus[8] can not be used here, therefore we make use of the energy stored up in a condenser by a small dry battery.

This device is shown at the left hand end of Fig. 1, and for simplicity that portion of the relay is represented in the diagram as an additional armature *r*. With this armature in the position shown, there is a circuit from the battery *B* thru the 6 mf condenser and the galvanometer back to the battery, so that the condenser is charged to the potential of the battery. When the armature of the relay is attracted this connection thru the battery is broken and another made around the battery, at *q*, causing the condenser to discharge thru the galvanometer. When the armature drops back against *p*, the condenser is again connected to the battery and the charging current flows thru the galvanometer but in the opposite direction to the discharging current. The polarity of the battery is such that there is in either case a momentary deflection of the galvanometer away from the particular contact thru which it has just caused the main relay to operate.

In practice the contacts seldom fail to close the circuit promptly. When they have failed, it has invariably seemed to be due to a coating of charred dust and the trouble has disappeared when this was removed.

[8] Roberts, op. cit., p. 402.

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The rectifier, Fig. 2, consists of a reed *R* of clock spring, clamped to a support at one end. The reed passes thru a coil *C* of about 1500 turns of No. 36 B. & S. gage (0.13 mm) copper wire wound on a hollow bobbin of vulcanized fiber. The free end of the reed swings between the poles of a permanent magnet *NS*. The coil *C* receives current from the line and induces an alternating magnetic field in the reed. This causes the end of the reed to be attracted alternately by the north and by the south pole of the magnet *NS*, so that the reed vibrates with a pitch equal to the line frequency. A short distance from the clamped end the reed carries a contact button of molybdenum which may make contact with either

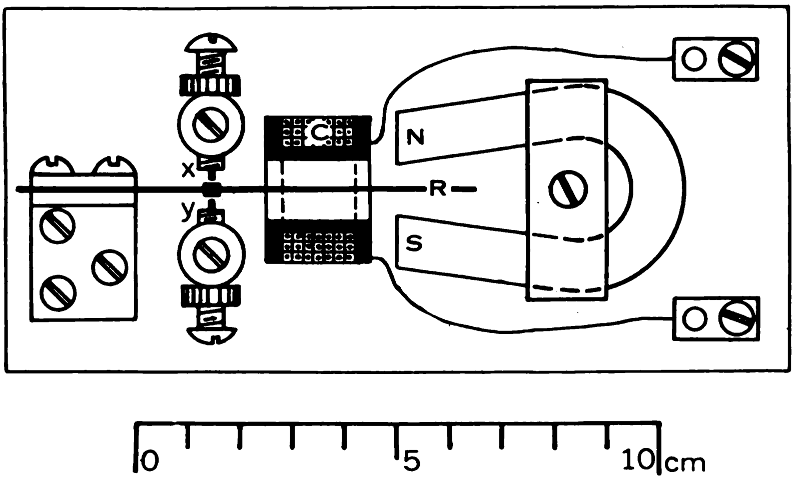

FIG. 2

of the molybdenum tipped contact screws x and y. Thus a circuit is closed thru one contact screw during the positive half of the reed's cycle and thru the other contact screw during the negative half.

The emf to be rectified is that of the split secondary of the transformer shown in Fig. 1. Here the emf's measured from the center z to the two ends are opposite in phase. Thus if the difference in phase between the vibration of the reed and the emf of the secondary as a whole is either 0° or 180°, a unidirectional emf is set up between the reed and the middle tap z of the

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split secondary and a pulsating direct current flows thru the galvanometer. The direction of this current depends on whether the difference in phase referred to above is 0° or 180°.

If the difference in phase between the vibration of the reed and this emf is not 0° or 180°, it can be shown that, while the *instantaneous* value of the rectified emf may change sign during its cycle, the *average* value is equal to that which would be obtained by rectifying only that component of the secondary in phase with or 180° from the vibration of the reed.

The transformer has a primary winding having 1,000 turns of No. 24 B. & S. gage (0.51 mm) copper wire and two secondaries having 1400 turns each of No. 28 B. & S. gage (0.32 mm) copper wire. Each of these is wound in two equal parts, one on each of the longer sides of a rectangular core of laminated silicon steel; the latter has a cross section 12 mm square and an opening 25 mm by 45 mm. This is a transformer of the "core type" and for this purpose is much less affected by the stray magnetic fields emanating from the rest of the apparatus than is the "shell type," where the winding is all on one branch of the core. In the former type the useful magnetic flux flows thru the two windings in opposite directions while the stray fields set up a flux having the same direction in each; thus the emf's resulting from the latter neutralize each other. With a particular shell type transformer the effect of the magnetic field caused by the regulating resistance was so large that it caused the relays to operate as this resistance was cut in and out even when the transformer was disconnected from the bridge.

Owing to the fact that the resistance of the primary circuit of the transformer is not low in comparison with its reactance, the phase of the secondary emf leads that of the emf impressed on the primary. It has already been shown that it is desirable that the vibration of the reed be in phase with the particular secondary emf that would be induced by a primary emf in phase with that impressed on the bridge. This may be brought about by inserting a condenser and parallel resistance in series with the magnetizing coil C and the phase may be adjusted by varying the amount of parallel resistance. The correctness of this adjust-

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ment may be tested while the regulator is running by placing a bar of iron in or close to any one of the bridge resistances. This changes the inductance of that particular coil and, unless the adjustment of the reed is correct, causes a halt in the otherwise steady clicking of the relay.

In the case of Rectifier *B* it seemed better to dispense with a transformer (chiefly because no suitable transformer was available) and to replace it with a condenser in parallel with the relay. Under these conditions the condenser supplies current to the relay during that half of the cycle when the rectifier contact is open. This particular rectifier works satisfactorily with only a condenser in series with the magnetizing coil, and under these conditions the relay receives about 80 per cent of the current it would take if connected directly to a direct current line of the same voltage as the alternating current line.

The "main relay," which is operated by direct current obtained from Rectifier *B*, is a converted "main line" telegraph relay. There are two spools of about 20 000 turns each of No. 40 B. & S. gage (0.08 mm) enameled copper wire on either leg. The two spools nearer the yoke, connected in series, form the left hand winding in Fig. 1; and the other two, also connected in series, the right hand winding. The resistance of each of the two windings is about 11 000 ohms. The separate windings must be thoroly insulated from each other and from the iron core on which they are wound.

The left hand armature of Fig. 1 is the original armature found on the relay, while what is represented in the figure as the right hand armature is in reality a strip of spring brass, attached to the original armature and insulated from it. This spring should be rather stiff and the contact screws should be so adjusted that there is considerable pressure on contact *m* when the relay is open. These contacts, *l*, *m*, and *n*, are of molybdenum, 1.5 mm square and may be used to break currents up to 10 amperes at 40 volts. For heavier duty it is better to break the current by means of a second, heavier relay operated by this one.

Where the occasional sticking of the galvanometer is not an objection, the contacts *p*, *q*, and *r* may be omitted along with their

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battery and condenser shown in Fig. 1. In this case the original armature may be used for *l*.

If the current thru the heating element exceeds 10 amperes, or the temperature must be controlled for periods longer than about 24 hours without supervision, it will usually be more satisfactory to turn over the control of the heating current to a second, more rugged relay. This may be one of the stock relays ("remote control switches") built for heavy duty, or a telegraph relay may be fitted with two or more sets of contacts connected in parallel through suitable resistances. By the latter means the shunt around the regulating resistance may be removed in several steps and the emf, available to cause sparking at the various contacts, reduced to a negligible amount. In either case the energizing current for the second relay passes through the contacts *l* and *m* of the differential relay in Fig. 1.

The use of this second relay reduces the current through the contacts of the main relay to ½ ampere or less, so that the main relay may be adjusted to operate on a much smaller current. This slows down the deterioration of the galvanometer contacts, and the apparatus may be run for longer periods without attention.

As was the case with the direct current apparatus a variable shunting resistance may be connected in parallel with one of the arms of the bridge in order to vary the temperature of the furnace or bath continuously in either direction.[9] In the present apparatus this takes the form of a rheostat giving steps of 10 ohms from 1000 to 2000 ohms and connected in parallel with the 75 ohm coil of the bridge. The heating and cooling curves obtained with this are not quite linear, but are entirely satisfactory for most purposes.

PERFORMANCE

The sensitivity of the regulator is about the same as that of the author's direct current apparatus described elsewhere. The hot resistance of the winding of a furnace wound with platinum and insulated with magnesia is held constant to the equiva-

[9] Op. cit., p. 409.

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lent of $\pm 0.1°$ at 1000-1400°. The actual temperature of the furnace falls less than 1° per day when it is maintained about 1200°. At 1400° it may fall as much as 10° per day. This fall in temperature is quite steady and seems to be due to changes in either the heating element or in the insulation of the furnace itself.

Although the regulator has been used with an oil bath, no numerical data of its performance with such a bath are available. There seems no doubt, however, that with a heater of large area and with effective stirring, temperature fluctuations should not exceed 0.05° at 200°.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C., JUNE 12, 1922.

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AN INTEGRAPH BASED ON PARALLEL DOUBLE TONGS

BY VLADIMIR KARAPETOFF

An integraph[1] is a mechanical device which draws a differential or integral curve to a given curve. Referring to Fig. 1, let $y = f(x)$ by a given curve plotted against OO as the axis of abscissae, and let $z = \phi(x)$ be another curve plotted against NN as the axis of abscissae. Let e and e' be two points on these curves corresponding to the same x. In other words, let the origin on NN be shifted by the amount A to the right, with respect to the origin on OO. The absolute positions of the two origins are of no consequence.

Let the curve $\phi(x)$ be such that its ordinate, say z, at e' be equal, on a certain arbitrary scale, to the slope, dy/dx, of the curve $f(x)$, at the corresponding point e. Then $\phi(x)$ is the ***differential curve*** of $f(x)$. Conversely, $f(x)$ is an ***integral curve*** of $\phi(x)$. If the curve $f(x)$ is given, then by tracing it with a stylus fixed at e, the integraph is made to draw the corresponding differential curve $\phi(x)$. If $\phi(x)$ is given, then by guiding the integraph stylus e' along it, the instrument is made to draw the integral curve $f(x)$.

The differential curve gives values of the slope, or rate of change, of the given curve. The integral curve gives areas between the differential curve and its axis of abscissae. The position of the integral curve with respect to OO depends upon the point at which the area is to be equal to zero. In other words, there is a constant of integration, denoted by C. This constant must be determined by some given initial conditions, like in any problem in integration.

The best known integraph is that invented by Abdank-Abakanowicz and described in many mathematical books.[2] This device

[1] The investigation upon which this paper is based was supported by a grant from the Heckscher Foundation for the Advancement of Research, established by August Heckscher at Cornell University.

[2] See, for example, A. Galle, Mathematische Instrumente, Teubner, 1912, p. 157.

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has proved to be of very limited practical use on account of its price and of delicate adjustments of parts. At the same time, there is a great need for an integraph in a number of engineering

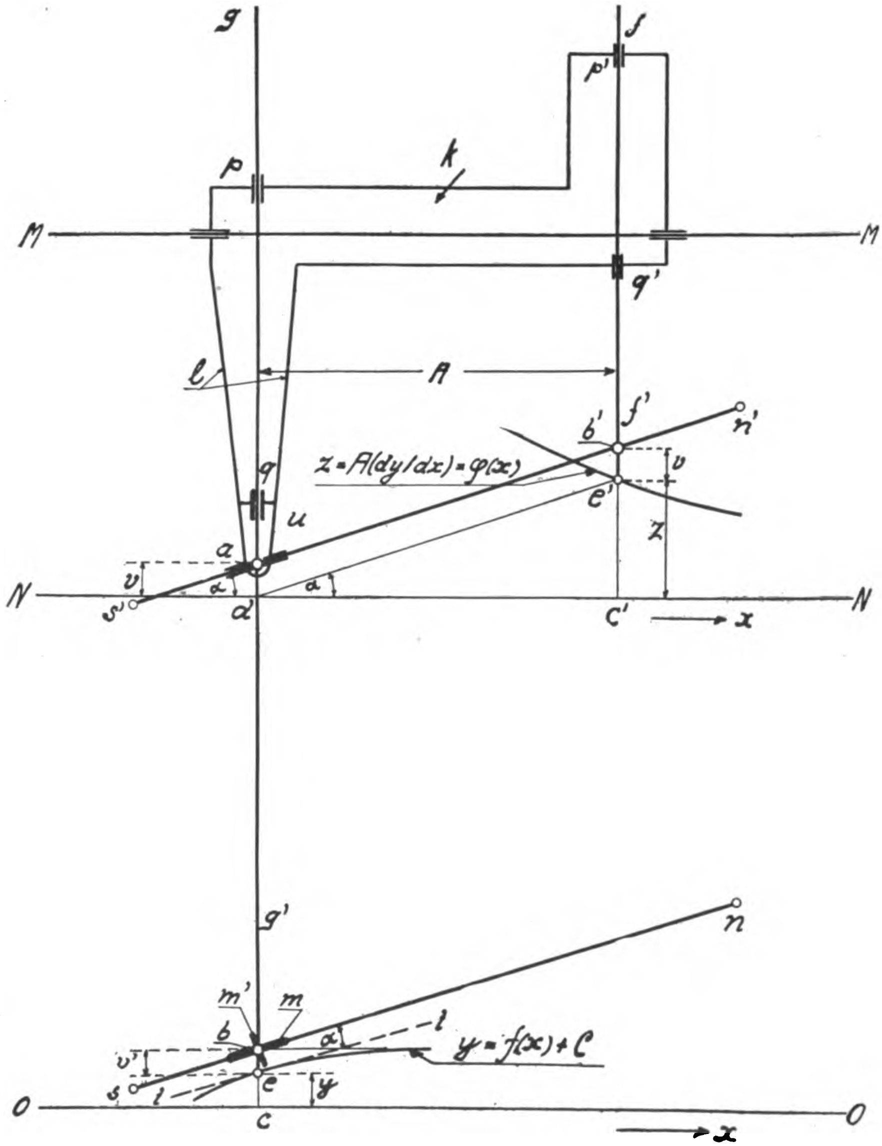

FIG. 1. *The principle of the Integraph.*

and scientific problems, involving an integration or differentiation of a given irregular curve.

The present writer was led to the development of his integraph through his interest in the hunting of synchronous machinery.[3] In this problem the determination of the size of the fly-wheel requires a double integration of the tangential-effort curve, an operation which in practice is quite tedious. He found that a simple and robust integraph can be built by using his parallel double tongs, previously described.[4] An integraph is also useful in determining the stability and flooding of ships, in plotting time-speed curves of electric trains, in computing losses in a machine by the retardation method, and in numerous other problems in which a differentiation or an integration of a given curve is necessary.

In Figs. 1 and 2, MM is a guide rail along which a carriage k can roll with very little friction. The rod gg' is movable at right angles to MM, in the guides p and q. The rod ff' is similarly guided at p' and q'. The bar sn may be rotated about point b as a center, and it carries a sharp-edged wheel m which can rotate about the axis m' perpendicular to sn.

The bracket l, firmly attached to the carriage k, has at its end a guide u, pivoted at a. The bar $s'n'$ passes through this guide and is pivoted at b' to the bar ff'. The stylus or pencil e is rigidly connected to the bar gg', and the pencil or stylus e' is rigidly attached to the bar ff'.

For the proper functioning of the integraph the bars sn and $s'n'$ must remain parallel to each in all their possible positions. This parallelism is preserved by means of a special kinematic linkage called the parallel double tongs, shown in Figs. 2 and 3, and described below. For the present it is sufficient to know that the bars sn and $s'n'$ are constrained to remain parallel to each other, but that both can be turned, and sn can be moved nearer to $s'n'$ or farther away from it, without being interferred with by the parallel double tongs.

[3] Sibley Journal of Engineering, v. *34*, No. 3:3, 1920.

[4] The American Machinist, v. *55*, p. 1050, 1921.

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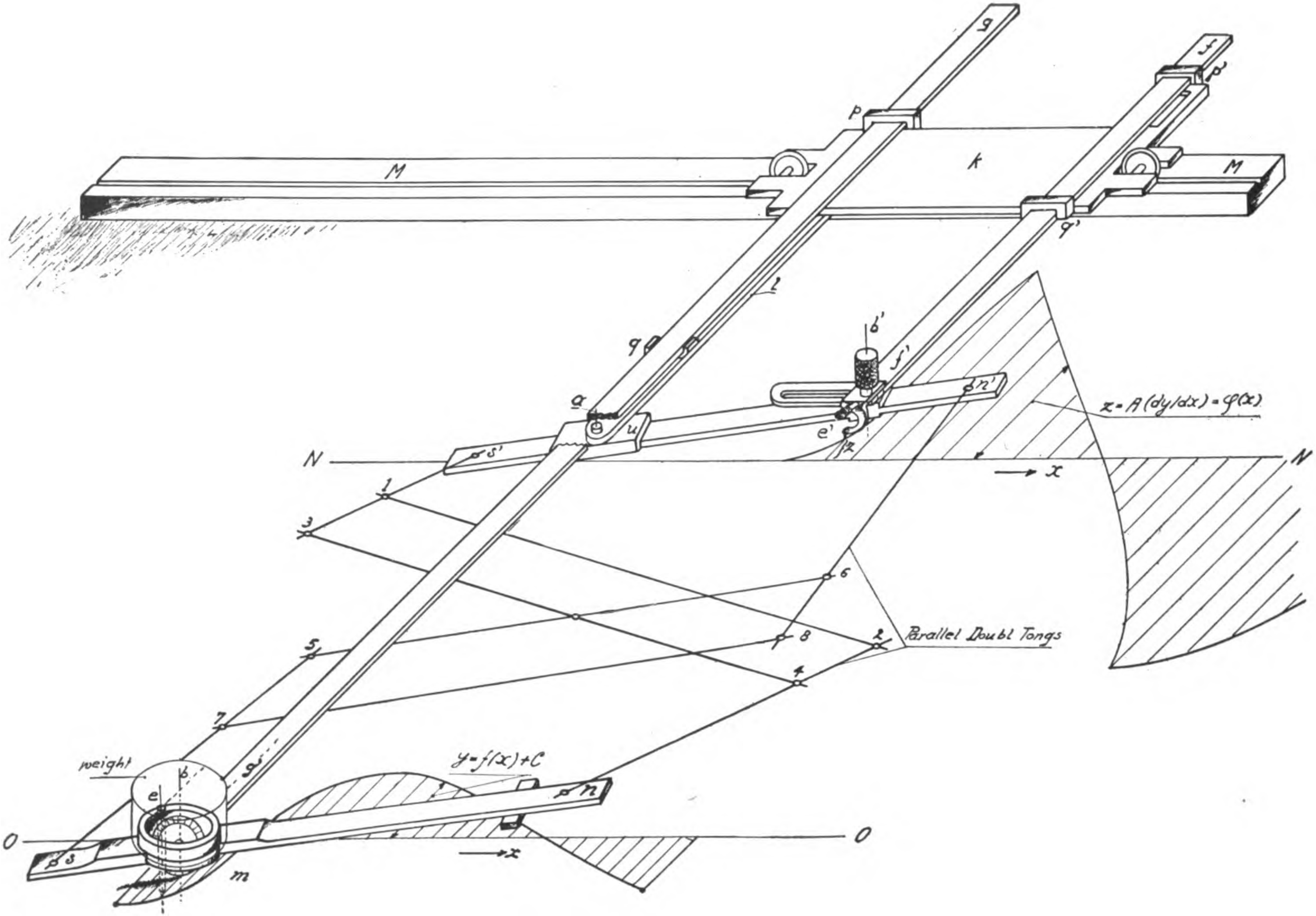

FIG. 2. *A perspective view of the Integraph.*

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To draw a differential curve $\phi(x)$, corresponding to a given curve $f(x)$, the latter is traced by the stylus e, say to the right, by so guiding the bar sn that the wheel m rolls on the paper, without slipping sidewise. The axis of sn is then parallel to the tangent tt to the curve at the point e which the stylus is touching at that particular instant.

Let, in the position shown, the bars sn and $s'n'$ form an angle a with the axis NN, so that

$$\tan a = dy/dx \quad (1)$$

The axis NN is drawn at the distance v from the center a, where

$$aa' = b'e' = v \quad (2)$$

Hence, from the triangle $a'e'c'$ we have

$$c'e' = a'c' \tan a \quad (3)$$

or

$$z = A(dy/dx) \quad (4)$$

In other words, the ordinate of the curve $\phi(x)$, at point e', is proportional to the first derivative or slope of the curve $f(x)$ at the corresponding point e. The coefficient of proportionality is A, equal to the distance between the centers of the bars ff' and gg'. In the actual device this distance is adjustable within certain limits, so that a convenient scale can be had for either curve.

The kinematic arrangement of the parts is convertible, that is, when the stylus e' is guided along $\phi(x)$, a pencil attached at e will draw an integral curve $f(x)$. The edge of the wheel m traces a curve identical with $f(x)$, at a distance v' above it. Since the bar gg' can be moved up and down in its guides p and q, without disturbing $s'n'$, the initial ordinate of the curve $f(x)$ is arbitrary, as it ought to be, because of a constant of integration. The distance of the center a from the rail MM is also adjustable within certain limits, so that the differential curve may be raised or lowered at will.

The device can be best checked by drawing some simple curves beforehand. For example, if the given curve is an inclined straight line, $\phi(x) = Bx$, where B is a constant, its integral curve is a parabola, $f(x) = \frac{1}{2} Bx^2$. The integral curve of a sine wave is a cosine wave; etc.

The parallel double tongs (Figs. 2 and 3) consist of two identical articulated parallelograms, *1234* and *5678*, with pivot joints at all

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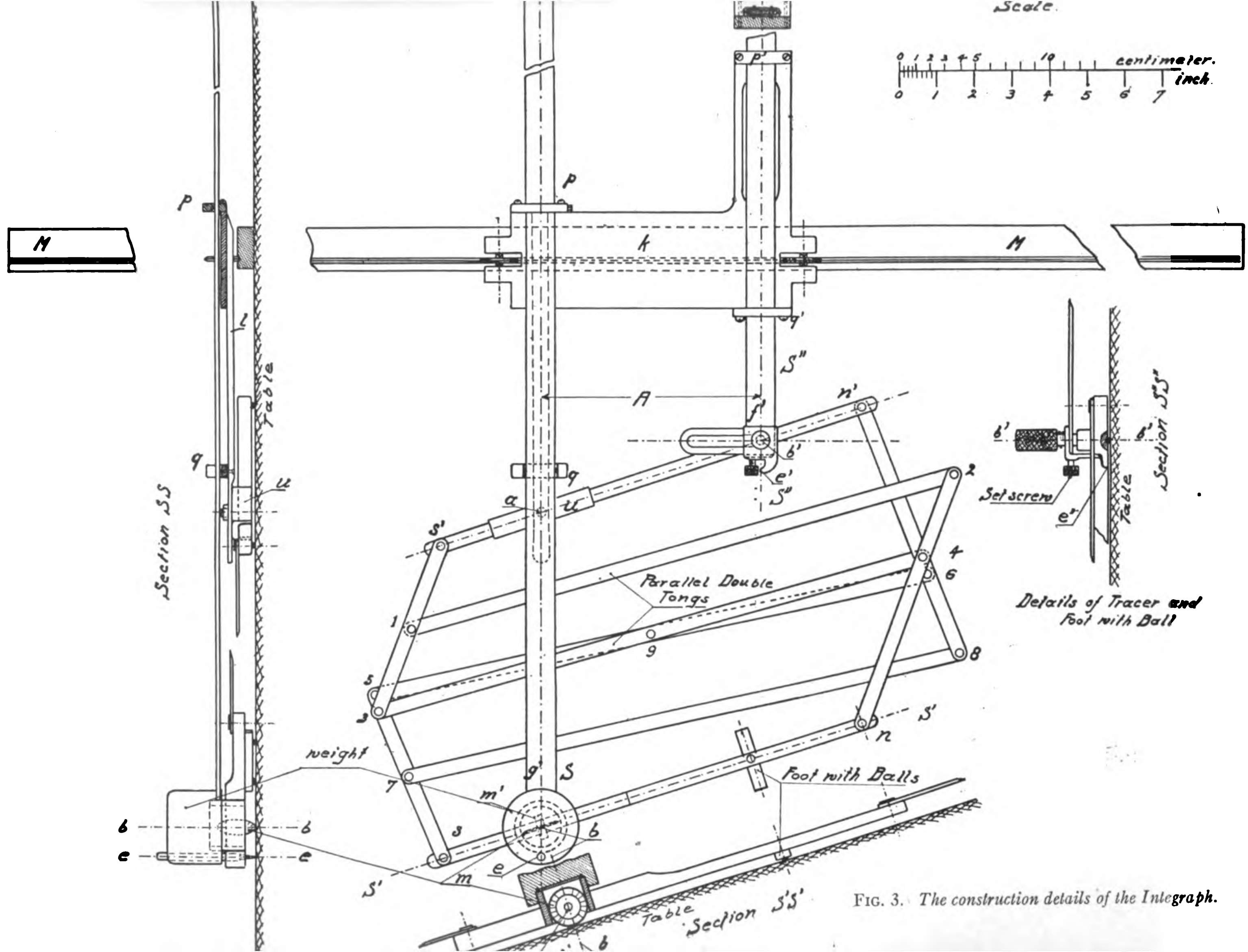

FIG. 3. The construction details of the Integraph.

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the vertices. The two parallelograms are pivoted together at the middle point *9*. The opposite short sides of the parallelograms are extended, and connected to the integraph bars at n, s, n', and s', the lengths ns and $n's'$ being equal. The extended lengths must satisfy the condition

$$n4 = s'3 = n'6 = s5 \qquad (5)$$

The bars ns and $n's'$ are then constrained to remain parallel to each other, without their position or motion being otherwise impeded by the parallel double tongs.

The construction details of the experimental integraph made in Cornell University are shown in Fig. 3, the lettering being the same as in Figs. 1 and 2. The device was constructed and the mechanical details worked out by Mr. O. K. Marti, to whom the author wishes to express his sincere appreciation for the valuable assistance rendered.

CORNELL UNIVERSITY,
ITHACA, NEW YORK.
SEPTEMBER, 1922.

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REVIEWS AND NOTICES

Patent Essentials for the Executive, Engineer, Lawyer, and Inventor. A Rudimentary and Practical Treatise on the Nature of Patents, the Mechanism of their Procurement, Scientific Drafting of Patent Claims, Conduct of Cases and Special Proceedings, Including Forms. John F. Robb of the Cleveland and District of Columbia Bars with Papers by G. P. Tucker, L. W. Maxson, E. C. Reynolds, L. A. Sadler, and Edward Collins, Patent Examiners. 436 pp., Funk and Wagnalls, New York, 1922.

This book was written for the layman and presents, in nontechnical, or at least readily understood, language, the fundamental principles of patent law with which any scientist, who intends to file a patent application, should be acquainted. The book is written in a most interesting style, not at all in the dry-cut form of most treatises on legal subjects. A few hours spent in reading this work will be saved many times in the elimination of unnecessary discussion and correspondence between patent applicant and his attorney.

PAUL D. FOOTE.

Isotopes. By F. W. Aston, 152 pp., Arnold (London) 1922.

Titles of the main chapters are as follows: The Radio-active Isotopes; Positive Rays; Neon; The Mass-Spectrograph; Analysis of the Elements; The Electrical Theory of Matter; Isotopes and Atomic Numbers; The Spectra of Isotopes; The Separation of Isotopes; Various Tables. The work summarizes all data and experiments to January, 1922. Every scientist interested in the subject should have a copy of this book for reference. The book is so clearly and simply written that one who is not specializing in science will be able to read it readily and to understand the greater portion of the subjects treated.

PAUL D. FOOTE.

OPTICAL SOCIETY OF AMERICA

MINUTES OF THE EXECUTIVE COUNCIL

A meeting of the Executive Council was called to meet in Washington, Jan. 28, 1922. Present: Troland, Ives, Foote, Priest. No quorum.

A meeting of the Executive Council was called to meet in New York, Feb. 25, 1922. Present: Troland, Ives, Lomb, Gale, Southall. No quorum. (Southall Secretary, protem.)

An informal conference of the following members was held in Washington, April 22, 1922: Foote, Forsythe, Gale, Merritt, Priest.

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As a result of these conferences and correspondence, President Troland on May 4, 1922, declared the following resolutions adopted by unanimous vote of the Council:

> To amend Article II, Section 2 of the By-Laws to read as follows: "No officer or member of the Society except the editor-in-chief and the assistant editor-in-chief and business manager of the Journal shall receive any remuneration for his services."
>
> To Amend Article V of the By-Laws by inserting after editor-in-chief the words "and the assistant editor-in-chief and business manager."
>
> RESOLVED that a ballot be mailed to the members of the Society to vote upon an amendment to the constitution to read as follows: "The assistant editor-in-chief and the business manager of the Journal shall be ex officio a member of the Executive Council."
>
> RESOLVED that Mr. Adolph Lomb, as treasurer of the Optical Society, is hereby designated by the Executive Council to sign the agreement between the Optical Society of America and the Association of Scientific Apparatus Makers relating to the financing and publication of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS.
>
> RESOLVED that the agreement between the Optical Society of America and the Association of Scientific Apparatus Makers relating to the publication of a Journal to be known as the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS for a period of two years beginning April 1, 1922, when signed by the authorized representatives of the said Society and Association, be hereby ratified confirmed and approved by the Executive Council acting for the Optical Society of America.
>
> RESOLVED that the financial transactions of the Optical Society of America and of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS shall hereafter be separately managed, those of the Society remaining in the hands of the treasurer whereas those of the Journal shall be entirely under the control of the business manager of the Journal, that the business manager of the Journal shall be responsible for all collections and disbursements involved in the publication of the Journal and that contributions or payments made by the Society towards the expenses of the Journal should be made by the treasurer to the business manager of the Journal in a manner mutually agreed upon by said officers.
>
> RESOLVED that the disbursement of the contribution of $2,500 made by the National Research Council towards the support of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND THE REVIEW OF SCIENTIFIC INSTRUMENTS shall be entirely in the hands of the business manager of the Journal, who shall be authorized to sign all vouchers and requisitions connected with this appropriation.
>
> RESOLVED that the business manager of the Journal shall make an annual report as of April 1st of each year and that this report shall be duly audited and published in the Journal.
>
> RESOLVED that the committee on membership be authorized to drop from membership the names of such persons as are in arrears in accordance with Article I, Section 5, of the By-Laws.
>
> RESOLVED that the business manager of the Journal be directed and authorized to revise the contract with the George Banta Publishing Company of Menasha, Wisconsin, and to submit the same to the president and the editor-in-chief for approval, and that after receiving such

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approval the business manager be empowered to sign the contract on behalf of the Optical Society.

RESOLVED that the printed reports of the sub-committees of the Optical Society committee on nomenclature and standards shall be advertised in the Journal as for sale at a price per copy depending upon the number of pages, the number of such reprinted reports and price to be determined by the publication committee, and the proceeds of such sales to go to the treasury of the Optical Society.

IRWIN G. PRIEST,
Secretary.

WASHINGTON, D. C.
AUG. 14, 1922.

OPTICAL SOCIETY OF AMERICA

NEW MEMBERS

The following new members have been duly elected by the Executive Council:

REGULAR

No. 325. William L. Benedict, Mayo Clinic, Rochester, Minn.
No. 326. George Walter Stewart, Hall of Physics, Iowa City, Iowa.
No. 327. Arthur Edward Ruark, 6010 Henderson Avenue, Govans, Baltimore, Md.
No. 328. Francis G. Pease, Mount Wilson Observatory, Pasadena, Calif.
No. 329. Frank Walter Weymouth, Stanford University, California.

ASSOCIATE

No. 330. Ting Supoa, 38 M. D. Hall, University of Chicago, Chicago, Ill.

BY TRANSFER FROM ASSOCIATE TO REGULAR

Reinhard A. Wetzel has been transferred to Regular Membership.

David Rines, who was elected to Associate Membership at Rochester, October 24, 1921, through lack of complete information as to his qualifications, has been made a Regular Member.

IRWIN G. PRIEST,
Secretary.

August 31, 1922.

NOTICES

OPTICAL SOCIETY OF AMERICA

TELLERS' REPORT

To Irwin G. Priest, Secretary:

We, the undersigned regular members of the Optical Society of America, have counted the ballots cast on the following proposed amentment to the Constitution:

"The assistant editor-in-chief and business manager of the JOURNAL shall be ex-officio a member of the Executive Council."

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We have verified our count and certify the following to be the true result:

In favor of the amentment.........95
Opposed to the Amendment........ 0

(Signed) K. S. GIBSON
M. K. FREHAFER
Tellers.

September 2, 1922.

In accord with the above report and the Constitution's provision for its amendment, the above amendment is hereby declared adopted.

(Signed) LEONARD T. TROLAND
President
IRWIN G. PRIEST
Secretary

September 5, 1922.

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Journal
of the
Optical Society of America
and
Review of Scientific Instruments

Vol. VI DECEMBER, 1922 Number 10

EXTRAORDINARY DIFFRACTION OF X-RAYS

BY L. W. McKEEHAN

The term "Extraordinary Diffraction" is here proposed for the directed emission of characteristic X-rays from the atoms of a crystal placed in a narrow beam of X-rays containing sufficiently short wave-lengths. The theory here presented was developed in attempting to explain the occurrence, on photographs taken for the crystal analysis of iron, nickel, and copper, of spots, evidently diffraction images of the source, at places, and in positions, quite inexplicable by the ordinary theory of X-ray diffraction in crystals. Before admitting the explanation here offered attempts were made to explain the observed effects by postulates less radical in their implications than those finally adopted, but it was found, for example, that no number of successive reflections within a single crystal, or within a twinned pair, could be made to account for the observed effects. The new physical hypothesis given below does permit an explanation of the new phenomena, and that, at present, is its sole justification.

The effects of "extraordinary diffraction" are always associated with the effects of what may, for distinction, be termed "ordinary diffraction." It seems clearest, therefore, to present the analysis in a form covering both sorts of diffraction at once, there being, of course, nothing novel in the results so predicted for the ordinary case.

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In order to avoid the wholly formal complexity inseparable from equations applying to the completely general case of a triclinic crystal, the analysis will be undertaken for the simplest possible crystal, in which the mean positions of the atom-centers are the points of a simple cubic space-lattice, and in which the atoms are all alike. There is no known crystal as simple as this, but crystals of potassium chloride approach it closely, and the crystals of many metals, including those which clearly show the new effects, can be regarded as composed of two or four interpenetrating arrangements of exactly this type. Further to simplify the mathematical expressions involved, the incident X-rays will be taken as forming a plane-parallel beam, and the dimensions of the crystal will be taken as negligibly small in comparison with the radius of the sphere, centered at the crystal, on which the diffraction effects are studied. To simplify the description of these effects it will be supposed that they produce a photographic record, so that it will be appropriate to speak of spots, lines, bands, and the like. The modifications due to non-parallelism of the incident beam, to lack of circular symmetry in it, and to the finite extent of the source of primary X-rays, will be discussed only qualitatively.

Take the origin, O, at any point of the space-lattice, and lay the axes of X, Y, and Z along the edges of that one of the eight cubical cells which meet at O which includes the prolongation of the incident ray through O. Let the orientation of the crystal with respect to this incident ray be unrestricted, so that its direction-cosiness l_1, m_1, n_1, in addition to being all positive as required by the choice of axes, are restricted only by the geometrical requirement that $l_1^2+m_1^2+n_1^2=1$. Let a be the parameter of the space-lattice, so that adjacent points along each of the three axes are separated by this interval.

Assume (1) that the incident beam contains wave-trains long in comparison with their wave-length λ_1, and (2) that each diffracted beam consists of wave-trains long in comparison with their wave-length λ_2. The wave-lengths λ_1 and λ_2 are, in the practically important cases, of the same order of magnitude as a.

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Assume (3) that at a time $t=\tau+\frac{n\lambda_2}{c}$ after the time $t=0$ when some particular wave of a primary wave-train had phase ϕ_1 at the lattice-point P, that a secondary wave, if emitted from P as a result of the primary wave having passed through it at time $t=0$, will have phase ϕ_2 at a distance $n\lambda_2$ from P. In the last assumption ϕ_1 may be called the primary phase at excitation, τ the delay between excitation and emission, and ϕ_2 the secondary phase at emission. The integer n is introduced to obviate the awkwardness of talking about the phase at the origin of a divergent beam, and c is the velocity of propagation in vacuo. Assume (4) that the values of ϕ_1, τ, ϕ_2 do not depend upon the coordinates x, y, z, of P, nor upon events at other lattice-points, and (5) that the variations among the individual values of $\phi_1, \frac{c\tau}{\lambda_1}, \frac{c\tau}{\lambda_2}, \phi_2$ are small in comparison with 2π. Assume (6) that secondary waves from P have appreciable amplitude in directions considerably inclined to the prolongation of the incident ray.

The only assumption which can be dropped in the case of ordinary diffraction is the first and this is therefore the new physical hypothesis here advanced. The third assumption can be suitably modified for the ordinary case so as to eliminate reference to a primary wave-train, the origin of time being changed to the instant when a particular singularity of the incident disturbance reaches the point P. In ordinary diffraction, also, $\lambda_1=\lambda_2$; in extraordiary diffraction $\lambda_1<\lambda_2'<\lambda_2$, where λ_2' is the wave-length of the absorption limit corresponding to the emission of the characteristic wave-length λ_2.

It is possible that the fifth assumption could be better expressed by requiring rigorous constancy of ϕ_1, τ, and ϕ_2, if these quantities were defined with respect to points P', P'', near P, where the absorbing and the emitting mechanisms concerned were at the times $t=0$ and $t=\tau$. As written above the phase differences, due to thermal agitation displacing atom-centers from the lattice-points, have been included in $\phi_2-\phi_1$.

The elementary theory of diffraction now states that appreciable energy in the form of secondary waves will only be emitted

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in those directions along which, at great distances, the secondary waves from all the points of the space-lattice agree in phase. These directions do not depend upon the mean values of ϕ_1, ϕ_2, and τ, so that all three of these quantities may conveniently be put equal to zero in discussing the geometry of emission. Referring to Fig. 1, which diagrammatically represents the incident and

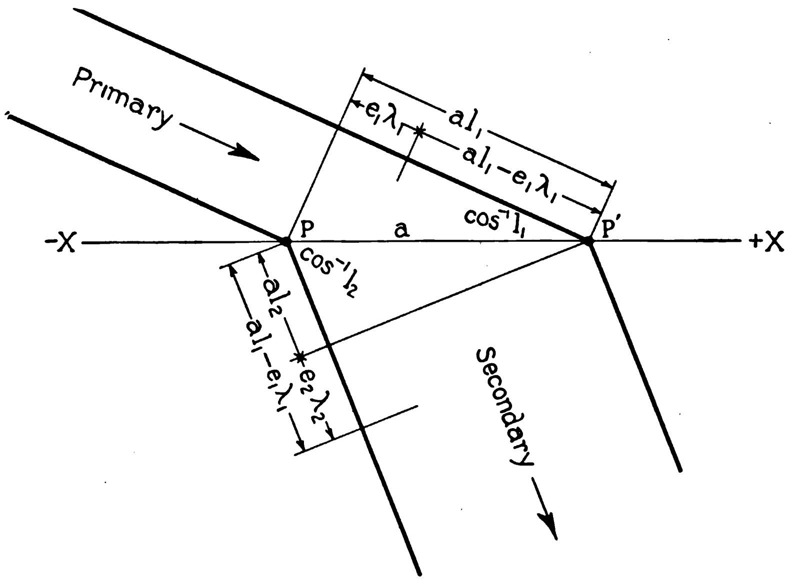

Fig. 1

Extraordinary Diffraction at Two Atom-Centers.

diffracted rays and wave-fronts near two adjacent atoms on the X-axis, it will be seen that the direction-cosines of the emitted secondary rays are given by

$$\left.\begin{aligned} l_2 &= l_1 - \frac{e_1\lambda_1}{a} - \frac{e_2\lambda_2}{a}, \\ m_2 &= m_1 - \frac{f_1\lambda_1}{a} - \frac{f_2\lambda_2}{a}, \\ n_2 &= n_1 - \frac{g_1\lambda_1}{a} - \frac{g_2\lambda_2}{a}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (a)$$

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In these equations l_2, m_2, n_2 are not limited as to sign, but $l_2^2+m_2^2+n_2^2=1$. The quantities, e_1, f_1, g_1 are integers positive, negative, or zero, which may be called the orders of incidence with respect to the three axes, and e_2, f_2, g_2 are integers which may similarly be called the orders of diffraction with respect to the axes. It will be noted that Fig. 1 is drawn for the special case $n_1=g_1=g_2=0$. In the general case the diffracted ray will not lie in a plane determined by the incident ray and one of the three axes.

For ordinary diffraction, putting $\lambda_1=\lambda_2=\lambda$ and $h=e_1+e_2$, $k=f_1+f_2$, $l=g_1+g_2$, where h, k, l are, of course, integers,

$$\left.\begin{aligned} l_2&=l_1-\frac{h\lambda}{a}, \\ m_2&=m_1-\frac{k\lambda}{a}, \\ n_2&=n_1-\frac{l\lambda}{a}. \end{aligned}\right\} \qquad \text{(b)}$$

These formulas, as they should, represent the directions of the transmission ($h=k=l=0$), of specular reflection in the various lattice-planes (h, k, l mutually prime) and of so-called reflection in higher orders (in order w if h, k, l have w as highest common factor). Examination shows that h, k, l are, in fact, the Miller indices of the reflecting planes in the last two cases. The appropriate wave-length for any particular choice of l_1, m_1, n_1 and h, k, l is found by eliminating l_2, m_2, n_2. This gives

$$\lambda=\frac{2a(l_1h+m_1k+n_1l)}{h^2+k^2+l^2}=\frac{2a\Sigma(l_1h)}{\Sigma(h^2)} \qquad (b')$$

In the case of extraordinary diffraction no simplification of the general formulas (a) is possible, and, using notation similar to that in (b')

$$\lambda_1=\frac{1}{\Sigma(e_1^2)}\left[a\Sigma(l_1e_1)-\lambda_2\Sigma(e_1e_2)\pm \sqrt{a\Sigma(l_1e_1)-\lambda_2\Sigma(e_1e_2)\ -\lambda_2\Sigma(e_1^2)[\lambda_2\Sigma(e_2^2)-2a\Sigma(l_1e_2)]}\right] \qquad (a')$$

The ambiguity of signs is resolved by the condition $\lambda_1<\lambda_2$. In both

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cases the number of real diffracted rays for a given range in λ_1 is limited by the condition that l_2, m_2, n_2 must be real and must lie between $+1$ and -1. The ray for which $e_1=f_1=g_1=e_2=f_2=g_2=0$ is coincident with the prolongation of the incident ray for all values of λ_1 and λ_2 and for amorphous as well as crystalline arrangements.

It is fairly obvious that the general case of extraordinary diffraction permits values of l_2, m_2, n_2 not possible in the special case of ordinary diffraction, but an arbitrary numerical example may serve to make this clearer.

Let $a=3.60\times10^{-8}$ cm $\quad e_1=1, f_1=0, g_1=0,$

$$e_2=0, f_2=-1, g_1=0$$
$$l_1=4/5,\ m_1=3/5,\ n_1=0$$

It is seen that $h=l$, $k=-1$, $l=0$.

In the case of ordinary diffraction, by (b')

$$\lambda=0.720\times10^{-8}\text{ cm}$$

and by (b)

$$l_2=3/5 \qquad m_2=4/5 \qquad n_2=0$$

The ray has clearly been reflected in the (110) plane. In the case of extraordinary diffraction we must take a value of λ_2, e.g., $\lambda_2=0.600\times10^{-8}$ cm. Substituting in (a') gives

$$\lambda_1=0.569\times10^{-8}\text{ cm}$$

and using this in (a) gives

$$l_2=0.642, \qquad m_2=0.767, \qquad n_2=0.$$

It will be found that both λ_1 and λ_2 must be less than λ and that the extraordinary ray therefore diverges less from the transmitted ray than does the ordinary ray. This is generally true.

The observed effects depend upon the range of wave-lengths present in the incident beam, and upon the number of crystals dealt with. If there is one crystal, fixed in position, and a wide range of incident wave-lengths, the ordinary diffraction gives the familiar spot-pattern (Laue pattern). The extraordinary diffraction gives additional spots which in the usual experimental arrangements wherein λ_1 is very much less than λ_2 would be relatively faint. The spots of the ordinary pattern are formed by beams of various wave-lengths, those of the extraordinary pattern by beams all of a single wave-length or of a few definite

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values corresponding to the strong lines in the characteristic X-ray spectra of the elements present. This case has not been experimentally tested.

If the crystal is rotated about any line as an axis the spots of the pattern move along paths which are, in general, curved. If attention is confined to a single direction of emission, and if the axis of rotation is perpendicular to this direction and to the incident ray, the conditions are those obtaining in the ordinary X-ray spectrometer. The customary orientations of the crystal are those in which the axis of rotation lies in one of its important planes. That both ordinary and extraordinary diffraction occurs in this case is apparently shown in results recently reported by Clark and Duane[1] for the case of *KI* crystals. The peak *X* which they obtain would be an extraordinary diffraction maximum in the sense of this analysis. Its location with respect to the ordinary maxima would, of course, depend upon the fixed sum of the angles of incidence and diffraction which would not, as in the ordinary case, be equal.

If the incident radiation is monochromatic there are no diffracted spots of either sort except for particular values of l_1, m_1, n_1, and no extraordinary diffraction for any direction of incidence if $\lambda_1 > \lambda_2'$. This case is of no practical importance, but if the single crystal is replaced by a great number, oriented at random these particular values of l_1, m_1, n_1 occur and the ring-pattern (Hull or Debye-Scherrer pattern) is obtained. Both ordinary and extraordinary patterns can occur. It was, in fact, phenomena observed in this case that led to the explanation here offered.

Both the ordinary and extraordinary spots obtained in the last mentioned type of experiment are, if the individual crystals are not too small, replicas of the source in the aspect which it

[1] Clark, G. L., Duane, Wm., N. A. S. Proc. *8*, pp. 90–96; May, 1922.

There is a curious error in this paper at the point where the spacing for lattice-planes making an angle of 17°.84 with the planes (100) is calculated, apparently by the formula $d = a \sin 17°.84$. There are no lattice planes with low indices inclined at this angle to the (100) planes of a cubic space-lattice and if we find integral values of h and k such that $\tan 17°.84 = h/k$ the spacing of these planes ($h\ k\ 0$) would be given by

$$d = \frac{a}{\sqrt{h^2+k^2}} = \frac{a}{h} \sin 17°.84.$$

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presents to the crystals, i.e., narrow elliptical outlines. The extraordinary spots are distinguishable from the ordinary spots, however, by several peculiarities. They are not so sharply defined, which may be attributed to the comparative rarity of atoms which emit characteristic X-rays as compared with those which merely scatter the incident beam. The greater complexity of the process in extraordinary diffraction may also account for greater variability in ϕ_1, ϕ_2, and τ, and consequent diffuseness in that case. The extraordinary spots can be inclined at greater angles to the lines joining them with the trace of the incident rays upon the film. This is due to the greater complexity in the extraordinary case of the expressions for l_2, m_2, n_2 which causes the direction of emission to vary less directly with the direction of incidence. The extraordinary spots frequently form parallel groups which are in fact characteristic emission spectra where the different values of λ_2 have been resolved by diffraction. This resolution, like all the extraordinary effects, is better for pure metals than for alloys, even if the ordinary diffraction for the two materials is equally sharp indicating crystals of similar size and regularity. The extraordinary spots are more absorbable than the ordinary spots, and are relatively more reduced in intensity by filters designed to improve contrast in the ordinary pattern. This prediction of the theory has been checked experimentally by omitting the filters and thereby reducing the exposure necessary to obtain marked extraordinary effects. The most striking peculiarity of the extraordinary spots is, however, that they can be found closer to the center of the pattern than can the ordinary spots. This closeness in itself enhances the intensity of the photographic effect by superposing the spots from various crystals within a more limited area, so that the first extraordinary ring of the combined pattern may even exceed in intensity the first ordinary ring. That this is not a spurious effect due to imperfect screening is conclusively shown by its complete absence under identical experimental conditions when the crystals contain no atoms of low enough atomic number to yield characteristic K-radiations when exposed to the molybdenum K-radiations available. No extraordinary diffractions of the molybdenum

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radiations have been observed with silver, palladium, or gold, while they are always present to some extent with iron, nickel, and copper.

If the proposed explanation be accepted as sound, it appears probable that all the entities causing characteristic secondary emission are exactly alike, and that such secondary emission only occurs when these entities meet the atoms under a very limited range of conditions. The evidence that extraordinary diffraction takes place is then in favor of the existence of spatially limited energy quanta in the incident beam, and also in favor of cyclic motions within the atom which cause occasional recurrences of configurations unstable when coincident in time with the presence of a passing quantum of sufficient energy. A quantitative study of the new phenomena may be expected to give valuable information regarding its possible dependence upon the direction of incidence, the nature of inter-atomic bonding and other factors of interest to students of atomic structure and the nature of luminous radiations of all wave-lengths.

I desire in conclusion to express my appreciation of the interest and helpful suggestions of my colleague, Dr. K. K. Darrow, in the analysis here presented.

RESEARCH LABORATORIES, OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND THE
WESTERN ELECTRIC COMPANY, INCORPORATED,
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ON THE QUANTITY OF LIGHT ENERGY REQUIRED TO RENDER DEVELOPABLE A GRAIN OF SILVER BROMIDE

By P. S. Helmick[1]

The work of Kinoshita[2] seems[3] to be one of the first published researches to permit a maximum estimate of the energy required to render developable a grain of silver bromide. His work shows: (1), that each silver bromide grain is rendered capable of development when struck by an alpha-particle from Radium C; and (2), that the reduction of developable grains cannot extend to neighboring un-ionized grains. As the kinetic energy possessed by an alpha-particle of Radium C is 1.31×10^{-5} ergs[4] this value gives a maximum limit to the energy per grain necessary to form a developable image.[5]

Two years later Einstein developed his photo-chemical Equivalence Law[6] which requires the absorption of a mean energy of effective radiation $h\nu$ for the photo-chemical decomposition of a molecule.[7]

By the application of the Equivalence Law, Henri and Wurmser[8] submitted the results of Leimbach[9] to a quantitative examina-

[1] National Research Fellow in Physics.

[2] Proc. Roy Soc., *83A*, p. 432; 1920.

[3] In 1891, Hurter and Driffield (Memorial Volume, Royal Photo. Soc., p. 151) calculated the energy per cm^2 necessary to produce a *latent image*. They were subsequently led to the conclusion (*ibid.* p. 228) that the light causes some change in the molecular structure of silver bromide.

[4] Smithsonian Physical Tables. 7th Ed. p. 396.

[5] Svedberg and Andersson (Phot. J. *61*, p. 325, 1921) have lately quoted the opinions of Michl, and St. Meyer and v. Schweidler, which indicate that only a certain fraction of silver halide grains in the track of an alpha-particle are made developable However in a recent critical review of the entire evidence, Muhlestein (Arch. Sci. Physiques et Naturelles, *5*, p. 38; 1922) comes to the conclusion supported by his own work, that the results of Kinoshita are in accordance with the facts.

[6] Ann. der Phys. *37*, p. 832; 1912.

[7] Plotnikow (Zeit. wiss. Phot. *21*, p. 134; 1922) calls the Einstein relation a *formula* rather than a *law*.

[8] Journ. de Physique. *3*, p. 305, 1913.

[9] Zeit. wiss. Phot., *7*, pp. 157, 181; 1909.

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tion in order to find the number of quanta necessary to render developable a molecule of silver bromide. Leimbach found that for monochromatic light made up of wave-lengths between 0.415μ and 0.475μ, 0.63 ergs/cm^2 were necessary for unit density.[10] Taking unit density as equivalent to 0.000103g of silver per cm^2 of plate surface,[11] or 7×10^{17} molecules of silver per cm,^{2} his results indicate that 9×10^{-19} ergs are required per molecule of silver bromide. Henri and Wurmser seem to consider the wave-length 0.415μ responsible for the photochemical process, consequently at this frequency a quantum is equal to 4.74×10^{-12} ergs. Thus it appeared that a quantum could render developable 4 million times as many molecules as Einstein's Law would predict.[12] Consequently Henri and Wurmser concluded that light seems only to act as a catalyser, placing the molecules in a state where they will react by themselves.

[10] Density $= \log_{10} \dfrac{\text{light incident on plate.}}{\text{light transmitted by plate}}$.

[11] Cf. Sheppard and Mees. *Investigations on the Theory of the Photographic Process.* Longmans, 1907, p. 41.

[12] Baly (Phil. Mag. *40*, p. 15, 1920) believes that this divergence is due to the reabsorbtion of energy which the molecules have radiated internally. However, an alternative explanation of this divergence might be based upon the fact demonstrated by Svedberg (Nature. *109*, p. 221, 1922), that from one to four nuclei or "centres," formed inside a silver bromide grain by the action of light, are effective in making developable all of the molecules in the grain. Assuming: (1), that by the action of a quantum, c "centres" are formed in each grain; (2), that each grain is a circular disc of thickness equal to 1/14 of its diameter,—this is the thickness found by Trivelli and Sheppard (*The Silver Bromide Grain.* Van Nostrand. 1921, p. 94); and (3), that the density of a silver bromide grain equals the density of precipitated silver bromide as found by Karsten (*Handbuch Anorgan. Chemie.* Friedheim u. Peters. 1914. Vol. 5, Div. 2, p. 108), 6.35 gm/cm^3, the following mean diameters (d) of grains result from Henri and Wurmser's figures, depending upon the number of "centres" (c) assumed present per grain: $c=1$, $d=0.16\mu$; $c=4$ (maximum number yet found by Svedberg), $d=0.25\mu$; $c=78$, $d=1.5\mu$ (mean diameter found by Trivelli and Sheppard). From the point of view of Svedberg's work, the results would seem more logical if it be assumed that more than one quantum at wave-length 0.415μ is necessary to form a "centre." For example, the observed quantity of silver would be accounted for, if an average of 6 "centres" were formed in each grain with energy equal to 13 quanta per "centre." However, the work of Eggert and Noddack which will be considered at a later point indicates that as many as several hundred "centres" can be formed in a grain.

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In the same year, Nutting[13] calculated the energy required to render developable a grain of silver bromide. He considered a photographic plate which required an energy of 10^{-7} ergs/cm^2 to produce a deposit of 1/10 mgm of silver per cm^2, or 10^7 grains 3μ in diameter. Therefore, each grain receives 10^{-14} ergs to make it developable, although to produce a gaseous ion only about 5×10^{-12} ergs are required. No data are given regarding the frequency of the light,[14] but the wave-length must lie outside the range 0.450μ to 0.650μ, for in that region Leimbach[15] and also the writer[16] have found that energies of the order of 1 erg/cm^2 to 4000 ergs/cm^2 are required. Consequently Nutting concludes that it is a reasonable hypothesis to consider a latent image as composed of a halide salt, from each of whose grains one electron has been liberated by exposure to light.[17]

Another determination of the energy required to make a grain of silver bromide developable is found in the work of O. H. Smith[18] with retrograde rays from a cold cathode. The power of a moving particle to affect a photographic plate seemed to be a function of the kinetic energy possessed by the particle. The minimum energy required to produce the faintest trace visible to the naked eye was about 7.4×10^{-9} ergs for the heaviest particles,—a value greater than the energy required to produce a gaseous ion. As the work of Kinoshita[19] and others[20] proves that the grain of silver bromide is the photo-chemical unit in the photo-

[13] Nature, *92*, p. 293, 1913.

[14] Mees (Jour. Frank Inst., *179*, p. 141, 1915), states that these figures refer to violet light.

[15] Loc. cit.

[16] Phys. Rev. *17*, p. 135, 1921.

[17] As opposed to this view, Renwick (Jour. Soc. Chem. Ind. Trans. *39*, p. 156; 1920) believes that the latent image is formed by a change in the highly unstable and light-resonant form of colloidal silver existing in a solid solution in the crystalline silver bromide of the emulsion.

[18] Phys. Rev., *7*, p. 625; 1916.

[19] Loc. cit.

[20] Joly Nature, *72*, p. 308; 1905.
Mees, Jour. Frank. Inst. *179*, p. 141, 1915; *191*, p. 631, 1921.
Slade and Higson, Roy. Soc. Proc. *98*A, p. 154; 1920.
Svedberg, loc. cit.

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graphic plate, the results of Smith would indicate that about 7.4×10^{-9} ergs or less[21] are required to render a grain developable.

Very recently Eggert and Noddack[22] made a preliminary measurement of the energy of wave-length 0.408μ necessary to produce a developable grain of silver bromide. In their first experiments it was found that after exposure to light a small amount of silver was separated from the silver bromide of the emulsion. It may thus be possible that these atoms of separated silver correspond to the "centres" mentioned by Svedberg.[23] Chemical analysis showed that for each quantum absorbed by the silver bromide one atom of silver was liberated. Other measurements indicated that in order to produce a developable grain, "a few hundred" quanta must be absorbed. The conclusion was reached that only those silver bromide grains would be developed in which the separated silver atom was directly located on the outside surface of the grain, and grains which possessed silver atoms in their interior would behave as if unilluminated. To support this theory they mentioned that one silver bromide grain of the kind found in their plates contained one surface molecule to 300 interior molecules. They have stated as their conclusion: not every quantum gives a grain of silver, but each grain of silver corresponds to one and only one quantum.

THE NATURAL ULTRA-VIOLET FREQUENCY OF SILVER BROMIDE

Before dealing further with the question of the energy necessary for the production of a developable grain of silver bromide, an attempt will be made to predict a natural frequency of silver bromide for the ultra-violet by calculating the ultra-violet maximum of the selective photo-electric effect.[24]

[21] It is possible that this value should be somewhat reduced because of the phenomenon mentioned by Sheppard and Mees,—(loc. cit. p. 279), that an image may be invisible to the eye, but still contain numbers of grains easily counted under the microscope.

[22] Physikal. Zeit., *22*, p. 673; 1921.

[23] Loc. cit.

[24] Cf. Pohl and Pringsheim, Verh. d. Deutsch. Phys. Ges. *13*, p. 474, 1911.

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It is possible to calculate the free period of silver bromide in the ultra-violet by making use of the quantum relationship given by Lubben:[25]

$$\nu_{\text{ion}} = \nu_{\text{undissolved salt}} + 2Q/Nh$$

where ν is the critical frequency, Q is the heat of solution of the substance, N is Avagodro's constant, and h is Planck's element of action. He states that in general the dispersion of colorless salt solutions in the visible and ultra-violet is attributable to the free periods of the anions. The kations possess free periods in the infra-red and farthest ultra-violet, whose influence very seldom extends into the visible or attainable ultra-violet regions. As a result of his measurements, the free period of the bromine anion was found to be 1.61×10^{15} sec^{-1}, which corresponds to 0.186μ.

Setting $Q = -20{,}100$ calories,[26] $N = 6.06 \times 10^{23}$,[27] and $h = 6.55 \times 10^{-27}$ erg sec.,[27]

$\nu_{\text{undissolved salt}} = 2.03 \times 10^{15}$ sec^{-1}, which corresponds to a wave-length of 0.148μ.

Another quantum relationship which can be used to obtain the ultra-violet frequency of silver bromide, is the expression $Q = Nh\,(\nu_{\text{resultants}} - \nu_{\text{reactants}})$. This expression may also be written Q equals critical increment of the resultants minus critical increment of the reactants. Haber[28] and Lewis[29] have shown that this formula is in good agreement with experimental results.

Following Lewis,—the observed heat of formation of liquid bromine and solid silver is 22,700 calories per gram molecule.[30] As the Haber-Lewis formula assumes that the bromine is in the gaseous form,[31] the heat of vaporization of bromine or 3470 calories[32] must be taken into consideration, consequently the heat of formation of solid silver and gaseous bromine is 26,170 cal per gm mol.

[25] Ann. der Physik. *44*, p. 977, 1914.
[26] *Thermo Chemistry*. J. Thomsen. Tr. by Burke. Longmans p. 137, 1905.
[27] Millikan. Phil. Mag. *34*, p. 1, 1917.
[28] Ber. Deutsch. Physik. Ges., *13*, p. 1117; 1911.
[29] Jour. Chem. Soc., *111*, p. 1086; 1917.
[30] Landolt-Bornstein. Tabellen. P. 868; 1912.
[31] Lewis, loc. cit., p. 1092.
[32] Tabellen. P. 834.

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No direct data seem to exist regarding the critical ultra-violet frequency of silver, so recourse must be had to calculation. Probably the best value of the mean infra-red frequency of silver is that given by the Nernst-Lindeman formula, $\nu_r = 4.5 \times 10^{12}$ sec^{-1}.[33]

A rule due to Haber[34] enables the characteristic ultra-violet frequency to be found:

$$\nu_v / \nu_r = \sqrt{M/m}.$$

ν_v is the characteristic frequency corresponding to the maximum of the selective photoelectric effect, ν_r is the characteristic infra-red frequency of the substance, M is the weight of an atom of the substance, and m is the mass of an electron.

Taking $M_{Ag}/M_H = 107.88/1.008$,

$M_H = 1.662 \times 10^{-24}$g,[35]

and $m = 9.01 \times 10^{-28}$ g.,[36]

As Lewis postulates that an ultra-violet quantum breaks the bond between two adjacent atoms, the critical increment for one gram atom of silver equals $Nh\nu_v/2$, or 94864 cal.

From a consideration of the heat of dissociation and the ultraviolet absorbtion band of bromine, Lewis[37] takes the value 28,500 calories for the critical increment of one gram-atom of gaseous bromine.

As substitution in the expression Q = critical increment of resultant—critical increment of reactants gives

$$\nu_{AgBr} = 1.58 \times 10^{15} \text{ sec}^{-1},$$

the wave-length corresponding to the critical ultra-violet frequency of silver bromide equals 0.190μ.

A more direct determination of the critical ultra-violet frequency of silver bromide can be made by the sole use of Haber's formula, as already stated:

$$\nu_v / \nu_r = \sqrt{M/m},$$

[33] Zeit. Elektrochem. *17*, p. 822; 1911.
Cf. Lewis.

[34] Ber. Deut. Physikal. Ges., *13*, p. 1117; 1911.

[35] Millikan. Phil. Mag. *34*, p. 1; 1917.

[36] Smithsonian Tables, p. 408; 1920.

[37] Loc. cit.

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or its equivalent,

$$\lambda_v/\lambda_r = \sqrt{m/M},$$

where λ_v and λ_r are the wave-lengths corresponding respectively to the critical ultra-violet and the critical infra-red frequencies of the substance, m is the mass of an electron and M is the molecular weight of the substance.

Taking $M_{AgBr}/M_H = 187.80/1.008$, assuming Rubens[38] mean value of 112.7μ for the characteristic residual ray from silver bromide, and giving to the other constants the same values as have been used in the preceding paragraphs, the wave-length corresponding to the critical ultra-violet frequency of silver bromide is equal to 0.192μ.

Finally, the ultra-violet frequency entering into the selective photo-electric effect will be calculated for silver bromide according to Lindeman's formula:[39]

$$\nu_v = \sqrt{ne^2/mr^3}/2\pi,$$

where n is the valency of the atom to which the electron belongs, m and e are the mass and the charge of an electron, and r is one-half the distance between two neighboring atoms.

By X-ray analysis, Wilsey[40] has found that silver bromide gives a diffraction pattern of a simple cube with sides of 2.89A, one atom being associated with each point of the lattice.

Taking $r = 2.89/2$ A, $m = 9.01 \times 10^{-28}$ g,[41] and $e = 4.774 \times 10^{-10}$ e. s. u.,[41] $\nu_v = 1.45 \times 10^{15}$ sec^{-1}, or $\lambda_v = 0.207\mu$.

To recapitulate, by means of the formulas given below the following values of the free ultra-violet period of silver bromide have been calculated:

1) Lubben's expression.

$$\nu_{\text{ion}} = \nu_{\text{undissolved salt}} + 2\,Q/Nh.$$
$$\lambda_v = 0.148\mu.$$

2) Haber and Lewis.

$$Q = Nh(\Sigma\nu_{\text{resultants}} - \Sigma\nu_{\text{reactants}}).$$
$$\lambda_v = 0.190\mu.$$

[38] Sitz. Akad. Wiss. Berlin, p. 513; 1913.
[39] Ber. Deutsch. Physikal. Ges., *13*, p. 1107; 1911.
[40] Phil. Mag., *42*, p. 262; 1921.
[41] Loc. cit.

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3) Haber's rule.

$$\nu_v/\nu_r = \sqrt{M/m}.$$
$$\lambda_v = 0.192\mu.$$

4) Lindeman's formula.

$$\nu_v = \sqrt{ne^2/mr^3}/2\pi.$$
$$\lambda_v = 0.207\mu.$$

Thus it appears that the critical frequency of silver bromide corresponds to a wave-length of about 0.190μ.

Some of the existing experimental work indicates that this value of 0.190μ is of the right order of magnitude. In 1893 Schumann[42] stated that the indications were that the absorbtion of silver bromide reaches a maximum at 0.210μ. Eight years later he found[43] that the ultra-violet sensibility of Schumann plates first becomes particularly noticeable at 0.220μ and increases very rapidly towards the ultra-violet. Thus his work shows a region of resonance in the neighborhood of the calculated value of 0.190μ.

Nutting[44] states that a photographic plate is practically uniformly sensitive from 0.5μ to the ultra-violet, and Mees[45] finds that the photographic sensitiveness of pure silver bromide is constant for wave-lengths less than 0.480μ.

Compton and Richardson[46] have shown theoretically and experimentally that if the wave-length λ_0 corresponds to the threshold photo-electric or photo-chemical sensitiveness, then the maximum of the selective effect should occur at wave-length $\lambda_m = 2\lambda_0/3$. In the case of silver bromide, this would indicate resonance at a wave-length of 0.320μ, a value which is not in very good agreement with previous calculations.

The latest work throwing light upon the critical ultra-violet frequency of silver bromide seems to be that of Slade and Toy.[47] Investigating the change in the extinction coefficient of silver

[42] Ber. Wien Akad. Wiss. *102* IIA, p. 465; 1893.
[43] Ann. der Physik. *5*, p. 373; 1901.
[44] *Outlines of Applied Optics*. p. 223; 1912.
[45] Jour. Frank. Inst., *179*, p. 141; 1915.
[46] Phil. Mag., *26*, p. 553; 1913.
[47] Proc. Roy. Soc., *97*A, p. 181; 1920.

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bromide with wave-length, they found that the coefficient increased from a value of 270 cm^{-1} at a wave-length of 0.450μ to a value of 6700 cm^{-1} at a wave-length of 0.360μ. Therefore, a resonance frequency for silver bromide must exist farther out in the ultra-violet than 0.360μ.

Thus all of the experimental evidence predicts a resonance frequency of silver bromide in the ultra-violet,—the work of Schumann fixes the corresponding wave-length at about 0.210μ or in the region of the calculated value, while the work of Mees indicates a greater value.

EXPERIMENTAL DETERMINATION OF ENERGY REQUIRED TO BLACKEN A GRAIN OF SILVER BROMIDE

Having this knowledge of the resonance frequency of silver bromide it was now thought of interest to measure the quantity of light energy which is necessary for the transformation of a grain of silver bromide to a developable condition. For a preliminary experiment, a wave-length of 0.540μ,—much longer than the wave-length corresponding to the critical frequency of silver bromide, was chosen, so that no particular difficulty would be experienced in the energy measurements.

To carry out the experimental manipulations, an ordinary photographic film was first exposed to a known quantity of approximately monochromatic light energy, then developed at constant temperature, fixed and washed, and finally the number of reduced grains was determined by direct microscopic counting.

A photographic film furnished the silver bromide grains. It must, of course, be borne in mind as Mees[48] has pointed out, that these elementary "silver bromide" grains contain some silver iodide, with possibly some absorbed gelatine and soluble bromide. Some workers, as for example Slade and Higson,[49] have prepared experimental emulsions by dissolving and diluting commercial plate-emulsions and coating glass slips with this diluted emulsion. This method produces a slide containing but a

[48] Jour. Frank. Inst., *191*, p. 631; 1921.
[49] Loc. cit.

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single layer of grains, but possesses according to Renwick,[50] through dissolving and diluting the emulsion, the grave danger of inducing chemical fog.

LIGHT SOURCE

The primary source of monochromatic light made use of in these experiments was a carbon strip-filament lamp operated with a constant storage battery current of 10 amperes. Light from this lamp traversed the optical system of a Hilger Ultra-Violet Monochromatic Illuminator[51] of aperture about $f/5.4$ at 0.540μ, and the emergent beam formed a spectrum in the focal plane of the instrument.

With a linear thermopile inserted in the arm of the illuminator the light intensity of a small portion of the spectrum was measured in absolute units. A low-power microscope with micrometer eyepiece focussed upon the rear of the thermopile strips, permitted the adjustment of the wave-length drum and enabled the accumulation of data used in calculating the amount of spectral overlapping. A very large slit of 4 mm width was used in order that its image should cover the whole receiving surface of the thermopile.

Interchangeable and of the same size as the thermopile was a film holder which could also be placed in the arm of the illuminator, allowing a piece of photographic film to be exposed in the focal plane of the illuminator. In order to reduce reflection from the back surface of the film, the holder was coated with a layer of gelatine and lampblack or in some cases black laboratory wax, and the film was attached in optical contact to this absorbing layer.

Exposures were made by hand with a sliding shutter, while listening to a watch ticking fifths of seconds. An exposure of two seconds was the shortest given, and for this most unfavorable case the maximum error of exposure amounted to about 8 per cent.

[50] Phot. Jour., *61*, p. 333; 1921.

[51] For the purpose of this work, Professor W. M. Clark of the Hygenic Laboratory, Washington has very kindly loaned this instrument through the medium of the National Research Council.

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ENERGY MEASUREMENT

The linear thermopile already referred to in connection with the monochromatic illuminator was also made by Hilger, and had a resistance of 13 ohms. Both before and after energy determinations it was directly calibrated against a standard lamp whose radiant flux[52] in a definite direction at a given distance had been certified to by the Bureau of Standards with an accuracy of 1 per cent.

By using a D'Arsonval galvanometer whose figure of merit was 2×10^{-7} amp/mm, and by taking the mean of ten or fifteen observations, it was possible to maintain a value of the relative error less than 1% or ½%.

An error far greater than that entering into the thermopile calibration was caused by the spectral overlapping due to the use of a very large slit width. A slit width of 4 mm was employed in order that the image of the slit should cover the whole receiving surface[53] of the thermopile.

This spectral impurity manifests itself both upon the photographic film and upon the thermopile.

In the measurement of the photographic film, only those silver bromide grains were chosen which were situated along a vertical line in the geometrical center of the photographic image formed by the spectral band. The illuminator was adjusted for the experiment so that when illuminated by light of wave-length 0.540μ, the entire image of the collimator slit was in coincidence with the thermopile strips.

Now when an exposure was made with the carbon lamp in front of the illuminator a small area ΔA in this median line received an *ensemble* of light energy approximately proportional to

$$E=\int_{\lambda=0.540-w/2k}^{\lambda=0.540+w/2k} E_\lambda d\lambda,$$

where $E_\lambda d\lambda$ is the energy in the spectral region between λ and

[52] The radiant flux for this lamp was $(63.8\pm0.64)\times10^{-8}$ watts/mm²

[53] 4 mm by 20 mm.

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$\lambda + d\lambda$, w is the width of the spectral image of the collimator slit, and k is the dispersive power of the instrument in mm. displacement of spectral image per μ of wave-length.[54]

For a rough application of this formula assume; (1), that the temperature of the carbon lamp is 1775° C[55] a normal temperature which corresponds to an efficiency of 3 watts per candle; (2), that the emissive power of the carbon filament is constant for each wave-length; and (3), that absorbtion in the apparatus and in the glass envelope of the lamp is constant for each wave-length.

Calculating the distribution of energy E_λ in the spectrum by Planck's formula[56] it is found that the relative energies of wave-lengths actually incident upon ΔA range as follows:

$$\begin{aligned}
\lambda &< 0.505\mu, & E_\lambda &= 0.00,\\
\lambda &= 0.505\mu, & E_{0.505} &= 0.57,\\
\lambda &= 0.540\mu, & E_{0.540} &= 1.00,\\
\lambda = \bar{\lambda} &= 0.549\mu, & E_{0.549} &= 1.14,\\
\lambda &= 0.580\mu, & E_{0.580} &= 1.71,\\
\lambda &> 0.580\mu, & E_\lambda &= 0.00.
\end{aligned}$$

The "mean" wave-length

$$\lambda = \int E_\lambda \lambda d\lambda / \int E_\lambda d\lambda$$

is found to be

$$\lambda = 0.549\mu.$$

Thus according to these data, the silver bromide grains were exposed to a band of wave-lengths comprised between 0.505μ and 0.580μ, the "mean" wave-length of the band corresponding to 0.549μ.[57]

To calculate the effect of spectral impurity on the readings of the thermopile, consider that the slit image caused by light of wave-length $\lambda \neq 0.540\mu$ is of the same size as the image $\lambda = 0.540\mu$, but displaced from it a distance $d = k(0.540 - \lambda)$. As the thermopile strips are the same width w as the 0.540μ image, for small

[54] At $\lambda = 0.540\mu$, the dispersive power is 56 mm/μ.

[55] Cf. J. A. Fleming. Art. "Lighting." Encyc. Britt. 11 Ed.

[56] $c = 14320$ micron degrees. Coblentz., Jour. Opt. Soc. of Am., *5*, p. 131; 1921.

[57] It is interesting to note that Toy (Proc. Roy. Soc., *100*A, p. 109; 1921) finds that for a spectral range of 0.030μ, radiations of different frequencies do not act independently in producing photo-chemical change, but probably as a total amount irrespective of any difference in quality.

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displacements the displaced image will overlap them by an amount $w-k\mid 0.540-\lambda\mid$,

where $\quad 0\leq \mid d\mid \geq w.$

Accordingly, the amount of energy between the limits λ and $\lambda+d\lambda$ which the thermopile receives is

$$E'_\lambda d\lambda = E_\lambda\{1-k\mid 0.540-\lambda\mid /w\}d\lambda.$$

As a consequence, when exposed to the whole slit image the thermopile receives energy equal to

$$E''=\int_{d=-w}^{d=w} E_\lambda(1-k\mid 0.540-\lambda\mid /w)d\lambda.$$

Taking the same values for constants as above, some of the relative energies E''_λ for the different wave-lengths as received by the thermopile are given by:

$$\begin{aligned}
\lambda &\leq 0.470\mu, & E''_\lambda &= 0.00,\\
\lambda &= 0.500\mu, & E''_{0.500} &= 0.21,\\
\lambda &= 0.540\mu, & E''_{0.540} &= 1.00,\\
\lambda = \bar{\lambda}' &= 0.558\mu, & E''_{0.558} &= 1.03,\\
\lambda &= 0.600\mu, & E''_{0.600} &= 0.60,\\
\lambda &\geq 0.620\mu, & E''_\lambda &= 0.00.
\end{aligned}$$

The "mean" wave-length $\bar{\lambda}'$ is found by taking the center of gravity of the curve in a manner corresponding to the previous example. Thus the thermopile received energy of wave-length from 0.470μ to 0.620μ, the mean wave-length of the band corresponding to 0.558μ.

It is interesting to note that with a value of $c_1=3.86\times10^4$ watts/cm^2,[58] assuming that the filament acts as a black body radiator, and with the above assumption as to temperature of filament, and no absorbtion in the apparatus, the calculated energy incident (E'') upon the thermopile was 2.0×10^{-7} watts/mm^2, whereas the energy actually measured was 1.9 times this figure, or 3.8×10^{-7} watts/mm^2. To account for this discrepancy, it is possible that the filament temperature has been assumed too low, but in any case, the figures above give some indication of the nature of the errors in both the photographic and the thermopile measurements.

[58] Smithsonian Physical Tables, 7th Ed. p. 247.

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USE AND CONSTRUCTION OF ABSORBING SCREENS

In order to reduce the intensity of light to a very small value, a series of lamp-black-in-gelatine absorbing screens was made for insertion between the collimator lens and the prism of the illuminator. These screens were prepared by coating levelled squares of plate glass 5.5 cm$\times$5.5 cm. with 0.2g. of gelatine dissolved in 4 cm^3 of water. A certain quantity of lamp-black suspended in alcohol was added to the gelatine solution, depending upon the opacity desired for the screen. A small quantity of carbolic acid was added as a preservative, and after drying, the two gelatine surfaces were cemented together with Canada Balsam.

The screens of greatest transparency were directly calibrated in the illuminator by means of the thermopile, but this proceeding was impossible with the denser screens. But by measuring once for all with a spectrophotometer the ratios of the transparencies T_2/T_1, T_3/T_1, . . . , T_n/T_1, of n screens to any wave-length, and then measuring T_1, T_2, and T_3 by means of the thermopile, for the wave-length to be used in the illuminator, the transparencies of the n different screens were determined for the illuminator wave-length.

For the purpose of this calculation assume three plane-parallel plates P_1, P_2 and P_3, of the same isotropic absorbing material of thicknesses d_1, d_2, and d_3. If I_0 is the intensity of incident light of wave-length λ, and I_1, I_2, and I_3 are the respective intensities of light transmitted by the three plates, the following equations hold:

For $\lambda=\lambda$, $I_n=aI_0e^{-kdn}$, $n=1, 2, 3,$

and for $\lambda=\lambda'$, $I_n=a'I'_0e^{-kd'n}$ $n=1, 2, 3,$

If we set $D_n=\log_{10}(I_0/I_n)$, $n=1, 2, 3,$

and $D_n'=\log_{10}(I'_0/I_n')$, $n=1, 2, 3,$

then (1), $D_3=D_1+(D_2-D_1)(D'_3-D_1')/(D_2'-D_1')$.

This formula can be applied to the case of the absorbing screens by the following argument. As there is the same thickness of glass, Canada Balsam, and gelatine in the case of each absorbing screen, it is possible to choose a's, and a''s which will

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take this condition into account. Now if the d's and d''s be considered "equivalent thicknesses," i.e., the number of cms. of standard substance which has the same absorbtion as a thickness d or d' of the carbon of the screen, the formula is directly applicable to the case of the screens. It is true that the formula assumes that the density measurements are made with monochromatic light, a condition approximated very closely in the case of the spectrophotometer, but not very closely in the case of the illuminator. However, the work of Toy and Ghosh[59] shows that lampblack has practically equal absorbtion for all wave-lengths within the range here used.

The spectrophotometer used in the calibration of the absorbtion screens was made up of two Pointolite lamps which furnished illumination for two spectroscopes. The spectroscopes were oriented so that the emergent pencils of light were at right angles to each other and came to a focus upon the silvered plane of a photometer head viewed with a suitable eyepiece.

To construct the photometer head, the hypotenuse of a right-angled prism was silvered in strips 0.75 mm. wide and the same distance apart. Another right angled prism was then cemented to the strip-silvered prism with the two hypotenuses in contact.

The field of view was consequently a set of adjacent slit-images situated one above the other. Each slit-image could be made to match its neighbors by the relative rotation of a pair of nicol prisms placed in the path of one of the light-beams. When an absorbing screen was placed in the path of one of the light-beams, its transmission could be found from the reading of the nicols.

The arrangement was such that there were no errors introduced as the nicols were rotated due to changes in angle between the plane of polarization of the light and the normal to any surface struck by the polarized light.

The accuracy attainable with the instrument was such that in a series of 25 readings the probable error in setting the nicols for extinction amounted to 3′ of arc.

[59] Phil. Mag. *40*, p. 775; 1920.

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DEVELOPMENT APPARATUS

After the film had been exposed, it was immediately developed and fixed in total darkness. The developer was of amidol, as used by Slade and Higson[60] who found that at 18° C. the exposed grains were completely developed in 3 minutes, but that unexposed grains were not affected in 5 minutes. Fresh developer was mixed up for each film. In this work it was decided to develop at a temperature of 23° C. for a period of three minutes.

In order to provide for a constant temperature, a metal developing box was constructed, which was attached inside a tank containing 50 litres of water,[61] in such a manner that the water was in contact with five sides of the box. The whole was enclosed in a wooden box provided with a door giving access to the developing box. To ensure uniformity of temperature throughout the bath, the water was kept in motion by an electrically driven stirrer.

A mercury regulator was arranged to control the temperature of the bath. A thin steel tube 1 cm. in diameter and 1 m. in length was filled with mercury and coiled about inside the bath, one end protruding from the tank. Expansion of the mercury forced a slender column of mercury through a constriction in the tube until electric contact was made against an adjustable nickel contact point, and electric lamps submerged in the water of the tank would operate until contact was again broken. Tests showed that the temperature regulation was carried out to an accuracy of at least 0.°01 C.

After developing and fixing, and thorough washing, the film was ready for a microscopic examination.

MICRO-PROJECTION

The determination of the number of silver bromide grains per unit area of film was made with the use of a micro-projection apparatus. An enlarged image of the film was cast upon a screen,

[60] Loc. cit.

[61] This developing apparatus was constructed by the writer at the State University of Iowa.

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and the grains through the various layers of the film were counted by progressively varying the focus of the microscope.

The source of light was a commercial reflecting 400 watt gas-filled microscope lamp. A 1.8 mm oil-immersion objective with 12.5 times eye-piece was used in the microscope.[62]

To absorb heat rays which would otherwise start immediate combustion of the film, a water cell with a 7.5 cm thickness of water was placed between the lamp and the condenser. A micrometer eyepiece ruled in 1 mm squares was placed in the eyepiece, and thus a co-ordinate system was furnished. A magnification of about 1000 was used for visual counting. One mm on the scale of the eyepiece micrometer represented 15μ upon the stage of the instrument.

With small blackenings it was a very simple matter to count the grains, for most of them lay near the surface, and they were not in close proximity. But with the greater densities there was a tendency for many of the grains to occur in groups, and because of the inevitable overlapping of grains as viewed in the microscope, it was sometimes difficult to decide whether one or more grains were visible.

It was assumed that the net number of developed grains could be approximated by counting the number of developed grains upon the unexposed portion of the film, and deducting this "fog reading" from the total number of developed grains as counted upon the exposed portion of the film.

The following table gives the results of the work. If it be assumed that equal light energies give equal blackenings,[63] then the number of quanta per grain should be the same, no matter whether the flux of light energy be great or small. If, then, the number of quanta per grain be averaged over all the experiments, it is found that about 2.3 quanta of incident light[64] of mean wave-

[62] The writer is indebted to the Department of Biology of Princeton University for the loan of this microscope.

[63] For references see, Helmick. Phys. Rev., *9*, p. 372; 1918.

[64] Preliminary results of work now in progress show that the diffuse reflection from a silver bromide emulsion is of the order of 10% for wave-length 0.540μ, while the amount of light transmitted by the film is roughly 5%. At this wave-length the absorption of light by the gelatine is negligible.

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length 0.549μ are necessary to render developable a grain of silver bromide.

EXPERIMENTAL RESULTS

Film number	Fog Reading		Exposed Reading		Net Grains Developed per mm²	Length of Exposure sec.	Light Energy in Quanta (at 0.549μ) per mm.	Quanta (at 0.549μ) per Grain	Fraction of Grains Fogged
	Grains counted	Grains per mm²	Grains counted	Grains per mm²					
27	277	49200	2629	467400	418200	2	213000 ± 2200	0.51	0.11
60	255	45330	2513	446800	401470	20	413000 ± 5600	1.03	0.10
61	218	40370	982	174600	134230	10	207000 ± 2800	1.54	0.23
62	240	35550	882	145200	109650	4	82700 ± 1100	0.76	0.24
63	239	42490	626	111300	68810	2	41400 ± 560	0.60	0.38
64	251	46630	526	93520	48890	2	41400 ± 560	0.85	0.48
65	242	43020	649	115300	72280	2	41400 ± 560	0.57	0.39
66	330	39640	958	115100	75460	10	207000 ± 2800	2.75	0.34
67	341	38860	889	106800	67940	4	82200 ± 1100	1.21	0.36
68	276	32270	884	106200	73930	4	82200 ± 1100	1.11	0.30
69	337	38410	499	56880	18470	2	41400 ± 560	2.24	0.68
70	381	43420	493	56180	12760	2	41400 ± 560	3.25	0.77
71	304	34650	526	59950	25300	2	41400 ± 560	1.64	0.58
77	361	41140	2539	304900	263760	64	730000 ± 14000	2.88	0.13
78	277	31570	1990	245700	214130	64	730000 ± 14000	3.30	0.13
79	300	34190	1219	146500	112310	32	366000 ± 5600	3.26	0.23
80	322	36700	1232	148000	111300	32	366000 ± 5600	3.29	0.25
81	349	39770	722	82280	42510	16	182000 ± 2800	4.28	0.48
82	298	33960	432	49230	15270	8	91000 ± 1700	5.97	0.69
							Mean	2.27 ±0.25	

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RECENT MEASUREMENTS OF STELLAR AND PLANETARY RADIATION[1]

By W. W. Coblentz

CONTENTS

In a previous report,[2] a summary and bibliography were given of measurements of thermal radiation from stars prior to 1920. The bibliography to the present report is given at the end of this paper.

I. Photoelectric Photometry: As indicated in the preceding report, the potassium hydride photoelectric cell is a useful instrument for photometering celestial objects, such as for example, variable stars.

Stebbins,[1] ,[2] after many trials has succeeded in developing a potassium photoelectric cell of fused quartz, which is much more sensitive than the selenium photometer previously used, thereby enabling him to study sixth-magnitude stars with a 12 in. telescope.

With this devise, Stebbins[1] was able to show that the variability in brightness of a certain star is not caused by eclipsing, but that the variation is caused by a change in the ellipsoidal shape of the components, resulting from their mutual attraction.

[1] Section of 1922 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

[2] Coblentz, Jour. Opt. Soc. Amer., *5*, p. 269 (see p. 276); 1921.

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In a recent photoelectric study of the variability of Algol Stebbins[3] found new results showing an effect due to the ellipsoidal shape of the components of the system.

Rosenberg[4] describes a stellar photometer consisting of a photoelectric cell and amplifying tubes. Measurements are given on several stars and planets.

II. STELLAR AND PLANETARY RADIOMETRY: Under this caption are described recent developments in nonselective stellar radiometers as well as recent measurements of the radiation from stars and planets.

1. *Reflecting Telescopes.*—Recent press dispatches tell of generous gifts of a 6 ft. reflector to be used primarily by students in a certain college in the state of Ohio, and of a 10 ft. reflector to be located in the state of Washington.

The gifts seem to be made through local pride for the hometown without consideration of the number of clear nights that will be available for observation and without thought of attainment of the maximum usefulness.

The needs of stellar radiometry are reflecting mirrors 15 or more feet in diameter, situated in a dry cloudless climate, such as obtains in Arizona and California. Some years ago, the writer had the temerity to inquire into the feasability of constructing such a mirror by piling up a number of sheets of polished plate glass, and bringing them to the annealing temperature, when they will coalesce. By placing them in a suitable mold, they will sag to the proper curvature, so that in the polishing there will be no cutting through of the first layer.

The production of a large disk of glass by this method would obviate the difficulty of obtaining a sufficient amount of molten glass for casting in one piece, which is homogeneous and free from local imperfections. From a discussion of the matter with a large-scale plate-glass producer, it appears that the suggestion of building up a glass disk from polished, selected sheets of plate glass may not be as foolish as it appears on first thought.

2. *Galvanometers and magnetic shielding.*—For stellar radiometry Abbott,[7] proposes to use a 2-coil galvanometer, in magnetic

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shielding, instead of the 16-coil instrument heretofore employed in his solar radiation work.

The use of a 2-coil non-astatic galvanometer is, of course, not new, similar instruments having been used by duBois and Rubens.[10] Modern conditions require so much magnetic shielding that the astatic magnet system is relatively unimportant. In his improved type of iron-clad Thomson galvanometer (in a vacuum) with the coils imbedded in blocks of soft iron and with an inner laminated shield of transformer iron, Coblentz,[12],[6] found that the old type of 4-coil galvanometer with its astatic magnet system can be displaced by a 2-coil galvanometer with a single set of magnets, reducing the weight of the suspension by one-half and thus practically doubling the sensitivity.

The use of an inner shield of laminated transformer iron separated by equal thicknesses of paper has recently been described by Wente.[14] In the interest of historical accuracy, it is relevant to add that Esmarch,[13] used shields consisting of fine iron wire wound upon cardboard instead of transformer iron as one would infer from the above quoted paper.[14]

The writer has tried the laminated shields, lightly wound (hence small air spaces between the lamina) without the intervening cardboard, and with the intervening spaces uniformly separated by cardboard, and has found no marked difference in the shielding efficiency. A considerable quantity of the metal close to the galvanometer, with the top of the laminated iron cylinder covered with a laminated lid seems most effective. In this connection, it is interesting to recall that from his theoretical calculations, of magnetic shielding, Rücker,[11] concluded that, under certain conditions, the resultant magnetic shielding obtained by the best use of a given quantity of material can be still further improved by filling up the spaces between the shells with additional material.

3. *Bolometers.*—From his theoretical studies, Abbot[8] concludes that contrary to the conclusions arrived at by him in determining the best dimensions for a solar vacuum bolometer, the most effective stellar vacuum bolometer must be exceedingly thin, say 0.0005 mm in thickness (length 8 mm, width 0.12 mm).

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Material of this thickness or even thinner, has been extensively used in black body spectral energy measurements. Owing to its small heat capacity, it is easily disturbed by air currents and hence must be used in a vacuum when refined measurements are attempted.

In view of the feeble intensity of the stellar radiation, it is evident that receivers of low heat capacity should be used. It seems to have been overlooked by users of bolometers that there is considerable loss of heat radiation from the rear side of a bolometer receiver, even when it is unblackened. By placing a second bolometer receiver directly back of the front one, Coblentz[9] found that the radiation sensitivity was greatly increased (amounting to 50 per cent in receivers which were blackened on both sides).

Johansen[15] has shown experimentally that heat conduction from the ends of a platinum bolometer strip (0.001 mm in thickness) greatly reduces the radiation sensitivity, extending out 3 to 4 mm from the electrodes. Coblentz and Emerson[16] using a platinum bolometer from one-half to one-third of this thickness, found appreciable heat conduction and hence loss in thermal radiation sensitivity extending 1.5 to 2 mm back from the ends of the bolometer receiver. From this and from Abbot's[8] calculations, it appears that a bolometer receiver for stellar radiometry cannot be made much less than 5 to 6 mm in length without loss in radiation sensitivity by heat conduction to the electrodes.

4. *Thermocouples.*—In a recent study[17] of thermocouple material to be used instead of a bolometer for stellar radiometry, it was found that the radiation sensitivity was closely proportional to the thermoelectric power (which was to be expected) but there was no great gain in sensitivity from reduction in thermal condution by reduction of the heat capacity of the material as compared with that previously employed.[5] The chief gain was in quickness of action and hence increased accuracy of observation.

The height of the spectrum of a star is only a small fraction of a millimeter. Hence, by placing the connecting wires of the thermocouple vertical, so as to coincide with the spectral image of the entrance slit of the spectroradiometer, it seems that the

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thermocouple could advantageously replace the spectro-bolometer, in view of the fact that the latter is heated by the battery current, which renders the galvanometer reading unsteady. For measuring the radiation from small areas, as for example the dark spots on Mars, the (vacuum) thermocouple seems more suitable than the (linear) bolometer.

5. *Stellar Spectroradiometry.*—The hundreds of photographs of stellar spectra, obtained by astonomers, show dark absorption lines and bands (also in some cases, bright emission lines) of hydrogen, helium, etc. From this, it is evident that a smooth spectral energy curve, without deep indentations, cannot be obtained. Our inferences as to the probable spectral energy distribution and inferred effective temperatures must therefore be obtained by smoothing over the depressions in the observed spectral energy curve. Then assuming that the stellar envelope radiates like a gray or black body, the effective temperature may be calculated.

Equipped with a spectroscope attached to a powerful telescope and the above mentioned improvements in radiometers, Abbot and Aldrich[8] expect to obtain the spectral energy curves of stars. Indeed, recent press dispatches announce this as now an accomplished fact. They find the effective temperature of a blue star (Betelgeux) to be 10 000°, confirming the observations of Coblentz,[17] which is to be expected in view of the fact the transmission screen method employed by the latter simply integrates wider regions of the spectrum in a single measurement.

While the discussion of stellar spectroradiometry has been mainly in connection with the determination of spectral energy curves, it should not be overlooked that the device may prove useful in mapping the spectral emission and absorption lines and bands of stars, in the infra-red spectrum to which the photographic plate is not sensitive. Spectroscopists will no doubt find use for any information pertaining to the wave-lengths and intensities of the bright and dark lines in this part of the infra-red spectrum.

6. *Stellar Spectral Energy Distribution by Means of Transmission Screens.*—At best a spectroscope is an inefficient device (Abbot,[8] *loc. cit.*, p. 59, places the loss at the slit jaws at 40 per cent) for

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utilizing the incoming stellar radiation, and recently Coblentz[17] determined the feasibility of obtaining the spectral energy distribution of a star by means of a series of transmission screens, placed in front of a vacuum thermocouple which was used as the radiometer.

Screens were selected which, either singly or in combination, had a uniformly high transmission over a fairly narrow region of the spectrum, terminating abruptly in complete opacity in the rest of the spectrum. By proceeding in this manner, the observations required no correction other than that for surface reflection, which amounts to about 9 per cent for the two surfaces of the screen. Corrections were made for absorption by the telescope mirrors; also for atmospheric absorption, using the spectral transmission factors for the sun, as observed by Abbot and Fowle.

By means of these screens (of red and yellow glass, quartz and water) it was possible to obtain the radiation intensity in the spectrum (from the extreme ultra-violet, which is limited by atmospheric transmission and the low reflectivity of the telescope mirrors) at 0.3μ to 0.43μ; 0.43μ to 0.60μ; 0.60μ to 1.4μ; 1.4μ to 4.1μ; and 4.1μ to 10μ.

By this novel means the distribution of energy in the spectra of 16 stars was determined, thus obtaining for the first time an insight into the radiation intensities in the complete spectrum of a star. From press dispatches, it appears that Abbot's recent spectrobolometric measurements verify the results obtained by Coblentz which is to be expected in view of the similarity of the principles involved.

This method may be open to criticism in view of the fact that it integrates the energy present in a certain spectral region and hence does not indicate the amount lost in the spectral absorption lines. The same criticism is true also of the direct spectroradiometric method which is no doubt limited to a few of the brightest stars, and if the dispersion is small, cannot properly evaluate the spectral intensities. Hence, the transmission screen method should prove useful in supplementing the spectroradiometric measurements, on fainter stars not measurable by the latter method. By integrating the spectrobolometric energy

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curves, a relation ought to be obtainable with the data observed with the transmission screens and in this way a comparison obtained of the spectral energy distribution of bright and faint stars of the same spectral class.

The measurements made by Coblentz[5] with the water screen show that, in blue and yellow stars, practically all the energy lies in that part of the spectrum to which the photographic plate is sensitive. Hence, since the effect on the photographic plate is cumulative, and the time for exposure is relatively unlimited, the spectral energy distribution of many weak stars and nebulae can be mapped, which will not be possible by any other methods known to us at the present day. Indeed such a beginning has been made by Plaskett.[19] But the measurements made through the water cell show that, if astonomers had known it, they could have used the photographic method long ago for determining the spectral energy distributions of blue and yellow stars. For this purpose, the photographic plate must, of course, be standardized by exposing it to the spectrum of a source of known spectral energy distribution as was done, for example by Ives and Coblentz[24] in determining the spectral energy distribution in the light of the firefly.

7. *Stellar Radiations Intensities.*—The thermocouple measurements of the total radiation from stars, made by Coblentz,[5, 17] in 1914, at Mt. Hamilton, Calif., (Lick Observatory, altitude 4000 ft.) were verified in 1921 at Flagstaff, Ariz. (Lowell Observatory, altitude 7250 ft.), showing that the early-type (Class M) red stars are losing heat 3 to 4 times as fast as the more dense, but hotter late-type (class B, A) blue stars. The least dense, class M, stars must therefore be losing heat by radiation, in which conduction does not contribute very materially in maintaining the surface at a given temperature.

In the dense stars the shape of the spectral energy curve, and hence our inferences of the effective temperature is determined by the spectral emissivity of the surface; while in the less dence stars the radiation emanates from great depths.

The water cell measurements open up a new line of thought on stellar evolution, showing, as already stated, that red stars are

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losing heat 4 times as fast as blue stars. From present theories, it appears that, in the Giant, red-star stage of evolution, a star may be rising in temperature, while it is decreasing in temperature on its return to the Dwarf (red star) stage.

A question of interest is whether, when the star is passing through the high temperature, blue, class B, A-stage of development (and is losing heat one-fourth as fast as when it is in the red, class M-stage) the interval of time of transition is relatively much longer (say 3 to 4 times longer) than when the star is in the low temperature, red, class M, N stage of evolution.

8. *Variable Stars.*—When the vacuum thermocouple and water cell were first used successfully,[5] in separating the infra-red from the visible radiation from stars, the usefulness of this device was apparent in studying the heat from double stars and from planets.

Recent reports from the Mt. Wilson Observatory, where Pettit and Nicholson[18] are using the 100 inch reflector, show that the vacuum thermocouple and water cell are rapidly becoming useful for routine study of variable stars of long period. However, from the viewpoint of the physicist it would seem preferable to tabulate the water-cell absorption in per cent of the total radiation received instead of in difference of stellar magnitude as given by these observers (or give both). For example it seems simpler to state that the water cell absorbs 25 per cent of the total radiation from Vega than to state that the water cell absorption, in difference of magnitude, is 0.3. The field of stellar radiometry is practically new and the adoption of a simple nomenclature and a simple tabulation of data should be begun now.

The water cell, having the property of absorbing the invisible infra-red rays, which are emitted by stars of low luminosity, will be a useful device for studying double stars, like Sirius, which have companions of low luminosity, and in searching for double stars which many have dark companions. This will form a new field of investigation.

9. *Stellar Temperatures.*—Data on the spectral energy distribution and temperature of stars as related to that of a black body are very meager. They are the results practically of the spectrophotometric measurements of Wilsing,[20] and of Nordmann,[21] and

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the spectral energy curves determined photographically by Plaskett,[19] all of which relate to the visible spectrum. From the data obtained on the radiation intensities in the visible spectrum, these experimenters have obtained estimates of stellar temperatures of 3000° for red stars to 25 000° or even higher for blue stars.

The temperature measurements of Wilsing, Scheiner and Münch[20] of various stars of class B (blue) vary from 7000° to 15 000° K; class A, from 8000° to 12 000° K; class F from 5000° to 8000° K; class G, (solar type) from 4000° to 7000° K; and class M (red) 3000° to 3500° K.

By means of his transmission screen device, Coblentz[17] found that, in the class B and class A (blue) stars, the maximum radiation intensity lies in the ultra-violet (0.3μ to 0.4μ) while in the cooler, class K and M (red) stars, the maximum emission lies at 0.7μ to 0.9μ in the infra-red. From this it appears that the black body temperature (i.e., the temperature which a black body would have to attain in order to emit a similar relative spectral energy distribution) varies from 3000° K for red, class M, stars to 10 000° for blue, class B, stars.

This estimate of the effective temperature of a star was obtained by comparing the observed spectral radiation components with the calculated values for a black body at various temperatures.

In Table 1, are assembled various determinations of the effective stellar temperatures by Nordmann,[22] and by Nordmann and le Morvan;[21] also by Wilsing, Scheiner, and Münch,[20] and calculated values by Saha[23] on the basis of ionization theory.

While it is to be expected that the various methods must give different results, it is interesting to find a rather close agreement in the estimated stellar temperatures. The agreement is especially close for stars of classes G, K, and M, that is, stars having a low temperature.

10. *Planetary Radiation Measurments.*—By planetary radiation is meant the emission of thermal radiation from a planet as a result of warming of its surface by exposure to solar radiation, including heat that may be radiated by virtue of a possible high internal temperature of the planet itself.

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TABLE 1.—*Stellar Temperatures*

Star	Class	Coblentz	Wilsing, Scheiner & Munch	Nord-mann & Morvan	Nord-mann	Saha
		°K	°K	°K	°K	°K
ε Orionis..........	Bo	13 000	....	....	....	18 000
β Orionis..........	B8p	10 000	....	....	....	....
α Lyrae...........	Ao	8 000	9 400	....	12 200	....
α Canis Majoris....	Ao	7 000	....	....	....	12 000
α Cygnii..........	A2	8 000	9 400	....	....	....
α Aquilae.........	A5	8 000	8 100	....	....	....
α Canis Minoris....	F5	6 000	7 200	....	....	....
α Aurigae.........	Go	6 000	7 100	....	....	7 000
α Boötis..........	Ko	4 000	3 700	....	....	....
β Geminorum......	Ko	5 500	4 900	....	....	....
α Tauri...........	K5	3 500	3 500	3 600	3 500	....
α Orionis..........	Ma	3 000	3 000	....	....	5 000
α Scorpii..........	Map	3 000	....	....	....	....
β Andromedae.....	Ma	4 000	3 200	4 300	3 700	....
μ Geminorum......	Ma	3 500	3 100	3 200	....	....
β Pagasi..........	Mb	3 000	2 800	....	....	....
Sun..............	Go	5 800[a] to 6 200	....	....	5 320	....

[a] Recalculated from Abbot & Fowle, Jour. Opt. Soc. Amer., **5**, p. 272; 1921.

The temperature of the surface, due to absorbed solar radiation and due to internal heat, at most, is probably not much higher than several hundred degrees centigrade and hence the reradiated energy will be predominately of long wave-lengths,—7μ to 12μ. Hence by means of a 1 cm. water cell interposed in the path of the total radiation emanating from the planet, this long-wave-length radiation can be separated from the reflected solar radiation, and in this manner a measurement obtained of the planetary energy radiated. If there is planetary radiation then the water cell transmission will be less than that of the direct solar radiation.

During the past June, (1922) the writer made further measurements (at the Lowell Observatory) of the thermal radiation emitted by the major planets.

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By comparing the transmission of the direct solar radiation, through a water cell, with the transmission of the radiation emanating from the planet, a measurement was obtained of the intensity of the planetary radiation.

Radiometric measurements were made on Venus, Mars, Jupiter, Saturn, and the Sun, and in cases where similar measurements had been made at Mt. Hamilton, Calif. in 1914, the data were found in good agreement.

The water cell transmissions of the radiations from Jupiter and from the Sun were practically the same, indicating (1) that the outer atmosphere of Jupiter does not become heated (either by the Sun's rays or by internal radiation) and emit long-wavelength infra-red radiation and (2) that any radiation emanating from its interior is entirely trapped by surrounding atmosphere. Hence, we cannot determine the internal condition of Jupiter.

The intensity of the planetary radiation increases with decrease in the density of the surrounding atmosphere and, as interpreted from the water cell transmission, is as follows: Jupiter (0), Venus (5), Saturn (15), Mars (30), and the Moon (80).

The intensity of the planetary radiation from the northern hemisphere of Mars was found to be less than from the southern hemisphere. This is to be expected in view of the observed cloudiness over the northern hemisphere which is approaching the winter season, and hence is at a lower superficial temperature.

The radiometric measurements on Mars are of especial interest in view of the question as to its temperature, etc. Lowell's[26] calculations of the surface temperature of Mars give values much higher than those obtained by Poynting,[25] who obtained a value of −38° C.

The calculations of Lowell, based on the heat retained, give a mean temperature of 9° C for the surface of Mars; while another calculation gives a temperature of 22° C. He points out that owing tocloudiness, only 60 per cent of the incident solar radiation is effective in warming the earth while 99 per cent is effective in warming the surface of Mars. Other meterological data of interest are that on Mars, water boils at 44° C; that the amount of air

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per unit surface is 177 mm (2/9 the earth's) and that the density of the air at the surface is 63 mm (1/12 the earth's).

In a recent discussion of climatic conditions on Mars, Pickering[27] inferred from phenomena generally observed on the planet, estimates the mean annual temperature at +20° F as compared with the mean annual temperature of the earth of +59° F (15° C). At night, the Martian temperature is below 32° F (0° C) and at noon it is perhaps 60 to 70° F (15° to 20° C). These estimates are arrived at from the appearance and disappearance of snow and frost during the course of the Martian day, and from the fact that snow is never seen on the equator at Martian noon.

The writer's radiometric measurements are in agreement with the calculations of Lowell, and with the arguments set forth by Pickering, showing a considerable rise in temperature of the surface of Mars.

Probably the most convincing experimental observations of the range of temperature of the moon are those of Langley and Very[28] and later, those of Very.[29] These measurements indicate inferred effective lunar temperature ranging from 45° C to over 100° C. The calculated value[30] using recent data on the solar constant, indicate a lunar temperature of 82° C.

When we consider that 30 per cent of the total radiation emanating from Mars is of planetary origin, as compared with 80 per cent from the moon, and that all the evidence shows that the lunar surface becomes appreciably warmed it appears that there is also a considerable temperature rise (10 to 25° C.) of the surface of Mars as calculated by Lowell. So whether or not we accept the view that vegetation can exist on Mars, the radiometric measurements confirm other meteorological data, showing that at Martian noon the snow is melted, which could not happen if the temperature were −39° C as some have calculated.

As for the views held by some, of the possibility of vegetation growing on Mars, much depends upon whether we think of palm trees growing in our tropics, or the mosses and lichens which thrive on the apparently bare piles of volcanic cinders of Arizona and under our Arctic snows.

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In conclusion, it may be noted that there is a divergence of opinion as to the spectral character and the intensity of the solar radiation, the "solar constant," outside of our own atmosphere. The unexpected observation of a considerable heating of the surface of Mars, raises the question whether the calculations of planetary temperatures are in error, or whether the solar radiation intensity (the "solar constant") outside our atmosphere is higher than the present accepted value.

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WASHINGTON, D. C.

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INSTRUMENT SECTION

THE LENSOMETER

AN INSTRUMENT FOR THE MEASUREMENT OF THE EFFECTIVE OR VERTEX POWER OF OPHTHALMIC LENSES

By Charles Sheard and E. D. Tillyer

Introduction

The significance of the difference between thin and thick lenses, as well as the meaning of the terms nodal point, principal point, focus, equivalent focus and back focal length of a lens are well known to those interested in physical optics. These points and foci may be determined in laboratories by various experimental methods more or less elaborate as the case may be. Their mathematical calculation, through the use of appropriate equations, has constituted for some time a part of the domain of science commonly spoken of as geometrical optics.

But the science of physiological optics, insofar as this field of scientific endeavor and research has to do with the problems of ocular refraction, has not until more recent years found it necessary to take cognizance of measurements of the true corrective or effective powers of ophthalmic lenses. This has been true for the reason that the prevalent forms of ophthalmic lenses worn until the opening years of the twentieth century were either double convex and double concave, or plano-convex and plano-concave, or were at best what are commonly called "flat" lenses in ophthalmic optics.

In the construction of the trial case lenses in our standard test sets, the forms of lenses used have been quite commonly of the double convex and double concave type. About 1899 the question as to why strong contrageneric (convex and concave) lenses of equal curvatures—but of opposite signs—did not neutralize was raised by Charles F. Prentice. As a result standard trial case lenses were so made that the *thin* biconcave lenses[1] con-

[1] Commercial "thin" negative lenses were compensated so as to agree with the theoretical thin negative lenses in their effects on the biconvex forms.

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stituted the master lenses and the positive, or biconvex, lenses were so modified in curvatures that they might be neutralized by these lenses. In this manner, therefore, the trial lenses were made such that they were at all times the equivalents of thin lenses whose powers were determined through neutralization. In reality, therefore, all ophthalmic lenses of the biconvex or biconcave form were so manufactured that their powers were measured in units of back focal length, or in other words their designation was in terms of effective or vertex refraction. For example, a plus 20 diopter lens was not made with a plus 10 diopter curve on each side, but with a reduced curvature, so that the lens marked +20.00 D. S. was neutralized by the −20.00 D. S.,—the thin, master lens. Fig. 1 illustrates the fact that

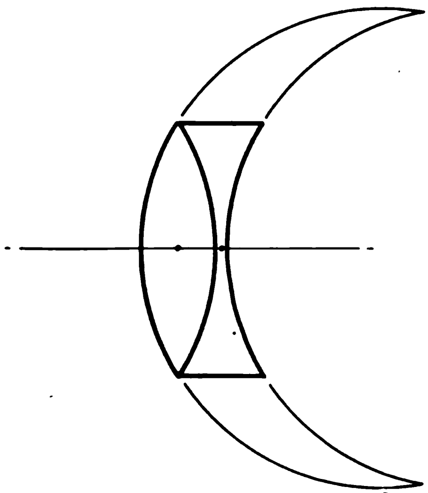

FIG. 1. *Illustrative of the statement that biconvex and biconcave lenses of the same curvatures cannot neutralize.*

biconvex and biconcave lenses of equal curvatures would give as a resultant a weak positive power lens.

With the advent of meniscus and so called toric lenses—in which various base curves might be employed and which, as a result, gave various shapes of lenses having powers which were said to be the same—the elements of *thickness* and *shape* of lens

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had to be taken into account. The application of neutralization methods—which involve the addition of such a quantity of opposite character or power of lens as will give a resultant of zero—to the determination of the true power of ophthalmic lenses of different thicknesses and shapes was not strictly scientific, for if the neutralization lens from the trial set was applied to these menisci and torics one or the other of two possible results was evidently obtainable: (1) the *front focal* length, or its true corrective effect or power if it were to be worn in a manner the reverse of that in which it is worn, and (2) its *back focal* length, which would be determined as the resultant of two lenses, one an air lens between glass surfaces and the second the supposedly correct biconcave neutralizing lens. Fig. 2 illustrates the method

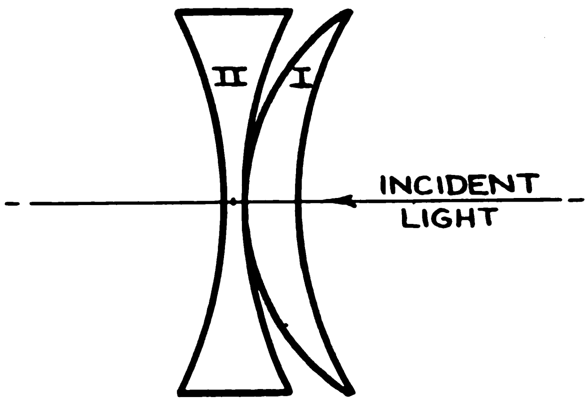

FIG. 2. *Neutralization from the front surface. This means that the front instead of the back focal length is determined.*

of finding the front focal length by neutralization. Fig. 3 shows the error in the method of neutralization as applied to finding the back focal length or the true corrective power of meniscus and toric shapes of lenses.

It is evident, therefore, that methods of neutralization as applied to the finding of the true corrective powers of ophthalmic lenses are in error except insofar as they are used when biconvex and biconcave forms of lenses are to be measured. In the case

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which is mentioned as the exception, the true corrective (i. e. back focal) power, or effective power or vertex refraction is determined. We prefer the term *effective* as it is self-explanatory.

Menisci and toric lenses are quite commonly prescribed at present for the general reason that the field of useful vision is greater and wider than in the earlier, equal curvature forms of lenses. At the present time the curvatures employed, or the base curves of toric lenses, are being quite widely varied—depending

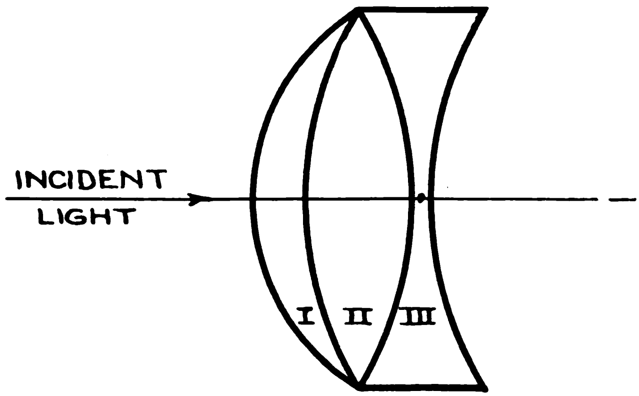

FIG. 3. *Neutralization of meniscus and toric forms of lenses from the ocular side of the lens means that three lenses will be involved in the procedure.*

upon the desired power of lens as a criterion—in order to give as wide-angled correct focus fields as possible. In ophthalmic optics it has therefore become imperative to devise an instrument which would quickly and accurately measure the *back focal power* of any lens, irrespective of its shape (i. e. curvatures), and thickness, as well as being capable of measuring the back focal length of any combination of spheres and cylinders that might be inserted in an ophthalmic trial frame. To accomplish the designing and perfecting of such an instrument the Research Division of the American Optical Company, and chiefly its physical opticist, Mr. E. D. Tillyer, have devoted a large amount of time and effort.

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THE LENSOMETER

The lensometer, therefore, measures a lens in *effective power* or *vertex diopters*. The term *vertex refraction* was introduced by von Rohr about a dozen years ago in writings in which he emphasized the well-known fact that the true unit of measurement of ophthalmic lenses should be in terms of the distance from the ocular side of the lens to its focal point, i.e., back focal length. The use of this term *vertex refraction* did not, however, introduce any new or novel conception as to lens measurement.

The optical system of the lensometer is given in the accompanying figure (Fig. 4). The observer's eye (A) is placed at the exit pupil of the telescopic system, composed of an adjustable Rams-

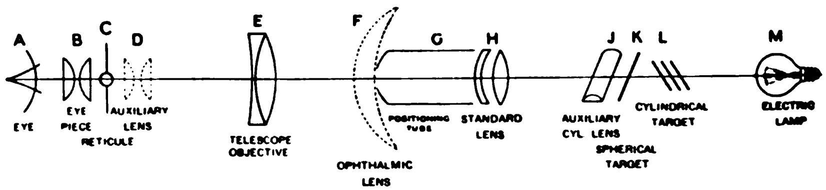

FIG. 4. *The optical system of the lensometer.*

den eyepiece (B) which is focused on the reticule (C) to compensate for errors in (A), while (E) is a telescope objective so adjusted as to bring the images of distant objects to a focus at (C).

The lens (D) is an auxiliary lens system[2] which may be inserted at the will of the operator. The optical system A, B, D, and E, is therefore a very low power compound microscope, imaging the spectacle lens (F) at the reticule (C) for the purpose of geometrically positioning the ophthalmic lens with high precision.

The positioning tube (G) is adjusted and set with lock screws for holding the ophthalmic lens (F) at the exact distance required from the adjusted standard lens (H) which projects the image plane of the targets (K) and (L) into the required back focal distance from the ocular surface of the lens (F) which is under test.

[2] This auxiliary lens system is free from certain angular errors which render it valuable in the optical laboratory for many purposes, such as aligning a spectrometer prism and so on.

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As ophthalmic lenses are in general composed of both spherical and cylindrical powers, two targets are used at right angles to each other. The cylindrical target (L) is permanently positioned at right angles to the spherical target (K) and for the sake of permanence both are made by milling fine slots in thin brass. The cylindrical lens (J) is permanently fastened to the target (K) to add a definite and constant cylindrical power and the scale reading of (K) is correspondingly shifted so that for a spherical lens at (F), targets (K) and (L) will not come into contact. For cylindrical lenses the separation will naturally be much greater.

The positioning tube (G) is adjusted so that the ocular surface of the ophthalmic lens (F) is held exactly in the principal focal plane of the standard lens (H), so that the distance (x) from this point to the image of the target is given by the optical equation

$$x\,x' = f^2$$

where x' is the distance of the target from the other focal plane of (H) and (f) is the equivalent focal length of the standard lens (H). This equation can then be put in the form

$$x' = f^2\left(\frac{1}{x}\right) = f^2D.$$

from which it is obvious that the reciprocal of the back focus of the lens (F) is read as a linear term, x', in units depending for their values upon the value of (f). It is thus seen that linear changes in position of the targets may be translated into dioptric values.

The standard lens (H) must be accurate. Even if the curves were to be exactly correct, two lenses made from different ends of the same piece of glass might differ in index sufficiently to produce an error which could not be permitted in the lensometer. The standard lens is consequently made adjustable in power by a slight change in the separation between the positive combination and an auxilliary negative lens, the cell of which is threaded into the positive lens cell. After accurately adjusting the power, the two cells are sealed together.

The precision of the protractor on the lensometer is tested by means of a marked plano-cylinder placed face up and then face

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down. The average readings will then give the 180° point regardless of the actual axis of the cylinder.

The centering is tested by positioning a slightly decentered lens in four ways, so that a marked point is first "in," then "out," and if the reading "in" is the same as the reading "out," the horizontal center is correct. The procedure for the "up" and "down" centering is similarly carried out.

The lensometer is so adjusted that readings made with each lensometer on the primary standard lens set agree with the values as certified by the Bureau of Standards. This makes the lensometer the duplicate of the primary standard lenses insofar as power is concerned.

The lensometer as a working instrument is pictured in Fig. 5.

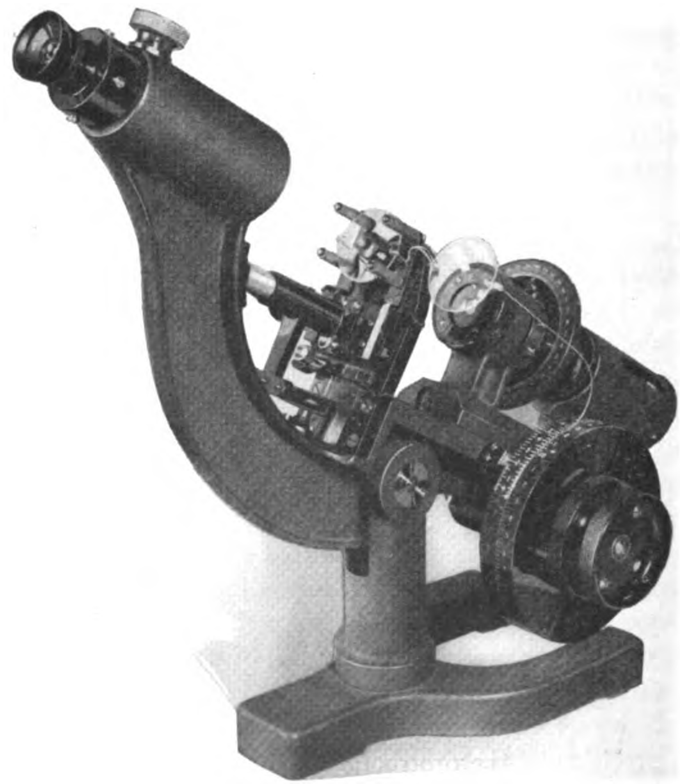

FIG. 5. *The Wellsworth lensometer.*

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Fig. 6 shows the wheels (L and K) by means of which the spherical and cylindrical targets (J, K, L in Fig. 4) are changed in position. The effective or vertex power readings are given by the division marks (both spherical and cylindrical) directly under the indicator line marked S.

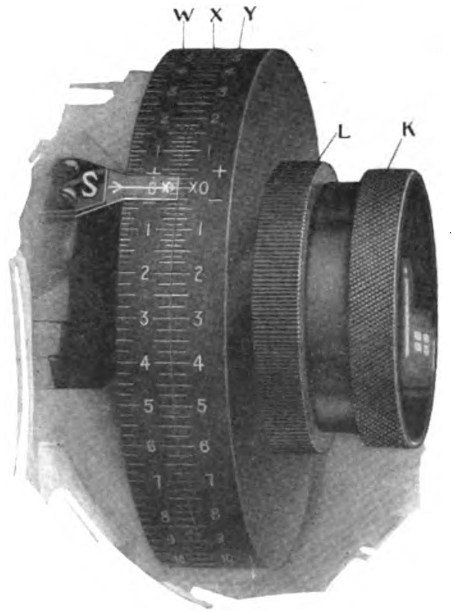

FIG. 6. *Showing the wheels and scales used in getting the spherical and cylindrical powers of lenses.*

Fig. 7 shows in (A) the dial, which may be revolved, for the determination of the cylindrical axis; the carriage for holding the lens (or lenses) is shown as D1, D2, D3, D4; at V and H are shown the two scales for determining the amount of decentration, or prism power, up or down, in or out; while B shows the ophthalmic lens in proper position for measurement just as illustrated in the diagrammatic sketch given in Fig. 4.

The reproductions which follow, and which are labelled as Figs. 8a and 8b, show a family of images of the targets obtainable under varying conditions of sphere, cylinder or both sphere and cylinder being in or out of focus.

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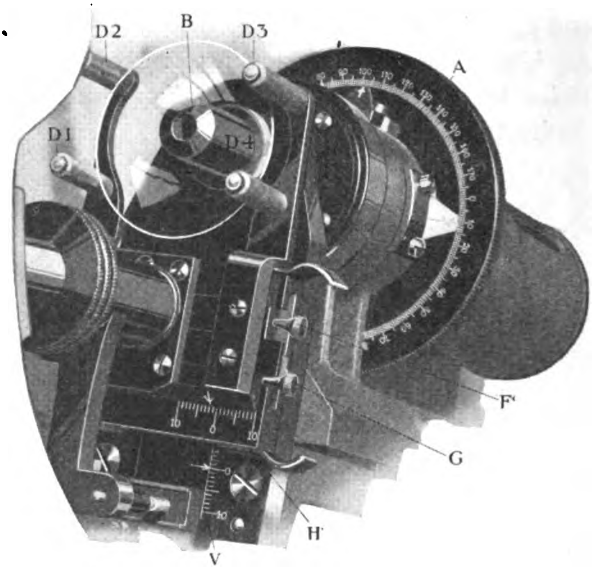

FIG. 7. *Showing the carriage for holding lenses, the protractor for determining the axis of the cylinder and the device for finding the amount of decentration or prismatic element incorporated.*

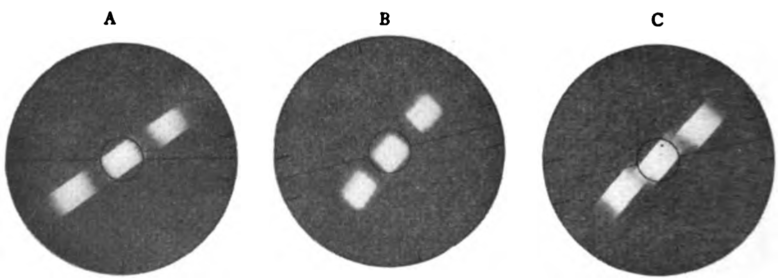

FIG. 8a. (*A*) *Sphere in focus: cylinder out of focus: axis correct.*
(*B*) *Sphere out of focus: no cylinder.*
(*C*) *Sphere in focus: cylinder out of focus: axis off a few degrees.*

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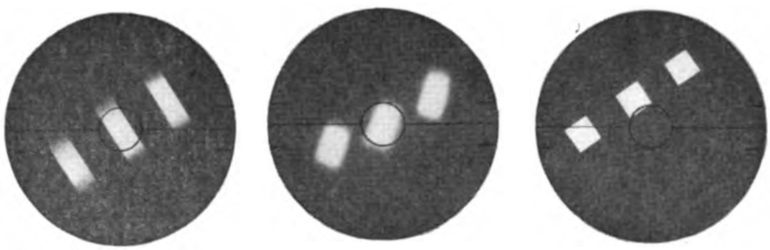

FIG. 8b. (*D*) *Sphere in focus: cylinder out of focus: axis 90 degrees off.*
(*E*) *Sphere out of focus: cylinder out of focus: axis off.*
(*F*) *Normal image, decentered.*

The lensometer measures the effective power (back focal length) of any lens or combination of lenses which falls within the range of plus 20 diopters (focus, 2 cm) or minus 20 diopters (focus,—2 cm) and a cylindrical power up to 8 diopters (focus 12.5 cm). The instrument may, therefore, be used for the determination of the back focal length or effective power of any single or combination lens or lenses, irrespective of the use to which the lens or lenses may be put, provided it falls within the limits of the instrument. Its chief value without doubt lies in the field of physiological optics as applied to ophthalmic lenses, for it provides a universal device for the measurement of the true corrective effects and powers of any ophthalmic lens, simple or compound, or any combination of such lenses.

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A HEMISPHERICAL PHOTOMETRIC INTEGRATOR

By Frank Benford

CONTENTS

SYNOPSIS

A study of the errors in spherical photometry due to the presence of the lamps and screens in the Ulbricht sphere led to the development of a fixed convex mirror to be used in place of the two larger screens. A further development of the theory of interference within the instrument led to the adoption of a hemisphere in place of the complete sphere. It is shown how the instrument is calibrated and used for the testing of

(a) Reflectors for indoor or street illumination
(b) Large searchlights
(c) Small searchlights, headlights, etc.
(d) Motion picture projectors
(c) High intensity arcs (for brilliancy).

The various accessories that go with the instrument are described and illustrated

INTRODUCTION

Some seven years ago the writer started an investigation of the optical action within an Ulbricht sphere when it contained a standard source of light, a test unit of considerable size as compared with the sphere and the three screens ordinarily used is this instrument. The investigation led to the development of a new type of compensating screen[1] and the elimination of some of the most troublesome variables encountered in practice. In order to construct the compensating screen, which was a convex mirror

[1] Benford, The Integrating Sphere and a New Type of Compensating Screen. Transactions I. E. S. *11*, p. 997–1005; 1916.

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placed close to the photometric window so that it concealed the opposite hemisphere of the Ulbricht sphere, it was necessary to remove the concealed hemisphere in order to get working room for the calibrating light. While engaged in the work of exploring the surface with a spot light and at the same time altering the mirror surface so that all parts of the hemisphere would give equal photometer readings for equal amounts of incident light, quite naturally there occurred the thought—If a hemisphere can be made to integrate properly for a light source at the center of the plane of the opening, would it not be possible to extend the optical corrections so that a searchlight beam could be received and photometered? If this is possible, then we have a solution for one of the most difficult and laborious problems in all photometry.

COMPENSATING MIRROR

If a small spot of light covering less than 10° arc on the spherical surface is moved around to various positions on the diffusing surface the photometer reading will vary, being greatest when the spot is farthest from the reading window and least when it is closest to the window. The variation depends upon the diffusing characteristics of the Keene cement used to surface the integrator and upon the transmission characteristic of the window. Thus, if the reading when the spot is 90° from the window is 100, it will be about 70 for a position as close as possible to the window. If the convex mirror is placed so that an image of the entire adjacent hemisphere may be seen it becomes possible to collect light from all parts of the hemisphere over two paths, that is, direct lighty from *A* to *C*, Fig. 1 and reflected light over path *ABC* in the same figure, and the sum of the two beams may be controlled by reducing the average reflectivity of the mirror at the zone *B* on the mirror.

Compensating mirrors have been made for the hemispherical integrator now in use and one of them is illustrated in Fig. 2. A star-shaped area in the center of the convex side is covered with a shell of oxidized copper spun to fit the convex surface. The reflectivity of the center of the mirror is perhaps as low as 3% and in the circular zones cutting the points there is a gradual

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increase of average reflectivity up to 86% where the mirror is entirely exposed. This form was obtained[2] by trial and error, using an exploring spot 5° in width. It was found to be impractical to attempt to compute the needed reflectivity of the various

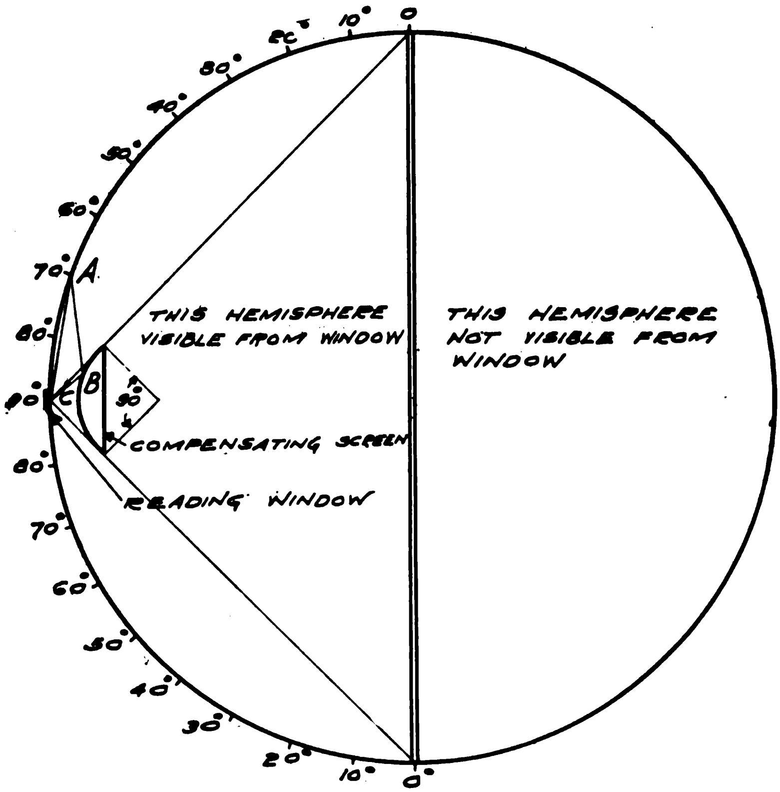

FIG. 1. *Diagram showing arrangement of compensating screen within sphere.*

zones on account of the strong secondary action of light reflected back and forth between the mirror and the adjacent surface of the hemisphere.

In Fig. 3 is shown how, after the mirror was finished and adjusted, the photometer reading varied as the spot was moved from the position of incidence on the back of the mirror outward to the edge of the hemisphere. At incidence on the back of the

[2] Benford—A Universal Photometric Integrator Transactions I.E.S. *15*, pp. 19–27; 1920.

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mirror (the spot really overlapped slightly) the reading was 67. At a point 7° outward the reading rose to 145 and then fell to 100 at 12°. At 14° it was 92 and then it rose to 100 at 30° and remained at this value out to the edge of the hemisphere. (If a complete sphere is used the identical form of star would be used

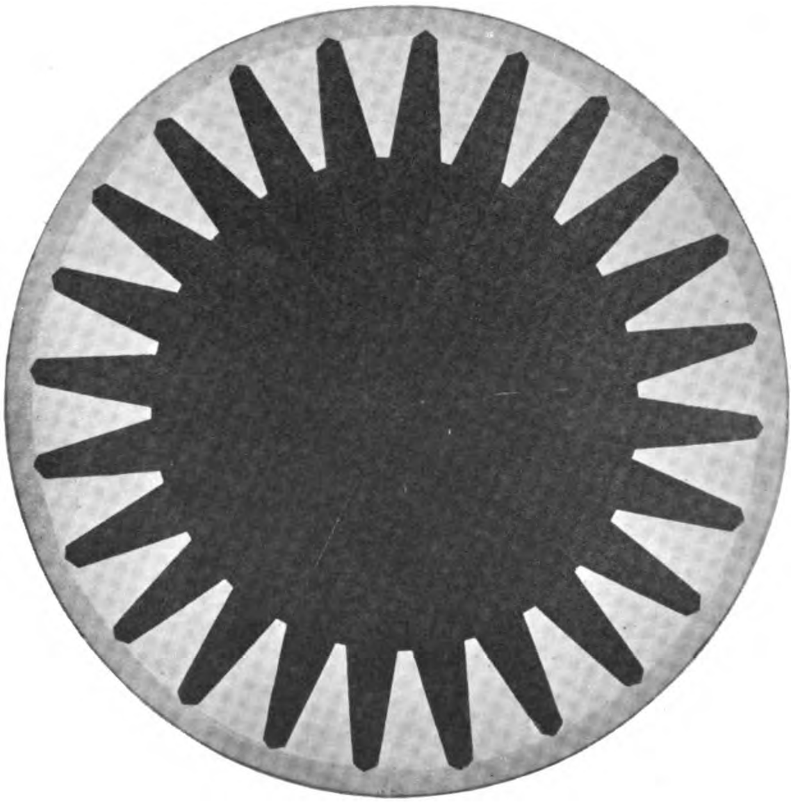

FIG. 2. *Compensating mirror for hemispherical integrator.*

and the variations from the average reading would all be scaled down to about one quarter). The areas of positive and negative errors around the screen are balanced against one another so that a spot of uniform illumination covering 25° around the axis will be read correctly.

INTERFERENCE BY TEST UNIT

In the photometering of a lighting unit in a sphere there must be an experimental determination of the light absorbed by the

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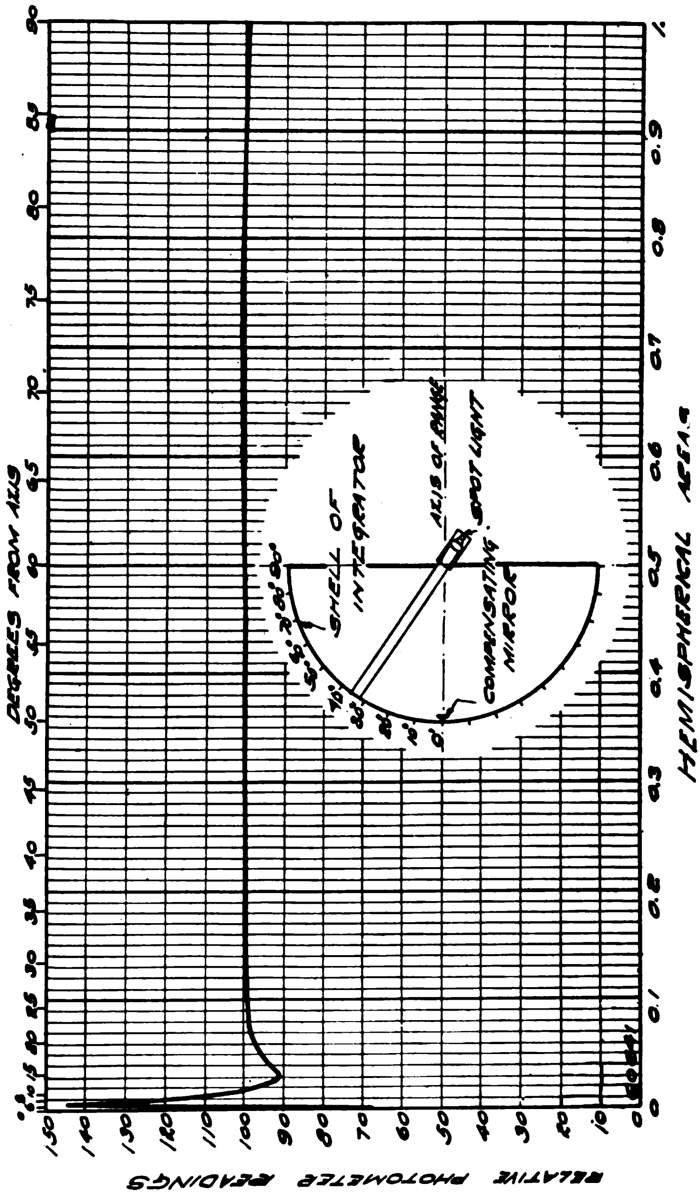

FIG. 3. *Spot calibration of hemispherical integrator for light originating in plane of opening.*

unit. This determination is not always exact because the absorption of light from the standard lamp is not the same as for light from the test unit. The difference here arises from the different distribution of brightness over the sphere wall given by the two sources. The action of the test unit results in a reduction of the reading below its proper value. If a hemisphere is used and the test unit is placed in the plane of the opening, as shown in Fig. 4, there are two opposing actions that under certain conditions

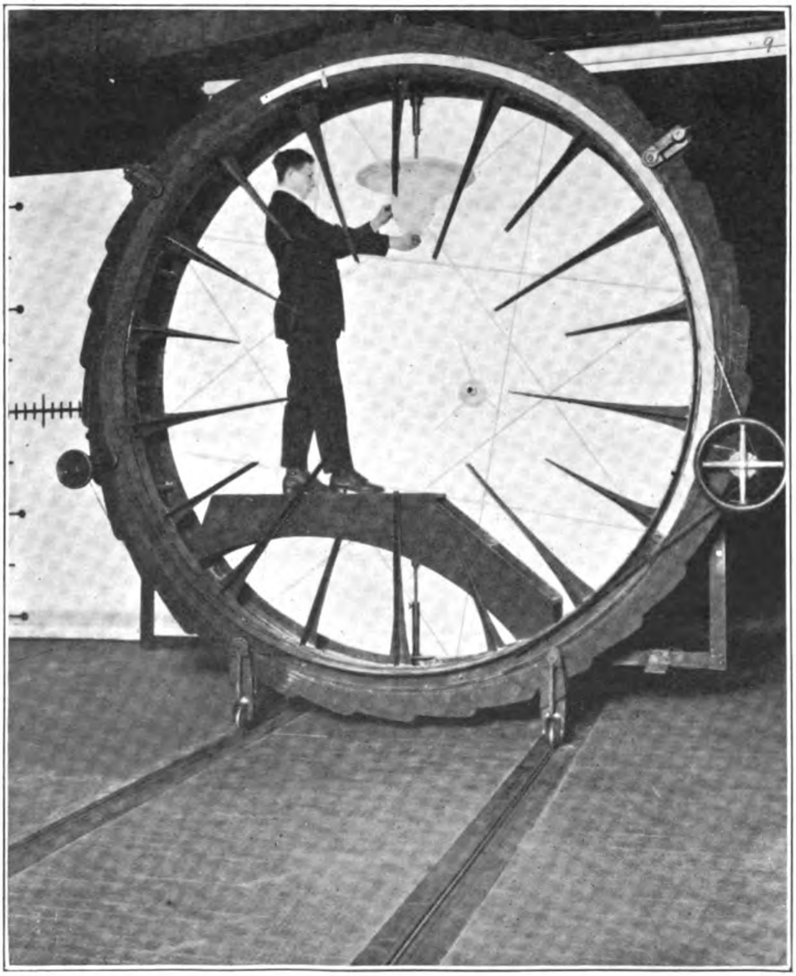

FIG. 4. *Front view of integrator showing location of compensating screen, optical wedges and bridge for use of operator in mounting test unit.*

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nullify one another. Light reflected from edge to edge of the hemisphere is interfered with by the test unit and the photometer reading is reduced, but light reflected outward through the plane of the opening is reflected by the test unit and returned to the instrument, increasing the reading.

The interference of the test unit depends upon its average coefficient of reflection and on its size relative to the instrument. Some theoretical brightness factors are plotted in Fig. 5, showing how the brightness and photometer reading varies for both sphere and hemisphere of equal diameter. The influence of the lighting unit is seen to be relatively small in the latter instrument, and this feature is important for two practical reasons: First, certain of the experimental errors of test are probably reduced in somewhat the same proportion as the interference, and second, the interference becomes so small that ordinarily it can be neglected, thus saving considerable test work.

A series of tests[3] made with both types of integrators (of the same diameter) and various lighting units gave the data tabulated in Table 1, where the positive signs indicate an increased reading due to the presence of the unit in the integrator, and the negative sign indicates a decreased reading. Note the change of sign in the data on the hemisphere.

TABLE 1

Reflectors	Change of Photometer Reading	
	Sphere	Hemisphere
	Per cent	Per cent
Group 1	−7.5	+1.6
2	−17.5	+2.3
3	−25.9	+0.7
4	−33.3	−0.3
5	−42.4	−2.3

POSITION OF TEST UNIT IN PLANE OF OPENING

The spot calibration for the mirror was carried out with light coming from the center of the plane of the opening. A number of

[3] Benford, An Integrating Hemisphere. Transactions I.E.S. *23*, pp. 323–356; 1918.

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Brightness Factors for Sphere and Hemisphere when containing Lighting Unit

$$\text{For sphere } F=\frac{1-K}{1-K\left[1+C\left(\frac{r}{R}\right)^2-\left(\frac{r}{R}\right)^2\right]}$$

$$\text{For hemisphere } F=\frac{1-\frac{K}{2}}{1-\frac{K}{2}\left[1+1.2C\left(\frac{r}{R}\right)^2-0.8\left(\frac{r}{R}\right)^2\right]}$$

where $K=$ Reflectivity of Integrator $=0.833$.
$C=$ " " Lighting Unit
$R=$ Radius of Integrator
$r=$ " " Lighting Unit.

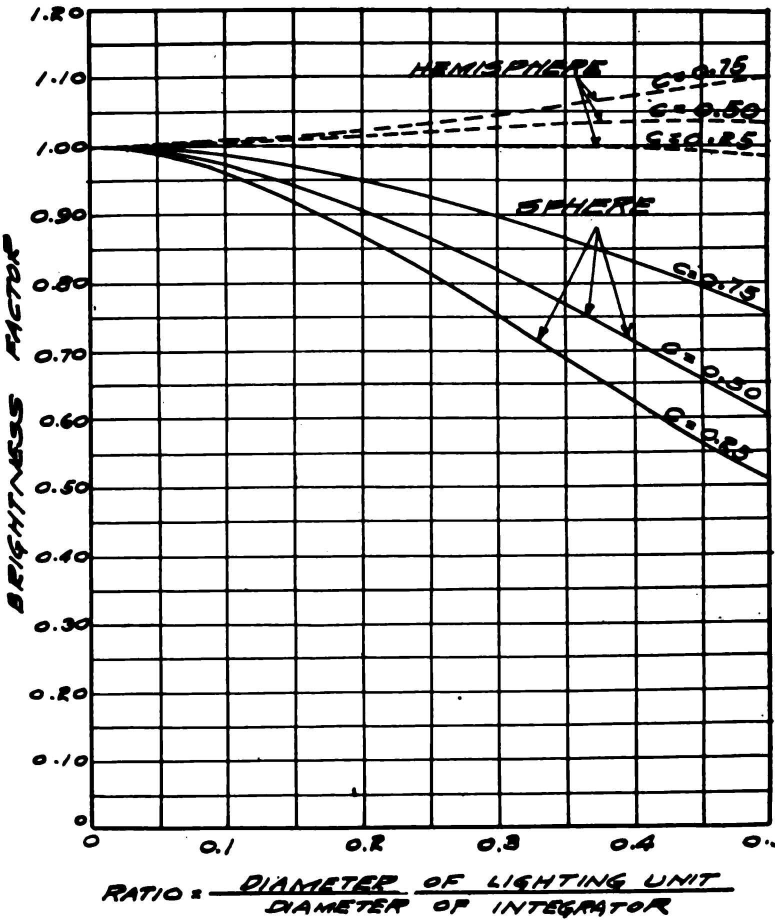

FIG. 5. *Brightness factors for Sphere and Hemisphere when containing lighting unit.*

tests since made with various lighting units show that they may be placed anywhere in this plane within 36 inches of the center without appreciable influence on the photometer reading. This allows the standard lamp and test unit to be well separated and a screen between them is not needed. In Fig. 4 the standard lamp, which is under the bridge upon which the operator is standing to adjust the test unit, is separated from the latter by 65 inches, and it is not necessary to have a screen between them.

The test unit is tested and then rotated through exactly 180° and tested again so as to include all the light; this rotation is made by means of the handwheel and gearing shown in Fig. 6. The handwheel for the standard lamp is on the platform but does not show in the illustration.

CALIBRATION FOR SEARCHLIGHT TESTS

The particular service for which the hemispheres in this laboratory were designed and built was the testing of searchlights. This type of photometry has many difficult features and ordinarily the cost is so high as to discourage any thorough investigation of the larger sizes of projectors. It is true that an indoor test on a searchlight has its limitations, principally because the beam does not get into its approximate final shape at short distances such as the 150 feet available in the indoor range, but on the other hand, the determination of the total quantity of useful light in the beam may be safely made at this distance. The smaller arc searchlights and incandescent 'floodlights' can be completely analyzed for total light and beam intensity at 150 feet, and there are many other types of projection units, such as motion picture machines, spot lights and automobile headlights that can be similarly tested in all details.

When light is projected into the hemisphere from a considerable distance, the angle of incidence is no longer nearly identical with that of the light used in calibrating for the compensating screen. In general, light projected along the axis will give readings higher than light radiated from the center of the instrument, and at the extreme edge of the instrument where projected light strikes near grazing incidence the readings will be as much as 33%

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high. After the surface and mirror were tested for action under parallel light the wedges shown in Fig. 4 were designed so as to reduce all circular zones to equality. It is assumed that the

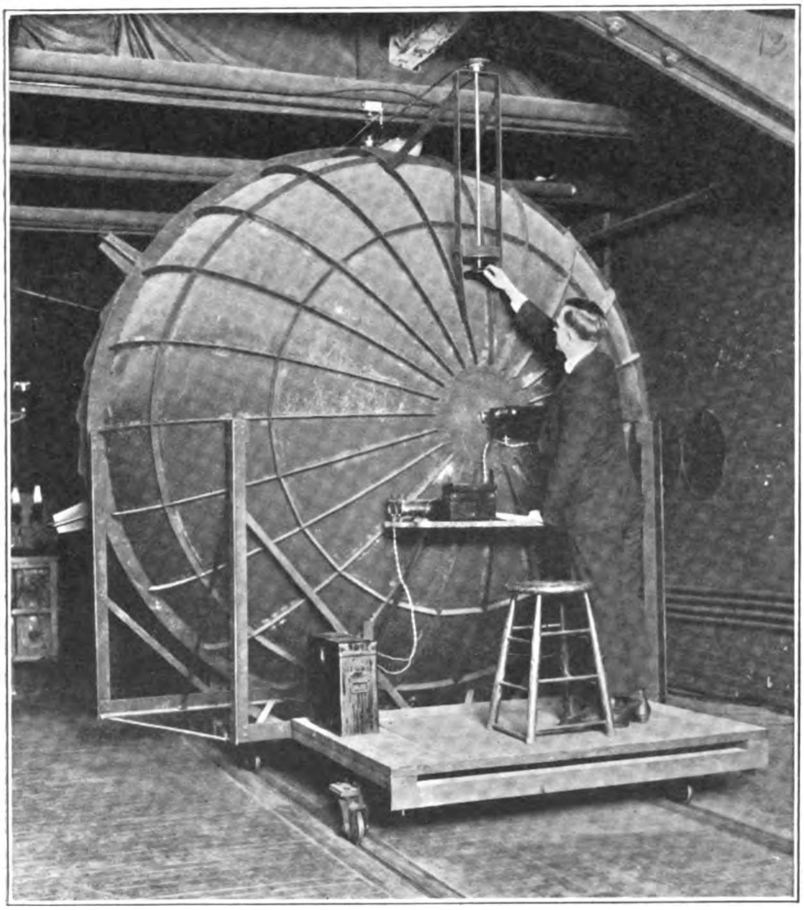

FIG. 6. *Rear view of integrator showing photometer and hand wheels for rotating test unit.*

test unit will give sufficiently uniform illumination so that each wedge will correct for the 20° arc on which it is centered. The illustration shows the wedges to be alternately long and short. This was done as a matter of expediency. The instruments

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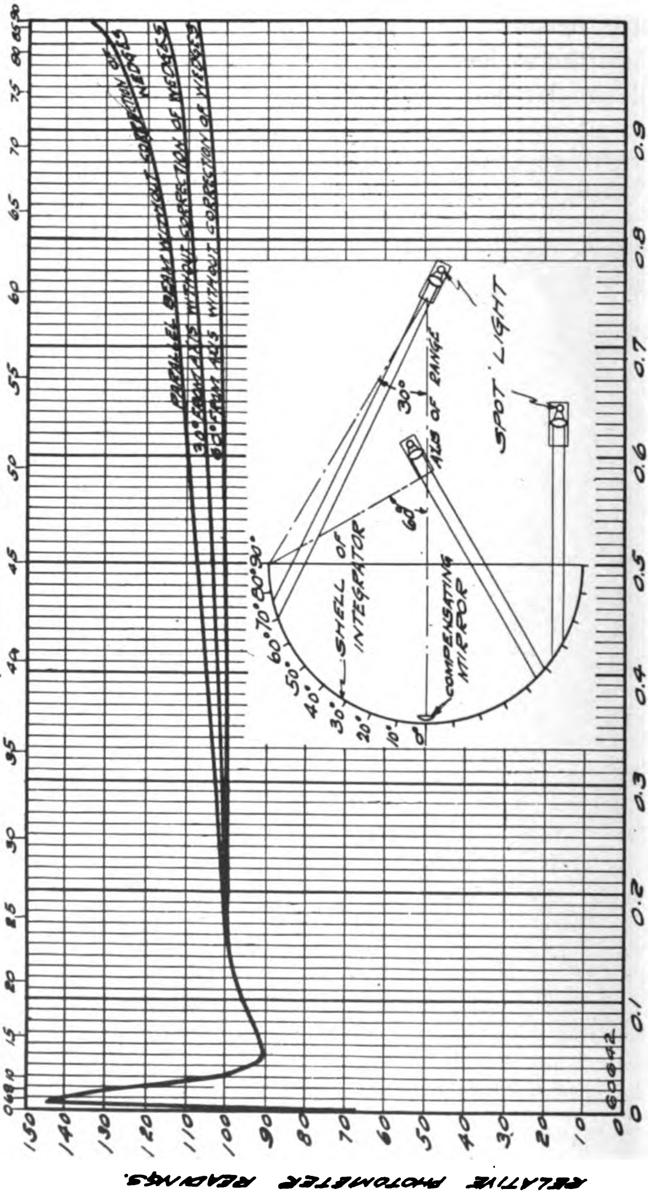

FIG. 7. *Spot calibration for parallel light, and for light originating on the axis at short range.*

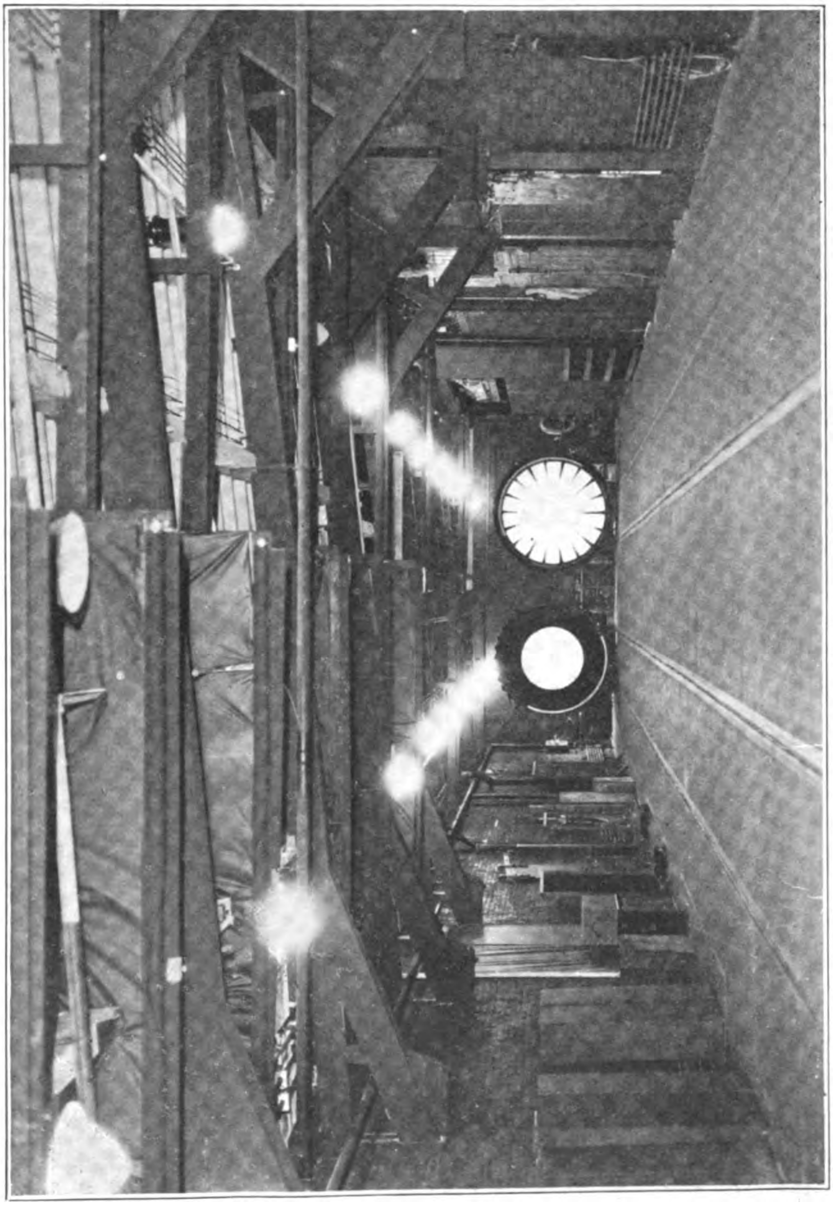

FIG. 8. *A view of the indoor searchlight range with the curtains raised. One integrator is diaphragmed down to half opening.*

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were constructed under great pressure during the war and the shop work was kept as far as possible abreast of the computations and calibration data. The wedges were originally all the size of the larger ones, following data on a previous small model with a paint surface, and the width in excess of the needs of the present plaster surface was removed from every other wedge. This not only saved time, but it made it mechanically possible to taper off the correction to a smaller value at the inner zones.

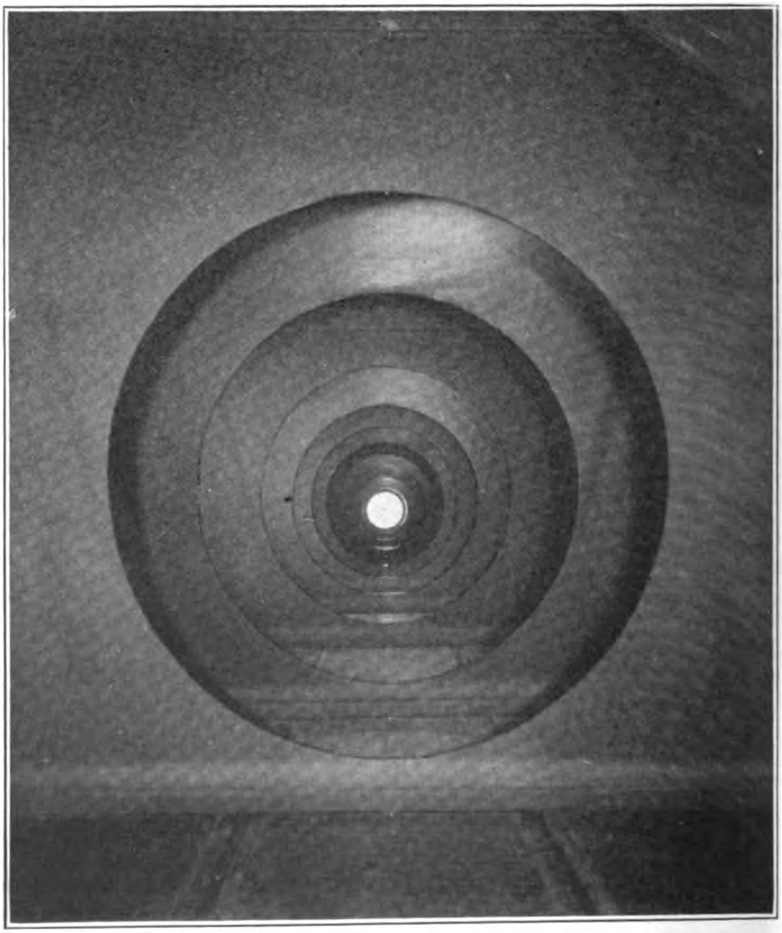

FIG. 9. *An integrator as viewed from the searchlight through the curtain diaphragm.*

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ACCESSORIES FOR SEARCHLIGHT TESTS

The indoor searchlight range is a room 170 feet long, 31 feet wide and about 13 feet high. The two integrators are mounted on parallel tracks that run the length of the room, and the distance between test unit and integrator may be any desired distance up to 150 feet. It is often necessary to use both instruments at the same time on work that requires an open iris shutter,

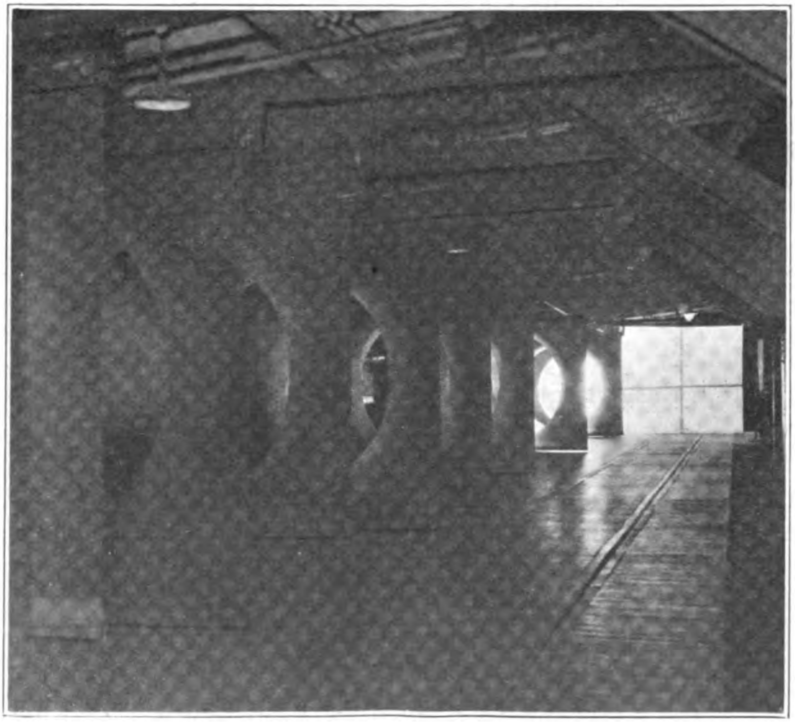

FIG. 10. *A view of the indoor searchlight range showing light tunnel, and integrator when testing a 60" searchlight for light outfit. The focusing curtain is down in front of the other integrator.*

and obviously unless some precautions are taken there will be an exchange of stray light, and also, a certain amount of light will be reflected from the floor and walls. A series of curtains with curcular holes in the centers are suspended above each track, and when these curtains are down the holes form a tunnel that

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effectually prevents the exchange of light and cuts off direct reflection of the floor and walls.

The floor presents the most favorable surface for undesired reflection, coming as it does closer to the light collecting shell than either the walls or ceiling, and to prevent light being reflected back into the instruments it is essential to have the floor as black as possible, or if not black then specular in reflection characteristic so that the light will not be diffused in the backward direction. This feature has worked out well, for as the floor becomes lighter through traffic it also becomes more specular. It has been found by trial with a constant amount of light directly incident on the shell that the reading is practically independent of the shutter opening, the greatest observable effect being about one half per cent. This shows that the effectiveness of the black inner surface of the iris shutter and the cylindrical shell between it and the hemispherical surface is not greatly different from the effectiveness of the room in returning light to the photometric surface.

As a preliminary to every searchlight test, it is necessary to see that the beam is circular in sections and properly focussed. To aid in this determination a white curtain with horizontal and vertical scales marked in degrees is lowered in front of the integrator and the illuminated area is then inspected for size, shape, regularity of outline and other characteristics that are related to the arc and its adjustment. (In taking the photograph of the indoor range in operation extra light was thrown on the white curtain in order to make it visible.)

The opening of the hemisphere is covered by an iris shutter that can be varied in opening from 4 to 110 inches. This shutter is operated by a handwheel which drives a sprocket wheel and chain that passes around the outer ring of the shutter.

BEAM ANALYSIS

There are a large number of short range units such as flood lights and motion picture projection arcs that may be completely analyzed for intensity at short range, and for this type of service the hemispherical integrators have been particularly valuable. A point-by-point analysis of any projected beam is always a

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physical possibility, except in the rare cases where the available supply of electrodes or the short life of an incandescent lamp prevents any extended test, so that it is hardly fair to claim that anything really new in the way of testing can be done, but the process of testing has been speeded up to the point where the time of actual test becomes a minor part of the whole time of preparation and time of burning necessary to establish an accurate average for varying arcs.

It was the former practice of this laboratory to test an incandescent flood light in at least four radial lines and occasionally in as many as eight lines about the center of the beam, and even then a repetition of the test would not always give a reasonably close check. There are two types of variations in any projected beam, a variation in space, as illustrated by the spots of high and low intensity in the beam from an incandescent lamp, and variations with time in an arc beam. It is evident that by taking zones completely around the axis of the beam the difficulty due to spots will be eliminated, but variations due to time can be overcome only by extending the test over a period that will allow the variations to average out. For this reason all arc tests are repeated from one to five times, but an incandescent test may be completed with a single exploration of the field of the beam.

In analyzing a projected beam six or more concentric zones are tested for average intensity and this average is plotted in the center of the zone on the curve sheet. Thus, a beam 10° in total width would be separated into zones as in the following table. The opening of the iris shutter and the distance from projector to integrator are given in the two last columns.

TABLE 2.—*Beam Analysis*

Zone	Zone Center	Iris Opening	Distance
Degrees	Degrees	Feet	Feet
0 to 0.5	0.33	2.62	150.0
0.5 to 1.5	1	7.86	150.0
1.5 to 2.5	2	9.17	105.0
2.5 to 3.5	3	9.17	75.0
3.5 to 4.5	4	9.17	58.2
4.5 to 5.5	5	9.17	47.6

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The angular opening of the integrator is increased by steps and the increment of reading is a measure of the increment of light in the added zone. The method thus depends upon successive differences and if the steps are too small the data points will be uneven and alternately above and below a smooth curve. With a proper selection of zone width this is prevented and it is not difficult to duplicate a test with good agreement between points.

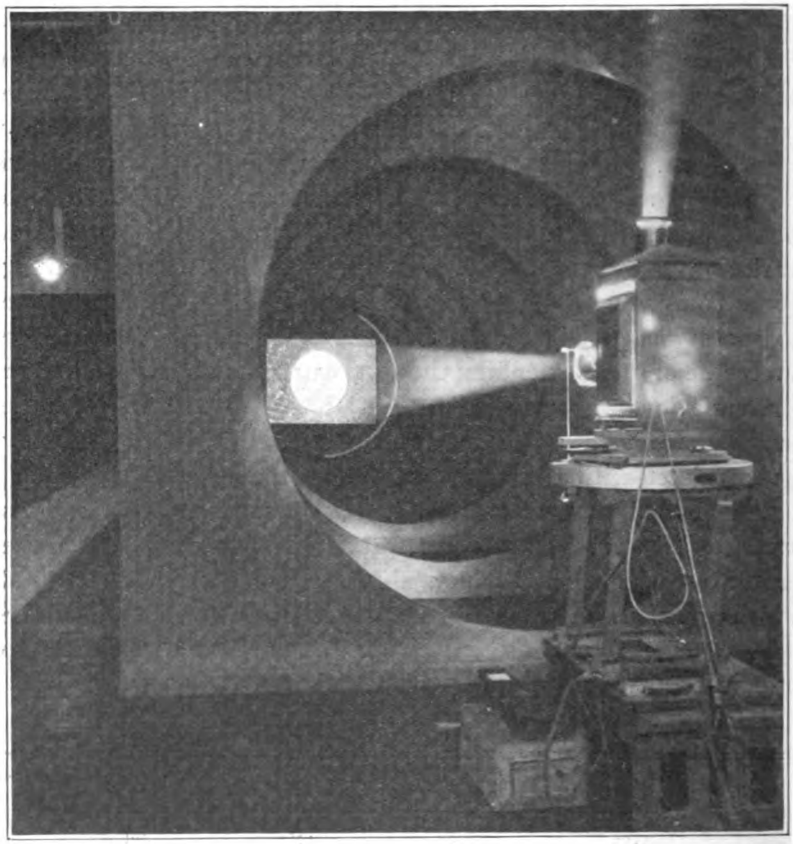

FIG. 11. *A test on the quantity of light in the central part of a motion picture projector beam.*

ANALYSIS OF MOTION PICTURE BEAM

A test on a motion picture lamp or lens is best made with all parts of the optical system in normal operating position. The

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collecting and projecting efficiencies of condenser and objective are obviously dependent upon the relations of the lenses to lamp and aperture, and the size of the integrator allows the measurement of the entire beam at nearly normal curtain distances, thus insuring data that apply directly to theatre practice.

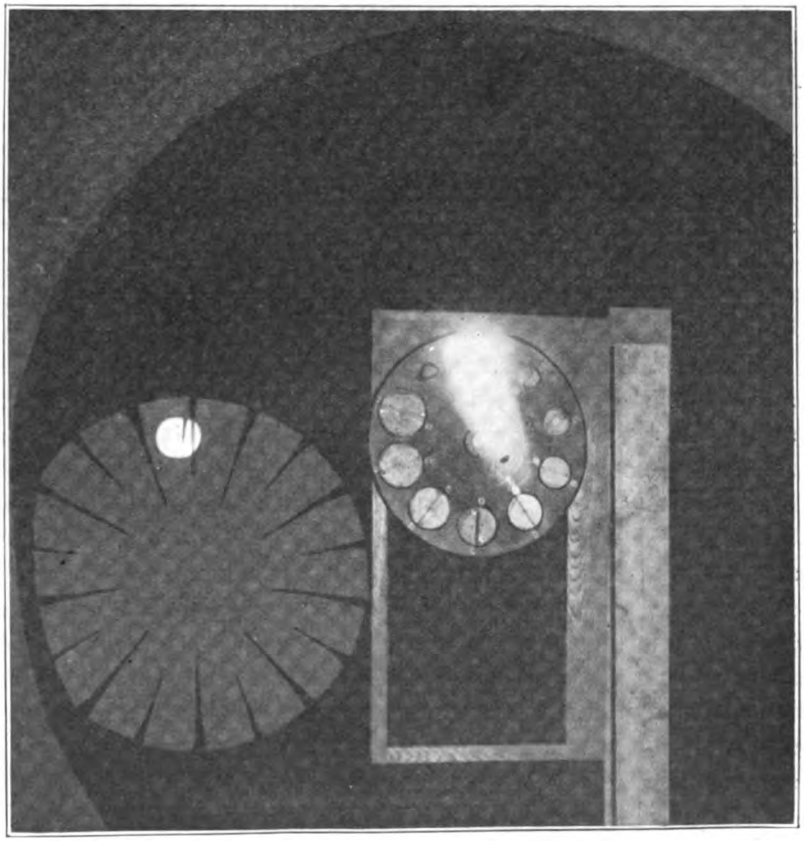

FIG. 12. *Integrator being used in the analysis of a high intensity crater for brilliancy in various zones.*

The analysis for distribution of intensity on the screen is carried out by concentric zones as in the case of the searchlight, but this method, while perfectly satisfactory for arc lamps, is not so well suited for incandescent projection lamps, and plans are

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now under consideration for providing each integrator with four curtains that may be arranged to diaphragm off any desired rectangular space and thus allow an analysis of the screen illumination in sections more suited to the general outline of the screen itself.

ANALYSIS FOR CRATER BRILLIANCY

The high intensity arc offers some peculiar problems in photometry, particularly if an attempt is made to find the distribution of brightness over the area of the gas filled crater. The gas is always in motion and while this motion is not at all violent it makes it rather difficult to read brightness for local points of small area. A method of testing for brilliancy in concentric zones (which may be circular or elliptical according to the purpose of the test) has been worked out in connection with the hemispherical integrators.

A lens of 18 inches focal length is set up in front of the bare arc and an image is projected onto an analyzing disk some 15 feet away. This disk is pierced around its periphery with ten annular zones of equal area, but different radii, so that if all were superimposed they would form concentric zones dividing the area of the outside circle into ten equal parts. The light of the image is allowed to pass through these zones one at a time and enter the integrator. The light through each disk zone divided by the corresponding zone area at the crater is obviously a measure of the zone brilliancy. This analysis can be carried out for any angle of radiation from the arc, and the resultant data give the true crater brilliancy and separates out all arc and flame light that originates outside of the area of useful generation.

ILLUMINATING ENGINEERING LABORATORY, GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.
SEPTEMBER 8, 1922.

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A TELEPHONE RECEIVER AND TRANSMITTER

By C. W. Hewlett

In previous publications [Phys. Rev., *17*, p. 257, 1921; and *19*, p. 52, 1922] the writer has described an instrument which lends itself to the reception and generation of voice currents. It consists of two coaxial pancake coils held by an insulating frame a short distance apart on either side of an electrically conducting diaphragm. Figs. 1 to 4 inclusive show reproductions of photographs of two of these instruments.

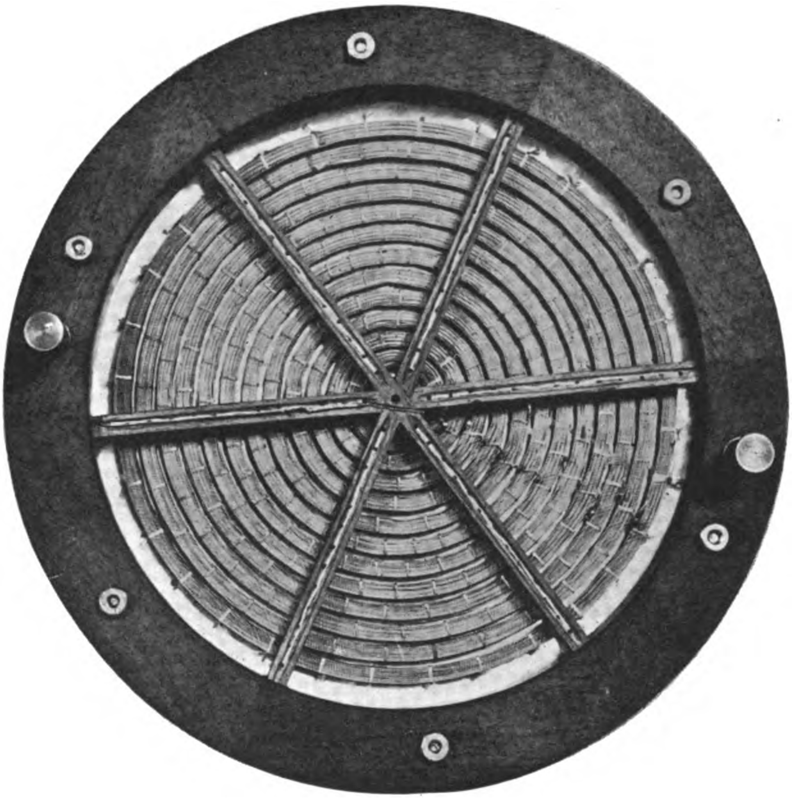

Fig. 1

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In order to use the instrument a direct current is sent through the coils in such a way that their magnetic fields oppose, the result being a radial magnetic field in the space occupied by the diaphragm between the coils. Fig. 5 shows an arrangement of circuits for using two of the instruments, one as a generator, and the other as a receiver of voice currents. In practice the circuit is somewhat simplified since one or two batteries can be used to supply all the currents. If generators are used instead of batteries, filters are necessary. LLLL are the coils, and DD the diaphragms of the voice current generator and receiver. AA are two amplifying tubes; *CC*, coupling condensers; *rr*, grid leaks; and *RR*, coupling resistances. C_1 is an output condenser coupling the second tube with the receiver of voice currents.

The generator of voice currents, shown on the left, functions as follows: Sound waves, striking *D*, cause it to vibrate across the radial magnetic field. This generates voice currents in the diaphragm which in turn induce voice electromotive forces in the coils *LL*. The induced electromotive forces in the two coils are in multiple supplying the grid-filament circuit of the first tube. The condensers *CC* should have an impedance small compared to the grid-filament resistance for all frequencies concerned. The twice amplified voice current is fed through the condenser C_1 to the receiver of voice current where it induces voice current in the diaphragm. The reaction between the voice current in the diaphragm and the radial magnetic field causes the diaphragm to vibrate and produce sound waves of the same character as the original ones striking the diaphragm of the generator of voice currents.

To avoid diaphragm resonance, which is one of the chief factors giving rise to distortion in the usual telephone apparatus, the diaphragm is thin and is held without tension between the coils. Calculation shows that the amplitude of motion of the diaphragm is nearly independent of its thickness, but increases with increase of its conductivity and decrease of its density. An aluminum diaphragm from 1 to 2.5 mils thick has been found suitable. Electrical resonance has been found to be of no importance for telephonic purposes, as with the low-inductance instruments

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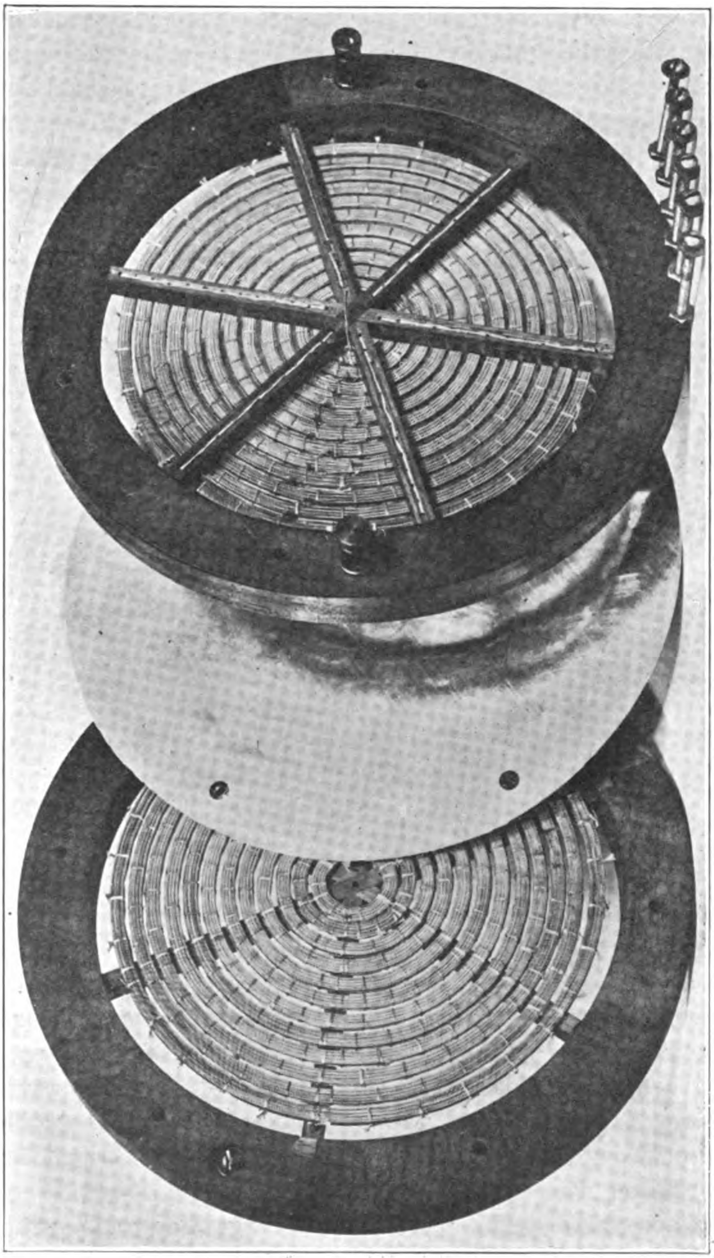

FIG. 2

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the frequencies of electrical resonance are usually above audibility, while with the high inductance instruments the resistance is usually so large that there are no sharp resonance peaks.

Another point of advantage in this type of instrument when used as a receiver of voice currents is that the diaphragm is acted upon fairly uniformly over its surface by the varying force due to the interaction between the voice current in the diaphragm and the radial magnetic field. This is unfavorable to the production of resonance as the diaphragm is urged to and fro somewhat

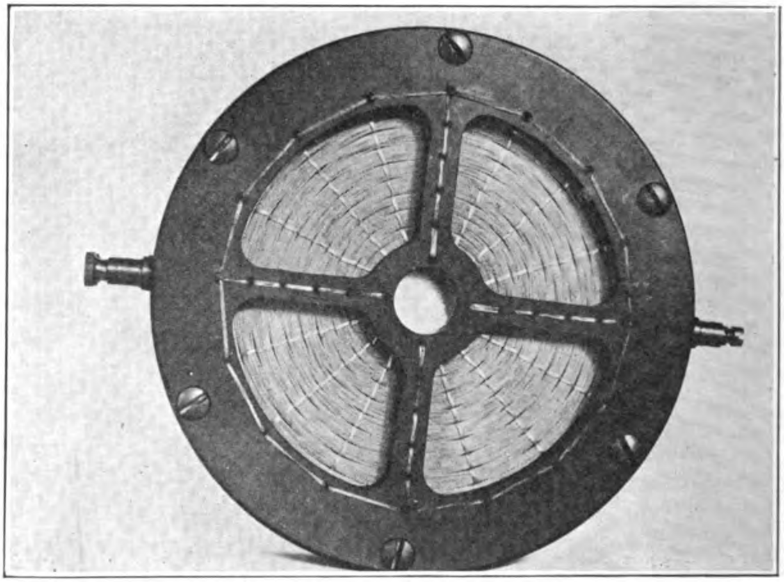

FIG. 3

like a piston. Calculation shows that the magnitude of the electrodynamic forces acting on the diaphragm is large compared to the reaction of the air due to the emission of sound waves.

In the ordinary forms of telephone apparatus there is a certain amount of distorsion which arises from the magnetic behavior of the iron used in its structure. The instrument described in this paper is free from this defect.

On account of being able to use a large diaphragm which is acted upon fairly uniformly over its whole surface, the use of a

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horn for intensifying the sound is of less advantage than in the usual forms of speakers. The use of a horn while greatly intensifying the sound always introduces distorsion due to resonance in the horn. The quality of the speech given by this instrument is remarkably like that spoken into the voice current generator when a properly designed amplifier connects the two.

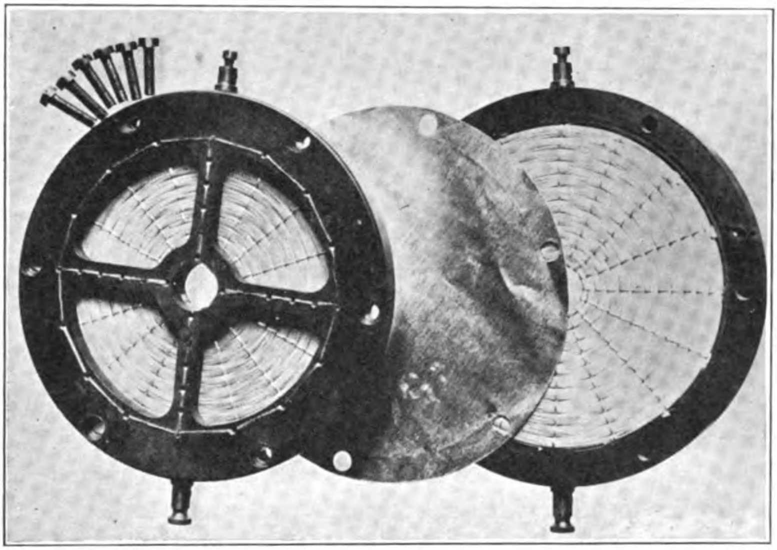

FIG. 4

The pancake coils of the instrument shown in figures 1 and 2 are each composed of twelve annular coils. Each annular coil is composed of eight layers five turns wide of No. 22 A. W. G. single silk covered copper wire, bound at intervals with silk thread. The outside diameter of each pancake is 7.5 inches. The resistance of each pancake is 8 ohms and the inductance 16 milliheuries. The diaphragm is 2.5 mil sheet aluminum, and is spaced 30 mils from each coil. The frame is of laminated walnut. This instrument is used as a speaker on receiver of voice currents. When excited by a direct current of 1.5 amperes and a voice current of 0.1 ampere it produces speech that is comfortably audible at all points in a room 20 ft. square. Another instrument wound

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with No. 34 A.W.G. double silk covered wire containing nine times as many turns, but with otherwise the same geometry and construction as the above is frequently used as a generator of voice currents. Each pancake has a resistance of 1200 ohms and an inductance of 1.2 heuries. This instrument may also be used as a speaker if the output of the amplifier has a high impedance.

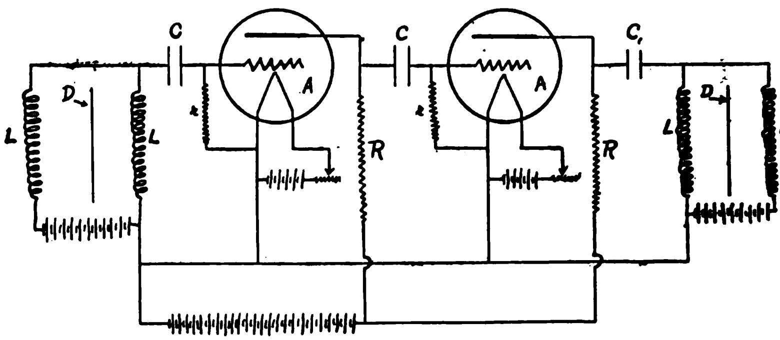

FIG. 5

The instrument shown in Figs. 3 and 4 consists of two compact pancake coils. Each wound of 3300 turns of No. 34 A.W.G. double silk covered wire. The coils are not impregnated, but are woven together with silk threads and are attached in the same manner to the hard fiber framework supporting them. Each pancake is 4 inches in diameter, is 0.2 inch thick, has a resistance of 540 ohms, and an inductance of 0.5 heury. The diaphragm is 1 mil sheet aluminum and is spaced 30 mils from the coils. The performance of this instrument differs only in magnitude from that shown in figures 1 and 2. It appears that the coils do not much cut off the sound waves to or from the diaphragm.

In case the amplifier used has an output impedance much different from that of the speaker at voice frequencies, it may be best to sacrifice quality of reproduction to a certain extent in order to obtain more intensity. This may be done by using a transformer as shown in figure 6. *P* and *S* are the primary and secondary of a transformer. The impedances of *P* and *S* are designed to fit the output of the amplifier and the coils *LL* of the speaker respectively. *B* is a direct current supply for polarizing the coils.

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It is possible by the use of choke coils and condensers inserted at the proper places to use the plate current of the amplifier tube as the polarizing current of the speaker, provided this plate current is large enough. This has not been found very useful, however, on account of the large choke coils required. A high impedance head set has been used in which the plate current of a

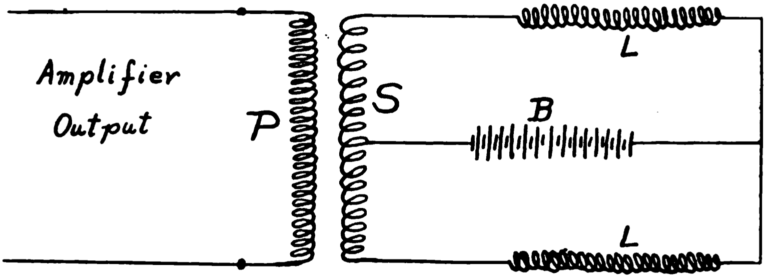

FIG. 6

single amplifier passing through the two halves of the instrument in series (one half held at each ear, and each half having a diaphragm) is sufficient to polarize the instrument. When the two halves are separated in this way, the choke coils and condensers mentioned above are of no use, and to avoid appreciable distorsion the variations of the plate current during amplification must be small.

Compared to other forms of receivers and transmitters, particularly the microphone and ordinary magnetic receiver, the instrument described above is very insensitive, but it gives an excellent quality of reproduction of speech and music.

The writer takes pleasure in acknowledging the help in the construction of these instruments and in the experimental work in connection with them rendered by Messrs. J. B. Dempster and Harlan Porter.

STATE UNIVERSITY OF IOWA,
IOWA CITY, IOWA,
JULY 20, 1922

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A SMALL HIGH INTENSITY MERCURY ARC IN QUARTZ-GLASS

By L. J. Buttolph

The unique value of the mercury arc as a source of monochromatic light has been known for some twenty-five years. Until recently it has been available only to the physical laboratory either as the large size low intensity Cooper Hewitt lamp, as the large capacity mercury arc in fused-quartz designed primarily for commercial use and for ultra-violet radiation, or as home-made Aron's types.

Fig. 1

A small quartz mercury arc lamp, Fig. 1, has been designed specifically to combine all the advantages of these lamps with special provisions for safe and convenient use in the laboratory.

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This lamp has an effective light source area of ¼ × 1¾ inches which is ample for the illumination of such slits or filters as are ordinarily used. It is enclosed in a metal casing to protect the observer from stray light and is provided with a removable mica filter to absorb the far ultra-violet when it is not needed. It emits relatively so little radiant heat that it may be used near to accessory optical apparatus. It has the same high intrinsic brilliancy as the larger quartz lamps sold for commercial use.

It is provided with an adjustable slit set close to the light source, but any standard slit with fine adjustment may be used where necessary. A light-tight removable filter holder is provided. The lamp outfit is made as a single standard unit for operation on 110 volts either alternating or direct current. The arc proper is connected to the auxiliary electrical equipment in the base by a separable connector and standard laboratory fixtures. Thus, it is easily adjustable, removable, and adaptable to a variety of set-ups by means of standard clamps and supports.

Two operating conditions are clearly defined. When running with sufficient additional resistance in series it is a low pressure arc and the light column appears to fill the whole arc tube uniformly. In this condition only the strongest spectrum lines show and no continuous spectrum is visible. When, on the other hand, the arc is adjusted for operation at high intensity it starts as a low pressure arc, but as it heats up, changes to the high pressure condition. This change may be observed through a dark filter and is indicated by an apparent concentration of the light into a narrow thread of great intensity in the axis of the arc tube. In this condition a continuous spectrum is superposed upon the mercury line spectrum, several additional mercury lines become visible and the ultra-violet is greatly intensified.

In this connection it is well to recall the kinds of radiation available with a mercury arc in fused quartz. This lamp emits the same characteristic radiation as the large quartz-glass lamps. The principal lines are a 10,140 A line in the infra-red; yellow, green, blue and violet lines in the visible for the isolation of which Wratten monochromatic filters have been developed; and a very

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complete series of ultra-violet lines extending out to the limits of transmission of clear fused quartz.

Because of its brilliancy, its high spectroscopic purity and its location at maximum visibility the green line at 5461 A is the most valuable source of monochromatic light now available. Its specific impurity of one part in a million is so low as to permit its use in interferometry.

The blue line of wave-length 4359 A is also unique in being the most powerful available source of monochromatic light for work at that end of the spectrum. While not so spectroscopically pure as the green line, the satellites are of so much lower intensity as to introduce no appreciable errors in polarimetric readings up to 100° rotation.

The two yellow lines are separated by some 21 A as compared with the 6 A separation of the sodium doublet. They are of value as reference points in spectrometry but superfluous for polarimetry as they are intermediate between the indispensable mercury green and cadmium red lines.

The 10,140 A infra-red line is of value to the physicist as a reference point for work in a part of the spectrum where instrument calibration is particularly difficult.

By removing the mica screen there are available some twenty-five ultra-violet lines so characteristic in their prismatic spectrum groupings and relative intensities as to be readily identified on a fluorescent screen or on a photograph without measurements. Of these, 3656, 3342, 3132–26 and 3025 are transmitted by the glass optical system of the ordinary spectroscope and may be identified by fluorescence on uranium glass or upon anthracene coated paper. The lines of wave-length less than 3,000 A are best studied with a quartz spectrograph or a monochromatic illuminator for the ultra-violet.

For the isolation of the various mercury lines two general methods are available. For ordinary polariscopic and general laboratory work filters will be found highly satisfactory. Three types of filters are available as listed in Table 1. Only the Corning glass or the liquid filters can be used close to the mercury arc because of the heat. The equipment recommended is the G555P

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and G34 combination for 5461 and the Noviol A and G585 combination for 4359. Gelatine and liquid filters must be kept as nearly as possible at room temperature. Table 1 and Fig. 2, show available filters and their transmissions. The liquid filters, all of which are two cell combinations, are best adjusted for the desired effect at the time of use. A spectroscope, preferably a small, direct-vision one, is a necessity in making this adjustment.

TABLE 1. *Selective Filters*

Radiation	Corning Glass	Eastman Wratten	Liquid Filters
5769	G34-5 mm red shade	22-E2 Hg Yellow 72%	Chrysoidine and Eosine
5461	G5552-10 mm and G34-5 mm yellow shade	62 Hg Green 12% or 77 Hg Special 72% or 77A. Hg Special 50% for Interferometry	Neodymium Ammonium Nitrate and Potassium Dichromate
4359	Noviol A-3.0 mm. and G585-L or M.	50 L.Hg Violet	Cobalt blue glass and Quinine Sulphate
4047–78	G586A-4 mm. and Noviol 0-5 mm.		Methyl Violet and Quinine Sulphate
3650 3656 3663	G586AW-10 mm	18 Ultra-Violet	Methyl Violet and Acid Green
Infra-red	G585-5 mm and G24 Red-5 mm.		
To absorb Infra-red	2% solution of Cupric Chloride or Corning Heat Absorbing Glass.		

For exact physical measurements, however, an optical method of spectrum line isolation and purification is necessary and several standard methods are described in the literature. There are on the market several highly developed monochromators of the constant deviation spectroscope type while the simplest device for use with the mercury arc is a direct-vision prism, calculated for

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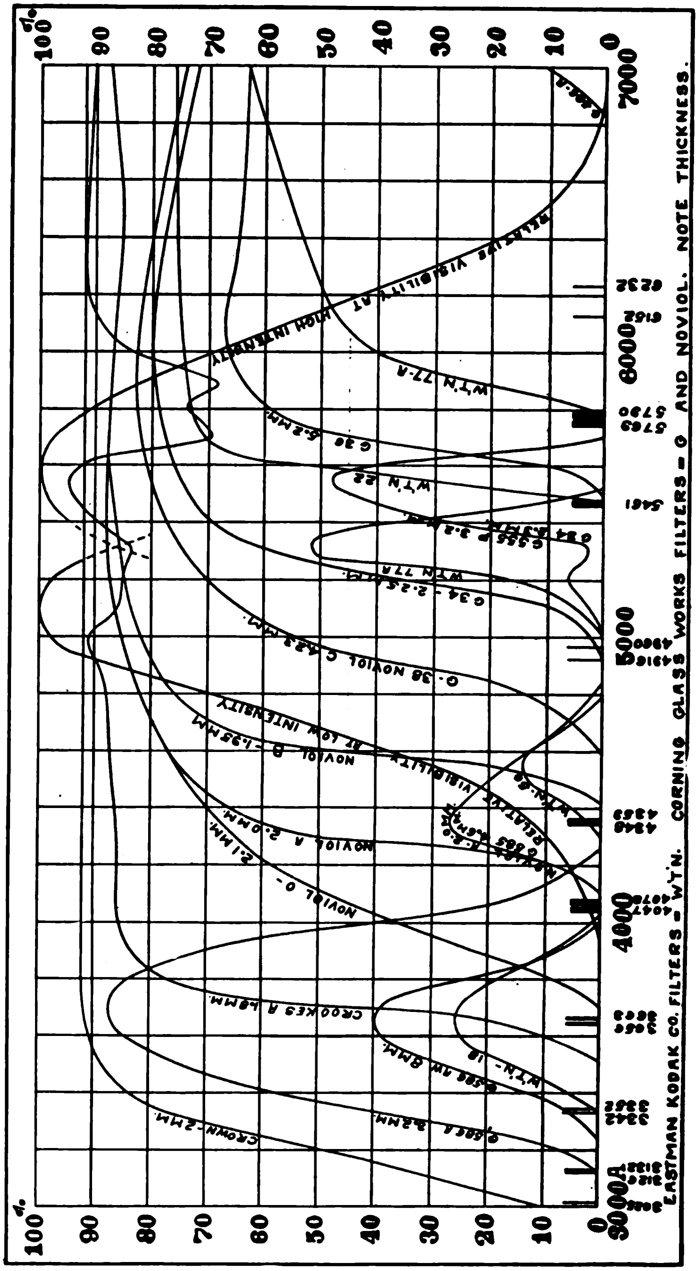

FIG. 2. *Transmission Curves.*

the green line, for example, and a long focus lens, or two collimating lenses for more exact work.

It is believed that this small high intensity mercury arc will have many applications in polarimetry, interferometry, and photomicrography, and while it was designed to meet these special requirements, its greatest field of usefulness is in the physical laboratory.

Through filters, it supplies a reliable source of monochromatic light for experiments with the diffraction grating, diffraction patterns from double slits, monochromatic polarized light, fluorescence and spectrometry. Also for general spectroscopic observation, where a wave-length scale is not available, the mercury spectrum can be used for comparison and for the determination of the limits of filter transmissions.

ENGINEERING DEPARTMENT,
COOPER HEWITT ELECTRIC CO.,
HOBOKEN, NEW JERSEY,
AUGUST 25, 1922.

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CORRELATION OF ELEMENTARY PROOFS OF THE FUNDAMENTAL PROPERTIES OF OBLIQUE DEVIATION BY PRISMS

By H. S. Uhler

The object of the present note is to indicate how the general properties of deviation produced by triangular prisms surrounded by a single medium may be deduced without making use of either spherical trigonometry or the calculus of infinitesimals. The chief advantage of so doing is to enable the authors of elementary books on optics to generalize the usual discussions of prismatic refraction without increasing either the grade or the difficulty of the text. The notation and symbolization will be those found in James P. C. Southall's "The Principles and Methods of Geometrical Optics" (2nd Edition).

(*a*) The formulas of oblique refraction at a single plane interface will be assumed.[1] For a single triangular prism,—or for any number of triangular prisms having all the refracting edges mutually parallel,—the condition that the refracting faces are in contact with the same medium leads at once to the fact that the incident and emergent rays are equally inclined to the plane of the principal section.

(*b*) Let D denote the oblique (or actual) deviation produced by the prism, that is, the smallest angle through which the emergent ray can be turned to bring it into parallelism with the incident segment of the same complete ray. Let E symbolize the projected deviation or, in other words, the angle between the orthogonal projections of the incident and emergent rays on a principal section of the prism. Let η_1 stand for the angle which the oblique incident (or emergent) ray makes with its projection on the principal plane (the angular altitude or elevation of the oblique ray). Then the formula

$$\sin\tfrac{1}{2}D = \sin\tfrac{1}{2}E\cos\eta_1$$

[1] Southall, pp. 28–31. Heath, p. 21.

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may be proved by very elementary methods.[2] It shows that, for $\eta_1 > 0$, the oblique deviation is less than the projected deviation ($D < E$, $\cos\eta_1 < 1$). Also, for η_1 constant, if either D or E passes through a stationary value (D_0 or E_0) so will the other member of the pair attain the same kind of value simultaneously.

(*c*) A new formula derived by the writer,—a proof of which will be given below after the general outline of the theory has been explained,—is as follows:

$$\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)} = \frac{1}{\nu^2} - \frac{4\sin\frac{1}{2}E\sin(\beta+\frac{1}{2}E)\sin^2\lambda}{\sin^2(\beta+E)},$$

where β means the refracting angle of the prism, λ is a convenient acute angle (defined in the proof), and ν represents the "artificial relative index of refraction" of the material of the prism or, in symbols, $\nu = n\cos\eta_1'/\cos\eta_1$.

The three terms or fractions in the formula are positive since the angles $\frac{1}{2}E$ and $\beta+\frac{1}{2}E$ are each less than 180°, and all the remaining factors are squared. When η_1 is assigned a definite value, ν is uniquely determined and fixed ($\sin\eta_1 = n\sin\eta_1'$). As λ decreases arithmetically the term subtracted from $\frac{1}{\nu^2}$ decreases and vanishes when λ becomes zero. Hence, when η_1 is kept constant, the right hand side of the equation attains its greatest value when $\lambda = 0$. Therefore, since $\sin\frac{1}{2}\beta$ is constant, the denominator of the left member of the formula must pass through a least value when $\lambda = 0$. Accordingly E experiences a minimum value E_0 when $\lambda = 0$. We then have[3]

$$\nu = \frac{\sin\frac{1}{2}(\beta+E_0)}{\sin\frac{1}{2}\beta}$$

which shows that the (partial) minimum value E_0 decreases as η_1 decreases, since ν decreases as η_1 becomes smaller,

$$[\nu = \sqrt{n^2 + (n^2-1)\tan^2\eta_1}\,].$$

[2] Southall, pp. 125–127. Uhler, Amer. Jour. Science, *27*, p. 224; March, 1909.

[3] The nagative root is not consistent with the definitions of the quantities involved.

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For $\eta_1=0$ we have the classical laboratory formula for a ray in a principal plane, namely

$$\nu_0=n=\frac{\sin\frac{1}{2}(\beta+\epsilon_0)}{\sin\frac{1}{2}\beta}$$

(*d*) The next step is to follow Southall's elegant proof[4] that $D_0>\epsilon_0$. Hence, [by (*b*) above], $E_0>D_0>\epsilon_0$. Thus ϵ_0 is an absolute minimum, that is, a minimum of all the partial minima E_0 of E.

(*e*) Finally, the relation

$$\sin\tfrac{1}{2}D=\sin\tfrac{1}{2}E\cos\eta_1$$

may be extended to a train of any number of prisms immersed in a single homogeneous medium by simply continuing the construction given in figure 1.[5] In fact the angles $B''\ OF''$ and B''' OF$'''$ may be interpreted respectively as the total oblique deviation effected by all the prisms of the train and as the algebraic sum of all the separate projected deviations.

To derive the formula of item (*c*):

$$\beta=\gamma_1'-\gamma_2 \tag{1}$$

$$E=\gamma_1-\gamma_2'-\beta \tag{2}$$

$$\sin\gamma_1=\nu\sin\gamma_1' \tag{3}$$

$$\sin\gamma_2'=\nu\sin\gamma_2 \tag{4}$$

For convenience, introduce an angle[6] λ defined by

$$\lambda=\tfrac{1}{2}(\gamma_1'+\gamma_2)$$

Then, by (1), $\gamma_1'=\lambda+\frac{1}{2}\beta$

and $\gamma_2=\lambda-\frac{1}{2}\beta$

hence (3) and (4) become

$$\sin\gamma_1=\nu\sin(\lambda+\tfrac{1}{2}\beta), \tag{5}$$

$$\sin\gamma_2'=\nu\sin(\lambda-\tfrac{1}{2}\beta) \tag{6}$$

Adding equations (5) and (6) we get

$$\sin\tfrac{1}{2}(\gamma_1+\gamma_2')\cos\tfrac{1}{2}(\gamma_1-\gamma_2')=\nu\sin\lambda\cos\tfrac{1}{2}\beta \tag{7}$$

Subtracting equation (6) from (5) we obtain

$$\cos\tfrac{1}{2}(\gamma_1+\gamma_2')\sin\tfrac{1}{2}(\gamma_1-\gamma_2')=\nu\cos\lambda\sin\tfrac{1}{2}\beta \tag{8}$$

[4] Pp. 127a, 127b.

[5] Amer. Jour. Science, *loc. cit.*

[6] For geometrical interpretations of λ see Uhler, Amer. Jour. Science, *35*, pp. 392, 393; April, 1913; and also Amer. Math. Monthly, *28*, pp. 6, 7, Jan. 1921; where λ is replaced by x.

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Replacing $\gamma_1-\gamma_2'$ by $\beta+E$, [comformably with (2)], and substituting from (7) and (8) in the identity.

$$\sin^2\tfrac{1}{2}(\gamma_1+\gamma_2')+\cos^2\tfrac{1}{2}(\gamma_1+\gamma_2')=1$$

we find

$$\frac{1}{\nu^2}=\frac{\cos^2\frac{1}{2}\beta\sin^2\lambda}{\cos^2\frac{1}{2}(\beta+E)}+\frac{\sin^2\frac{1}{2}\beta\cos^2\lambda}{\sin^2\frac{1}{2}(\beta+E)}$$

$$=\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}+\left\{\frac{\cos^2\frac{1}{2}\beta}{\cos^2\frac{1}{2}(\beta+E)}-\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}\right\}\sin^2\lambda$$

$$=\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}+\frac{4\sin\frac{1}{2}E\sin(\beta+\frac{1}{2}E)\sin^2\lambda}{\sin^2(\beta+E)}$$

YALE UNIVERSITY,
NEW HAVEN, CONNECTICUT,
SEPT. 18, 1922.

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